DOCUMENT RESUME

ED 180 609 PS 011 086

AUTHOR	Klausmeier, Herbert J.: And Others
TITLE	Cognitive Learning and Development: Information-Processing and Piagetian Perspectives.
INSTITUTION	Wisconsin Univ., Madison. Research and Development Center for Individualized Schooling.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C.
PUB DATE	79
NOTE	375p.
AVAILABLE FROM	Ballinger Publishing Company, 17 Dunster Street, Cambridge, MA 02138 ($25.00)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Children: Cognitive Ability: *Cognitive Development: *Cognitive Processes: *Cognitive Tests: Comparative Analysis: *Concept Formation: Developmental Stages: Learning Processes: *Learning Theories: Logical Thinking: Perspective Taking: Theories
IDENTIFIERS	*Cognitive Learning And Development: *Piaget (Jean)

ABSTRACT

This book deals with cognitive development from the perspective of the author's theory of conceptual learning and development (CLD) and from a Piagetian perspective. Chapter I provides an overview of CLD theory and also identifies and discusses the Piagetian constructs of stage, groupement structures, and equilibration. Chapter II focuses on the sampling design and the tests used in three studies of cognitive development (Kessen 1960, Nesselroads & Reese 1973, and Wohlwill 1973) that were carried out simultaneously using data from the same four longitudinal groups of studies. To supplement the condensed version of CLD theory, given in Chapter I, the major descriptive principles of cognitive development are discussed in Chapter III. The factor structure based on CLD theory is presented in Chapter IV. Also presented in this chapter are the results performed on the CLD tests and on educational achievement tests in grade 6, and of CLD tests and selected Project Talent tests in grades 9 and 12. Results from three studies of the Paigetian concrete operations stage in middle childhood are presented in Chapter V. Predictions that are based on expected relationships not only between the CLD measures and both the Piagetian and the groupements measures, but also between and within the Piagetian and the groupement measures, are tested in Chapter VI. Chapter VII presents a synthesis of the preceding findings regarding the developmental progression of the CLD, Piagetian, and groupement items during the school year. Chapter VIII discusses the relative importance given to the developmental factors and to learning in the Piagetian and CLD approaches. The final chapter presents an analysis of the relationship between psychological theories in general and their possible application to educational practice. Data from the studies discussed in this book are summarized in tables and presented in the appendix. (Author/MP)

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Cognitive Learning and Development: Information-Processing and Piagetian Perspectives

Herbert J. Klausmeier and Associates

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International Standard Book Number: 0-88410-188-6

Library of Congress Catalog Card Number: 79-11448

Printed in the United States of America

Library of Congress Cataloging in Publication Data

Klausmeier, Herbert John, 1915-
Cognitive development.

1. Cognition in children. 2. Piaget, Jean, 1896- 3. Human information processing. I. Title.
BF723.C5K58 155.4'13 79-11448
ISBN 0-88410-188-6

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Preface

This book deals with cognitive development from the perspective of my theory of conceptual learning and development (CLD) and from a Piagetian perspective. The aims of the two theories are similar in many ways but there are also major differences between them. CLD theory is cognitive and information processing in nature, whereas Piagetian theory is cognitive and structural. CLD theory attempts to describe and explain the short-term learning and long-term development of concepts, principles, structures of knowledge, and problem solving, particularly during the school years. Piagetian theory attempts to describe and explain all aspects of intellectual development from birth into adulthood. There are many other differences, not the least of which for our purposes here is that the Piagetian theory is very well established and has dominated developmental research for decades. The outline of CLD theory first appeared in 1971.

Because of the recent origin and the tendency of words to convey different meanings it may be well to indicate what the terms *cognitive* and *information processing* imply in CLD theory. I use the word *cognitive* as it is used generally to indicate a view different from the various association theories. *Information processing* implies that the attainment of concepts at successively higher levels and the formation of increasingly comprehensive conceptual cores can be accounted for in terms of acquiring, organizing, storing, and retrieving information. Acquiring and organizing are regarded as active constructive processes, not as passive reception processes. Cognitive information processing does not imply that the maturing human being is viewed as a complex machine or device, analogous to an electronic computer, that accepts and uses the programs provided by external agents for passively receiving, storing, processing, and retrieving information.

In Chapter 1, I provide an overview of CLD theory and also identify and discuss the Piagetian constructs of stage, groupement structures, and equilibration. These three constructs correspond to the CLD constructs of levels of concept attainment, cognitive operations and conceptual cores, and executive control. Although the constructs are corresponding or parallel, they imply very different explanations of the progression of the same developmental items, such as concepts and principles.

Three interrelated studies are reported in this book. They are referred to as the CLD study, the study of the concrete operations stage, and the comparative study. The method of all three studies is referred to as longitudinal and cross sectional. The studies are interrelated in that they had the common objective of describing and explaining development and used different samples of the same groups of students. Tests based on CLD theory were constructed and administered to four groups of students in the CLD study that extended from Grade 1 through Grade 12. In the study of the concrete operations stage three smaller samples of students from the first groups, kindergarten through Grade 9, were administered traditional Piagetian tasks and Brainerd's recently constructed tests of the Piagetian groupement structures. In the comparative study, the tests from all three domains were administered to the same samples as in the study of the concrete operations stage. The specific methods used in each study and the technical characteristics of each test are given in Chapter 2.

The discussion in Chapter 3 of the major descriptive principles of cognitive development supplements the condensed version of CLD theory given in Chapter 1. This is needed to understand the CLD study reported in Chapter 4, where the factor structure based on CLD theory is presented. Also presented are the results of correlational and factor analyses performed on the CLD tests and on educational achievement tests in Grade 6, and of CLD tests and selected Project Talent tests in Grade 9 and Grade 12. These two smaller studies were carried out to relate development of the CLD items to educational development more generally.

In Chapter 5, Hooper, Swinton, and Sipple give the results of their study of the Piagetian concrete operations stage, or logical reasoning in middle childhood. They were particularly interested in empirically testing the synchronic aspect of the concrete operations stage and to a lesser extent the diachronic aspect of the transition from one stage to another.

In Chapter 6 predictions are tested that are based on expected relationships not only between the CLD measures and both the Piagetian and the groupement measures, but also between and within the Piagetian and the groupement measures. The predictions were tested by analyzing the mean scores and also by performing correlational and factor analyses on the data for each annual measurement and for the longitudinal measurement intervals. Conclusions are drawn in terms of comparing the CLD and Piagetian perspectives and confirming and disconfirming elements of them.

Chapter 7 presents a synthesis of the preceding findings regarding the developmental progression of the CLD, Piagetian, and groupement items during the school years. The developmental progressions that are identified are accounted for by CLD constructs inasmuch as support was not found for orthodox stages as descriptions of development nor for the groupement structures as explanations of the concrete operations stage.

Brainerd analyzes the relative importance given to developmental factors and to learning in the Piagetian and CLD approaches in Chapter 8. He argues that whereas development and learning are separable processes in Piagetian theory, they are not in CLD. He also reports his learning experiments concerned with whether children's ability to learn conservation and perspective-taking concepts is constrained by what they already know about these concepts.

In Chapter 9, Murray presents an analysis of the relationship between psychological theories in general and their possible application to educational practice. Drawing upon his expertise in Piagetian theory and education, he evaluates the claims made for the application of Piagetian theory to curriculum and to instruction.

As would be expected when reporting three large original studies based on longitudinal data, only summary data are presented in tables in Chapters 2, 4, 5, and 6. Much of the detailed information on which the summary tables are based is given in tables in the appendix; the massive correlation matrices appear in a separate technical report by Sipple, Allen, and Klausmeier. The data were handled in this manner to provide a more readable and less costly book and at the same time to make the information available to anyone who may wish to perform other analyses or to examine it to answer particular questions of interest. Each student's scores are held in computer storage at the Wisconsin Research and Development Center for Individualized Schooling and also are available upon request.

I am especially appreciative of the contributions of my associates who were solely responsible for three of the nine chapters: Frank Hooper, Spencer Swinton, and Thomas Sipple for Chapter 5; Charles Brainerd for Chapter 8; and Frank Murray for Chapter 9. In addition to writing the chapters, they made other essential contributions. During the years 1972-1978, Thomas Sipple was a project associate for both Frank Hooper and me. He served as the liaison person with the participating schools, coordinated the data gathering and the computer data processing, prepared the tables and figures, and checked the edited manuscript for the accuracy of the numerical data that appear in the chapters. I was particularly pleased that he took primary responsibility for describing the methodology and the data gathering instruments for the studies as reported in Chapter 2.

Frank Hooper and I planned the interrelated studies in 1971. My responsibility was to carry out and report the CLD study while Frank took a similar responsibility for the Piagetian study. Spencer Swinton and Charles Brainerd were consultants on the Piagetian study from the outset. Charles Brainerd and Frank Murray reviewed my chapters and provided constructive criticisms.

The CLD and comparative studies took me into several highly specialized fields of inquiry. I was fortunate that persons were willing to serve as consultants and to contribute their specialized knowledge so generously. The suggestions of Margaret Harris, Coordinator, Santa Barbara County Schools, and Chester Harris, University of California, Santa Barbara, were followed in carrying out the factor analyses. The interpretation of the results of the CLD study and the comparative study rests largely on the methods used in identifying the factors based on the annual data and also the factors and factor loadings based on the longitudinal data.

Lawrence Hubert, University of California, Santa Barbara, and Michael Subkoviak, University of Wisconsin, Madison, provided assistance on the CLD test construction. Four tests of each of four batteries were constructed to measure performances related to the mental operations that are necessary for attaining concepts at each of four successively higher levels. The year at which a CLD item initially becomes functional and also fully functional is inferred from the results of these tests.

Benton Underwood, Northwestern University, consulted on the design of key laboratory and in-school experiments on concept learning. Results of these experiments are reported in Klausmeier, Ghatala, and Frayer, *Conceptual Learning and Development: A Cognitive View* (1974).

Recognition is also due Patricia Allen, my co-author of *Cognitive Development of Children and Youth: A Longitudinal Study* (1978), which gives the empirical bases of the principles of cognitive development discussed in Chapter 3 of this book. Patricia served as a research associate on the CLD study from 1973 to 1977.

I received funding through the Wisconsin R & D Center from the United States Office of Education from 1964 to 1972 and from the National Institute of Education thereafter to support the initial experiments on concept learning and to carry out the CLD study and comparative study reported in this book. Without this large-scale, continuous funding the studies could not have been done.

The participation of the children and youth, their parents, their teachers, and the school administrators is acknowledged with the deepest gratitude.

Anyone familiar with the accounting and reporting requirements of federal and state agencies and also with the practical problems involved in gathering and processing large amounts of data will be aware of the many contributions of individuals whom I have not acknowledged here by name. I do wish to recognize the continuing assistance provided by the administrative and support staff of the Wisconsin R & D Center.

Arlene Knudsen, my dedicated administrative secretary, not only coordinated the preparation of my chapters but also put together the reference section for all the chapters and checked all the citations.

February 1979
Herbert J. Klausmeier

Chapter 1

Introduction

Herbert J. Klausmeier

Developmental psychology continues to expand at a rapid rate. There is growing interest not only in describing but also in explaining the psychological changes that occur from infancy into adulthood. The mechanisms, structures, and processes that may account for cognitive learning and development have come under increasingly close scrutiny, particularly since the 1960s. In addition to theorizing, many descriptive studies and controlled experiments dealing with a vast array of learning and developmental phenomena have been conducted.

Most developmental psychologists have turned to Piaget's theory as a framework for describing and explaining human development and also to guide their own research. Other developmental psychologists, some educational psychologists, and many educators have identified Piagetian theory as their primary source of ideas for improving education in the expectation that an understanding of development should transfer to educational improvement. While the earlier efforts in these directions were accompanied with uncritical acceptance of Piagetian theory, more recently serious questions have arisen concerning the explanatory constructs employed in the theory (e.g., see Brainerd, 1978b; Flavell, 1977). Also because of the many diverse and contradictory applications to education purportedly based on Piagetian theory and the difficulty in reliably estimating a student's stage of development, there are increasing doubts about the applications of stage theory to education.

Interest in obtaining a better understanding of cognitive learning and development during the school years from an information-processing view as represented in a theory of conceptual learning and development (CLD) (Klausmeier, 1971; Klausmeier & Allen, 1978; Klausmeier, Ghatala, & Frayer, 1974) and in clarifying Piagetian constructs and propositions, particularly during the stage of concrete operations (Hooper & Klausmeier, 1974), resulted in the studies reported in this book. Getting measures of the same children and youth based on each of the two views enabled us to answer key questions regarding the theories such as these. How well do they take into account interindividual differences in the rate of development and also intraindividual differences in the organization of cognitive abilities? How well do the empirically derived descriptions of cognitive development coincide with the descriptions as stated in each theory? How well are the descriptive propositions explained by the theoretical constructs? What similarities in developmental progressions can be identified from both perspectives?

A brief overview follows of Piaget's structural theory and the theory of conceptual learning and development (CLD). CLD theory is based on information-processing constructs. Its assumptions about learning and development are in the cognitive tradition and have many similarities with those of Ausubel (1963, 1977), Bruner (1964, 1973), and Miller and Johnson-Laird (1976). Since Piagetian theory is so well known, only a few of its main constructs and propositions are presented here. In Chapter 5, however, Hooper, Swinton, and Sipple provide detailed information concerning the period of concrete operations, the stage that was studied intensively by them. A more complete introduction to CLD theory is provided in this chapter.

PIAGETIAN THEORY

The Piagetian system has been under formulation and refinement for some sixty years. It is acknowledged as the major theory of human development in Western society. Piaget's formulations concerning cognitive development have provided the basis for the research and theoretical activity of many developmental psychologists during the past three decades. Excellent translations of Piaget's own accounts of his theory are found in Piaget (1964, 1970a). Flavell (1963, 1970, 1977) continues as one of the friendly critics and primary interpreters of Piagetian theory. Brainerd (1978b) evaluates the theory from the standpoint of the inconsistencies between predictions based on the theory and their lack of empirical support. The reader, if not already familiar with these accounts, may wish to study them in order to understand Piagetian constructs more clearly. The brief overview presented in this chapter and the more detailed presentation in Chapter 5 will aid the reader to understand the results of the relationship study given in Chapter 6, but they are not intended to be a comprehensive account of the theory.

The Piagetian system is child-centered and purports to explain why development occurs the way it does and to describe the sequence of development. Adaptation, involving assimilation and accommodation, and organization are regarded as the invariant functions that are operative throughout the life span. There are also psychological structures. These psychological structures take different forms at the different stages through which the individual progresses before attaining adult intellectual functioning. The functions and structures, according to Piaget, are inextricably intertwined in that the successive psychological structures are created or constructed as a product of the adaptation-organization functions, and they are also employed by the individual in adapting to the environment. Equilibration is the construct employed by Piaget to indicate the internal self-regulating force or mechanism that controls the rate and direction of development. The analogous construct in information-processing theory is executive control.

The four successive stages in cognitive development are sensory-motor, birth to age 2; preoperational, age 2 to 7; concrete operations, age 7 to 11; and formal operations, age 11 through adulthood. All individuals from infancy through adolescence progress through these four stages in an invariant sequence. Each of these stages has its unique psychological structures. Flavell (1977) emphasizes Piaget's views regarding assimilation and accommodation when treating the stages of developmental progression; whereas Brainerd (1978b) points to the psychological structures as the key explanatory construct. In the next part of this chapter we shall see that CLD theory posits levels of concept attainment rather than stages of concept development.

Three other features of the Piagetian system (Piaget, 1964, 1970a) are important for the purposes of this overview.

First, new structures and also changes in the form or expression of the invariant functions characterize each successive stage. Each successive structure is constructed from and incorporates the prior one. Piaget recognizes that individuals progress through the stages at different rates; however, the amount of time that is required to progress from one stage to the next is relatively small in comparison to the amount of time within a stage. The transition periods between stages are critical periods since during this time the cognitive structures of the two stages exist in parallel (Brainerd, 1977). This in turn produces an unstable condition.

Second, developmental factors are given relatively greater weight than are learning-environmental factors in determining the emergence of the stages and their underlying structures. Specifically developmental factors precede and are prerequisite for learning. Piaget (1964) states that the *development* of knowledge is spontaneous and is tied to embryogenesis, including the development of the nervous system, and also to the invariant functions and the psychological structures discussed earlier. In contrast, learning is "provoked" or instigated by an external situation and is limited to a single problem and a single cognitive structure. Learning is subordinate to development and not vice versa. (See Brainerd's remark in Chapter 8 for a more complete discussion of this point.) Piaget holds the same view regarding the relationship between development and language acquisition.

Third, psychological structures rather than psychological processes, including symbolic, are used by Piaget to account for intellectual development from infancy to maturity. For example, eight mathematico-logical groupings are employed as the fundamental structure to account for the interiorized actions that characterize the concrete operations stage. They include four groupements involving classes and another four involving relations. (The tests constructed to measure these structures are discussed in Chapter 2.) The analogous constructs employed by Bruner (1973) to account for cognitive growth are the enactive, ikonic, and symbolic representation of experience. CLD theory employs the construct of mental operations, including attending, discriminating, remembering, generalizing, hypothesizing, and inferring, and also a dual coding system involving imagery and language.

The preceding discussion is intended to reflect Piaget's own views regarding development (Brainerd, 1978b; Flavell, 1977; Piaget, 1964, 1970a). Elaborations of these and other major propositions of the Piagetian system as they pertain to the stage of concrete operations will be presented in Chapters 5 and 6.

CLD THEORY

The theory of conceptual learning and development is concerned with the attainment of concepts at four successively higher levels and with the uses of the concepts when attained at each level in understanding principles, understanding taxonomic relations, and in solving problems. Although the interest is throughout the life span, the school years, Grade 1 through Grade 12, receive most attention in this book. The formulations were first presented in rudimentary form in 1971 (Klausmeier, 1971) and extensive elaborations appeared in Klausmeier, Ghatala, and Frayer (1974) and in Klausmeier and Allen (1978).

The principles of learning are based in part on laboratory experiments on concept learning begun in 1961. This experimentation involved Bruner-type concept attainment tasks (Klausmeier, Harris, & Wiersma, 1964; Klausmeier, Harris, Davis, Schwenn [Ghatala], & Frayer, 1968). After 1968, the controlled experiments dealing with learning were conducted mainly in school settings with children of age 3 to 18. The principles of development to be presented in Chapter 3 are drawn mainly from a longitudinal study carried out with four groups of students of ages 7-9, 10-12, 13-15, and 16-18 (Klausmeier & Allen, 1978).

The main propositions regarding learning and development draw also on the work of other cognitive theorists, including Ausubel (1963, 1966, 1968, 1977); Bruner (1964, 1973); Bruner, Goodnow, and Austin (1956); Bruner, Oliver, and Greenfield (1966); Gibson (1969); and Gibson and Levin (1975); and also on the information-processing views of Miller, Galanter, and Pribram (1960) and Miller and Johnson-Laird (1976). The CLD theory is essentially a cognitive, information-processing approach to learning and development, and the assumptions regarding learning and development differ markedly from those of Piaget presented earlier in this chapter.

Development is regarded as a product of the reciprocal interaction of learning and maturation that occurs across relatively long time spans, not as preceding learning. It is not presumed that neurological maturation combined with the spontaneous acquisition of knowledge can account for intellectual development adequately, except perhaps during infancy. Nor is cognitive development merely an additive accumulation of discrete items of information that are learned, as S-R theorists have proposed. Rather, new and increasingly powerful mental operations, or processes, emerge with learning and maturation under normal environmental conditions, environmental conditions that many persons experience and that stimulate and nurture their intellectual development. The processes do not emerge solely as a function of maturation or of learning but in the interaction of the two. In infancy and the preschool years, the years characterized by a rapid rate of physical-neurological maturation, much learning occurs in informal daily life situations without guidance. However, the quality of the guidance during the school years and the amount of schooling are important determinants of when and also whether the highest level operations, such as inferring, will be performed on some kinds of symbolic content. Neurological maturation is complete by age 18 or earlier for most individuals; however, improved functioning of the mental operations extends into adulthood in individuals who continue to learn.

As the new mental operations, or processes, emerge and become functional, learning itself becomes more economical and effective. The cognitive structure, that is, all the organized knowledge of the individual, develops slowly during the developmental years as a product of the continuous interaction between learning and maturational processes. Furthermore, long after physical maturation has ceased, the cognitive structure continues to develop in individuals as they continue to learn. (It is noted that this use of term, cognitive structure, follows that of Ausubel (1963) and is different from that employed by Piaget; also, use of the term, operations, follows that of Guilford (1967).

The Cognitive Structure

The cognitive structure of the individual at any time includes everything the individual has learned. As noted in the preceding section, what the individual is capable of learning at any time is related to the mental operations that have emerged and are functional. The products, or outcomes, of learning may be categorized as perceptual information, verbal information and skills, concepts, principles, structures of knowledge including taxonomies, and problem-solving skills, including strategies of learning and of remembering. These products are learned initially, represented internally, organized, and stored in the form of images, symbols, meanings, and relations among images, symbols, and meanings.

The total cognitive structure changes from birth onward as do also its substructures. The structures of interest here are those that emerge developmentally. These structures start as the meanings that the individual has of individual concepts. As more individual meanings are learned they are organized into increasingly comprehensive conceptual cores corresponding to external structures of knowledge, including taxonomies and other hierarchies. The reader should be aware that this conception of cognitive structure is very different from that of Piagetian logico-mathematical structures, including the groupement structures used to explain the stage of concrete operations.

Concepts. A concept is both a mental construct of the individual and the societally-accepted meaning of one or more words that express the particular concept. Concepts as mental constructs are the critical component of a maturing individual's continuously changing, enlarging cognitive structure and are the basic tools of thought. Concepts as societally accepted meanings of words comprise much of the subject matter of English, mathematics, science, and social studies that children and youth learn during the school years.

There are a number of different ways to classify concepts. One is in accordance with the six form classes of words: nominals, adjectivals, verbals, adverbials, prepositionals, and conjunctivals. Words and word groups that can be categorized as belonging in each of these form classes represent concepts.

Nominals include nouns, pronouns, and noun phrases. Examples of these are as follows: nouns—*chair, Americans, thought,* and *symmetry;* pronouns—*she, who, themselves;* noun phrases—*the broken chair, the ugly Americans, a brilliant thought.* The meaning that you have for each example given is your concept of the class of things it represents. The meaning you have of noun, pronoun, and noun phrase is your concept of each of them as a subclass of nominals. It is recognized that the meaning ascribed to words, and particularly to pronouns, is related to the context in which they are experienced.

Verbals include verbs and verb phrases that refer to events, relationships, or states whose location or distribution in a time dimension can be specified. Examples of verbs are *hit, love, throw, is,* and *was* and of verb phrases, *has been hit, will be hit.* We might examine adjectivals, adverbials, prepositionals, and conjunctivals, but assume that the reader can do so independently. It is noted in passing that the concepts represented by prepositionals (e.g., *between, in*), conjunctivals (e.g., *but, because*), adjectivals (e.g., *red, smooth*) differ in many respects, one of which is the number and kind of attributes that define each form class.

Another means for classifying concepts is in terms of whether the defining attributes can be perceived. Concepts having perceptible attributes (e.g., *animal, swim, color*) are concrete, whereas concepts whose attributes cannot be directly perceived (e.g., *nominals, imagine, however*) are abstract.

Principles. A principle may be defined formally as a relationship between two or more concepts. Understanding the concepts embedded in principle is necessary but not sufficient for understanding the principle. Like a concept, a principle serves both as a mental construct of the individual and as the societally accepted meaning of the statement that expresses the principle. Principles, like concepts, are powerful tools of thought that are necessary for interpreting many phenomena in the physical and social world and for solving problems.

Four basic types of relationships expressed in principles are *cause-and-effect*, *correlation*, *probability*, and *axiomatic*. An example of each of the first three follows:

Tuberculosis is caused by the organism *Myobacterium tuberculosis*. (Cause-and-effect)

Sample correlations between two sets of scores on standardized reading achievement tests of the same children taken at yearly intervals during the successive elementary school years range from .70 to .90. (Correlation)

The probability of giving birth to a boy during any one pregnancy is .52. (Probability)

Axioms are universally accepted truths or conditions. They represent the largest class of principles. The class of axiomatic relationships may be divided into five subclasses: fundamentals, laws, rules, theorems, and axioms. What each class of axioms is called and how they function vary among disciplines such as mathematics, physics, and psychology.

Structures of Knowledge. Persons throughout the recorded history of humanity have been adding to their individual and collective knowledge, organizing it, and putting it into communicable forms, primarily symbolic. Taxonomies, other hierarchies, and theories are three important frameworks for organizing an individual's cognitive structure, and also for organizing the communicable knowledge of a group who share the same language and cultural experiences. Concepts and principles are the building blocks of these structures of organized knowledge.

Problem Solving. The ability to solve problems is perhaps the most important product of learning inasmuch as a person who is capable of solving problems can learn independently. The means of attacking problems and of acquiring, processing, and remembering information may be called strategies. Concepts, principles, and strategies are used in solving problems.

Despite the importance of principles, structures of knowledge, and problem solving in human cognitive functioning, psychologists during the 1970s gave relatively little attention to these products of learning. Neimark and Santa (1975), in their review of 266 publications dealing with thinking and concept attainment, did not report any research dealing with the learning of principles or of structures of knowledge. They indicated a declining interest in problem solving.

One exception to this trend was the comprehensive information theory of problem solving formulated by Newell and Simon (1972). More recently, Bernard (1975) reported pioneering experiments on learning and teaching multiple concepts and taxonomic relations while Tiemann and Markle (1978) dealt more directly with coordinate concepts.

Mental Operations and the Successively Higher Levels of Concept Attainment

One way of explaining the internal conditions of learning across short time intervals and to relate maturation and learning across long time periods is to identify the mental operations that are necessary for bringing about changes in observable and inferrable behavior. Scientific methods of accomplishing this include the behavioral analysis of learning tasks, observation as the learning tasks are performed, controlled experimentation, computer simulation of learning processes, and factor analysis based on tests of the mental operations. Guilford (1967) employed task analysis and factor analysis in formulating a structure of intellect that includes five main operations: cognition, memory, productive thinking convergent, productive thinking divergent, and evaluation. He constructed tests to measure the operations. Miller and Johnson-Laird (1976) drew heavily on information processing by computers in formulating their theory of perception and language. They explain all human intellectual functioning in terms of five basic operations: perceiving, intending, remembering, knowing, and feeling.

Neither Guilford nor Miller and Johnson-Laird dealt directly with changes in the internal conditions of learning that may occur developmentally from birth through adolescence. We recognize, of course, that the structure of intellect of the young child is very different from Guilford's 120-factor structure of the educated adult. There are also great changes in perception and language from infancy into adulthood.

One way of explaining cognitive development across long time spans involves identifying the mental operations that individuals employ in completing tasks, constructing and administering tests that measure the operations, observing the changes that occur between measurements, and inferring the conditions that account for the changes. Controlled experimentation is required along with the longitudinal approach both in identifying the operations and in inferring the necessary and sufficient conditions of learning. Problems encountered are the identification of the mental operations, devising tasks that require use of the operations as specified for maturing individuals of different ages, and constructing the measuring instruments and procedures.

The basic constructs of CLD theory to be described in this chapter involved identifying the levels at which the same concept is learned from its earliest manifestation through its most mature understanding and then identifying and describing the mental operations involved at each level. Early formulations of this theory (Klausmeier, 1971, and Klausmeier, Ghatala, & Frayer, 1974) provided the guidelines for conducting the CLD part of the study, the results of which will be reported in Chapters 3 and 4.

The four successive levels of attaining the same concept are designated *concrete*, *identity*, *classificatory*, and *formal* (Figure 1-1). The ability to learn a concept at each of the four successive levels is accounted for partially by the individual becoming capable of certain mental operations. The initial manifestation of a particular mental operation is presumed to be a product of both maturation and learning, or more broadly, of development.

The proposition that the same concept is attained at these four successively higher levels applies to concepts that (a) have more than one example; (b) have observable three-dimensional examples or representations in the form of drawings, words, or other symbols; and (c) are defined in terms of intrinsic attributes, function attributes, or both. Not all concepts are of this kind. Some concepts have only one example, for example, *the earth's moon.* Some do not have observable examples, for example, *atom*, *eternity*, *soul.* Still others

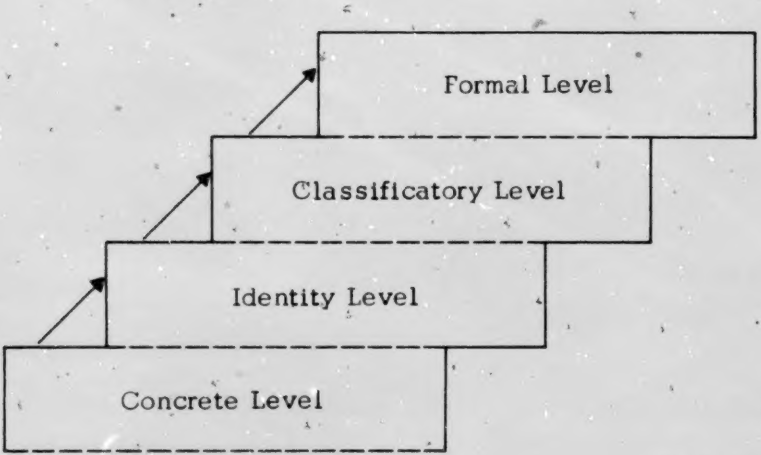
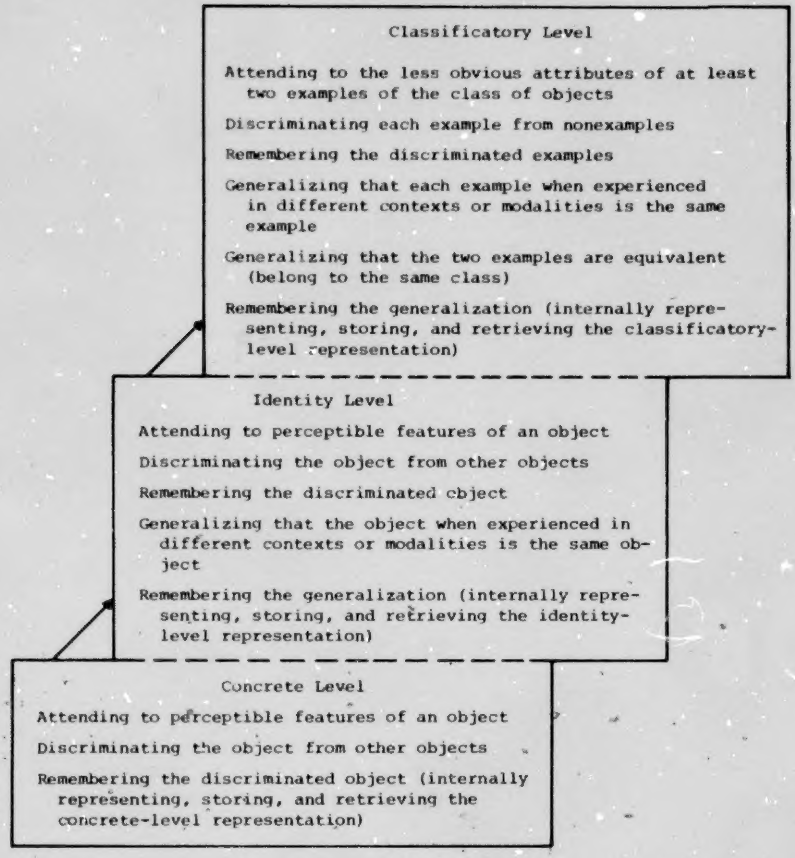


Figure 1-1. Four Levels of Concept Attainment

are defined in terms of a single dimension or quality, for example, *rough, thin;* or in terms of an invariant relation, including spatial—*south, between, above;* temporal—*before, after;* and deictic—*they, whom.* Attainment of all four successive levels is not applicable to these concepts. However, the attainment of any concept that has only one example or that has examples of an identical form may be described in terms of the concrete, identity, and formal levels; the classificatory level does not apply to concepts having only one example or only two or more identical examples. Attainment of some concepts defined by an observable single dimension, for example, *rough*, or in terms of an invariant relationship, for example, *before*, may be described primarily by the classificatory and formal levels, not the concrete or identity level. Concepts that do not have observable, classifiable examples, for example, *heaven, intelligence*, are learned only at the formal level. The remainder of this chapter is devoted to concepts that are attainable at the four levels and are defined by intrinsic and/or functional attributes rather than by a single dimension or by one or more invariant relations.

Concrete Level. A young child attends to a clock on the wall, discriminates it from other objects in the environment, represents the image of the clock internally, maintains the image (remembers), and then, after a period of time, attends to the clock and again recognizes it as the same thing attended to earlier. This child has attained a concept of this particular clock at the concrete level, not necessarily as a clock, but as an object with permanence. The operations involved in attaining this level, as shown in Figure 1-2, are attending to an object, discriminating it from other objects, representing it internally as an image, and maintaining the representation (remembering). A good deal of recent evidence suggests that even human infants are capable of this level of conceptualizing (e.g., Bower, 1974).

Identity Level. Attainment of a concept at the identity level is inferred by the individual's recognition of an object as the same one previously encountered when the object is observed in a different situational context, from a different spatio-temporal perspective, or sensed in a different modality, such as hearing or seeing. For example, the child who recognizes the clock as the same one when it is removed from the wall of one room and placed in another room has attained the concept of the particular clock at the identity level. Similarly, the child who recognizes the family poodle whether seen from straight ahead, from the side, or from various angles has



Classificatory Level

Attending to the less obvious attributes of at least two examples of the class of objects

Discriminating each example from nonexamples

Remembering the discriminated examples

Generalizing that each example when experienced in different contexts or modalities is the same example

Generalizing that the two examples are equivalent (belong to the same class)

Remembering the generalization (internally representing, storing, and retrieving the classificatory-level representation)

Identity Level

Attending to perceptible features of an object

Discriminating the object from other objects

Remembering the discriminated object

Generalizing that the object when experienced in different contexts or modalities is the same object

Remembering the generalization (internally representing, storing, and retrieving the identity-level representation)

Concrete Level

Attending to perceptible features of an object

Discriminating the object from other objects

Remembering the discriminated object (internally representing, storing, and retrieving the concrete-level representation)

Figure 1-2. Mental Operations in Concept Attainment at the Concrete, Identity, and Clasificatory Levels

attained the concept of the particular poodle at the identity level. As shown in Figure 1-2, the operations of attending, discriminating, and remembering are involved in attainment at the identity level as well as at the concrete level. However, concept attainment at the concrete level involves only the discrimination of an object from other objects, whereas attainment at the identity level involves not only discriminating various forms of the same object from other objects, but also generalizing the forms of the particular object as equivalent, or generalizing across the different contexts in which the same object is experienced. Generalizing is the new operation postulated to emerge as a result of learning and maturation that makes attainment at the identity level possible.

Most psychologists, even at the present time, treat concepts at the concrete and identity levels as discriminations, not as concepts. Miller and Johnson-Laird (1976) treat them as percepts. Piaget (1970a) does not differentiate between the concrete and identity levels but refers to object concepts. Flavell (1977) presents a lucid account of Piaget's six phases in an infant's attainment of an object concept during the first two years of life; whereas Nelson (1974) theorizes that infants of 12 months of age, or even younger, attain object concepts. Our interest here is less in concept learning during infancy and more in attainment of the concrete and identity levels by kindergarten and primary school children. We should recognize, too, that as adults we meet many new and different persons for very short time periods. We are continuously attaining concepts of individual persons at the identity level; however, we already have the concept *person* at both the classificatory and formal levels so our task is much easier than that of the young child.

Classificatory Level. As shown earlier in Figure 1-2, the new mental operation that makes possible attainment of concepts at the classificatory level is generalizing that two or more things are equivalent. The lowest level of attainment of a concept at a classificatory level is inferred when the individual responds to at least two different examples of the same class of objects, events, or invariant relations of objects or events as equivalent. For example, the child who treats the clock on the wall and the one on the desk as equivalent in one or more ways, such as both have the same shape, have moving parts, and are used to tell time, has attained a concept of clock at a beginning classificatory level. At this beginning level children seem to base their classifications on some of the intrinsic and function attributes of the concept examples that they have experienced; but they cannot state the basis of their classifications.

Individuals are still at the classificatory level when they can correctly identify a large number of things as examples and others as nonexamples but they cannot use the defining attributes of the concept in evaluating examples and nonexamples. At this higher phase in attaining concepts at the classificatory level, children discriminate some of the less obvious attributes of the concepts and generalize correctly to a greater variety of examples, some of which are very much like the nonexamples. They are also able to make the basis of their classification more explicit but it is still incomplete.

Formal Level. Persons demonstrate a concept of *tree* at the formal level, if when shown some examples of trees, shrubs, and herbs, they properly identify the trees and call them "trees," discriminate and name the defining attributes of *tree*, give a societally accepted definition of *tree*, and evaluate how examples of trees differ from examples of shrubs and herbs in terms of the defining attributes of *tree*. When individuals can do these things, it is inferred that they have learned the concept at the formal level through performing the inductive or the reception operations shown in Figure 1-3.

The inductive pattern involves formulating, remembering, and evaluating hypotheses regarding the attributes of the concept and then inferring the concept definition if the concept has already been attained at the classificatory level, or inferring the concept itself if it has not been learned at the classificatory level at an earlier time. (A high school student encountering the concepts *trigonometry* or *valence* for the first time illustrates the latter.) The operations involved in the strategy of inductive hypothesis testing characterize individuals who secure information potentially available to them from both examples and nonexamples of the concept. These individuals apparently reason like this: "Thing 1 is land totally surrounded by water. It is a member of the class. Thing 2 is land that is only

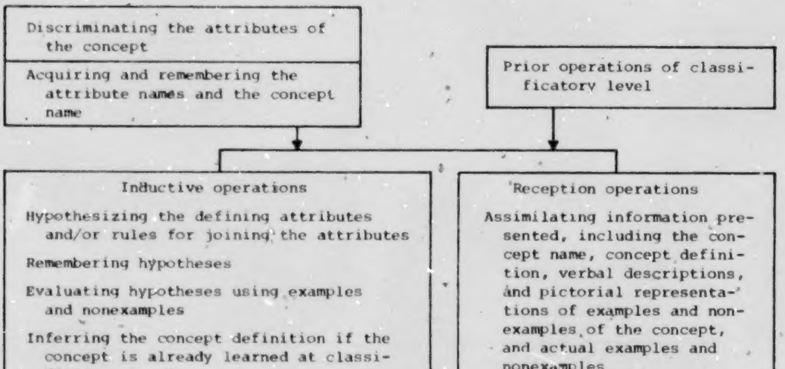

Discriminating the attributes of the concept
Acquiring and remembering the attribute names and the concept name
Prior operations of classificatory level
Inductive operations
Hypothesizing the defining attributes and/or rules for joining the attributes
Remembering hypotheses
Evaluating hypotheses using examples and nonexamples
Inferring the concept definition if the concept is already learned at classificatory level
Inferring the concept if not already learned at classificatory level
Evaluating examples and nonexamples of the concept on the basis of the presence or absence of the defining attributes
Reception operations
Assimilating information presented, including the concept name, concept definition, verbal descriptions, and pictorial representations of examples and nonexamples of the concept, and actual examples and nonexamples
Remembering the information
Evaluating examples and nonexamples of the concept on the basis of the presence or absence of the defining attributes

Figure 1-3. Mental Operations in Concept Attainment at the Formal Level

partially surrounded by water. It is not a member of the class. Therefore, lands totally surrounded by water belong to the class, but lands only partially surrounded by water do not." The individual has hypothesized a defining attribute, remembered it, evaluated the hypothesis, and is making progress toward inferring a definition of island in terms of its defining attributes.

A reception strategy, rather than an inductive strategy, is often employed in school settings with older students to enable the students to move from merely being able to classify examples of concepts to being able to understand the concepts at a mature level. In expository instruction using a meaningful reception strategy, students are given the name of the concept and its defining attributes. Explanations and illustrations are provided by the teacher; information may also be made available in books and visuals. The students' main task is to attend to, receive, and process the information that is provided and then to retrieve it when needed.

Much effective concept learning at the formal level in school settings involves some combining of the inductive and reception operations. It is very time-consuming for students to infer the defining attributes independently through a hypothesis-testing approach. However, to be told everything and not to do any hypothesizing or evaluating may result in lack of understanding. It may be well to emphasize that learning a definition of a word is not learning a concept at the formal level. Furthermore, verbal instruction in school settings that does not utilize examples and nonexamples of the concepts leads to rote memory of verbal information, not to being able to classify or to understand concepts fully.

In passing we note that in the earlier literature on concept learning the term "attribute" was used narrowly to refer only to the intrinsic properties of objects. Not included as attributes were the functions of objects, events, and processes or the invariant relationships among them. Furthermore, attributes were considered either as relevant or irrelevant for identifying particular concepts (Bruner, Goodnow, & Austin, 1956). In this book attribute is defined to include intrinsic properties, functions, and invariant relations. Three kinds of attributes are dealt with: *defining, critical,* and *variable. Defining attributes* of a concept include all the attributes that are necessary to determine whether an instance is or is not an example of a concept. The defining attributes of *equilateral triangle* are plane, simple, closed, three equal angles, and three sides of equal length.

Critical attributes of a concept are those necessary to distinguish members of one coordinate class from members of other coordinate classes. The critical attributes of *equilateral triangle* are three equal angles and three sides of equal length. These attributes are necessary and sufficient for discriminating between equilateral triangles and triangles of coordinate classes, including right triangles.

Variable attributes are those that are necessary for discriminating among members within a given class (e.g., black, green, and blue spruce trees). The same variable attribute, such as size or color, may be found across coordinate classes and also across supraordinate and subordinate classes. For example, equilateral triangles vary in size from very small to very large; right triangles (coordinate class) do also. A more complete discussion of defining, critical, and variable attributes may be found in Markle (1977) and in Tiemann and Markle (1978).

Transfer

Concepts are the building blocks of cognitive development, not only because they enable one to understand and relate many otherwise discrete phenomena, but also because they provide the basis for vertical and horizontal transfer. There is vertical transfer from one successive level to the next as individuals attain the successively higher levels of the same concept. There are four directions of horizontal transfer, corresponding to the four categories of uses that may be made of concepts.

The individual who has formed a concept at any of the four levels may use it as shown in Figure 1-4. A concept attained only at the concrete or identity level may be used in solving simple perceptual problems that do not require thinking with the use of symbols. Concepts learned at the classificatory and formal levels can be used in generalizing to examples and discriminating nonexamples when encountered, in forming structures of knowledge including the understanding of taxonomic and other hierarchical relationships involving the concept, in understanding principles, and in solving problems. As a concept is used in any of these ways, the meaning of the concept itself is extended. For example, when a principle or a taxonomy is understood, new relationships among each of the concepts embodied in the principle or the taxonomy are also understood, thus extending the meaning of the concepts. A more complete account of transfer may now be examined.

Historically, three theories have attempted to account for transfer from one learning situation to another—identical elements, generalization, and transposition (Klausmeier & Davis, 1969). There is no lack of agreement among the theories that transfer occurs; the difference lies in what transfers. According to the identical elements theory, specific facts, skills, and attitudes transfer from one situation

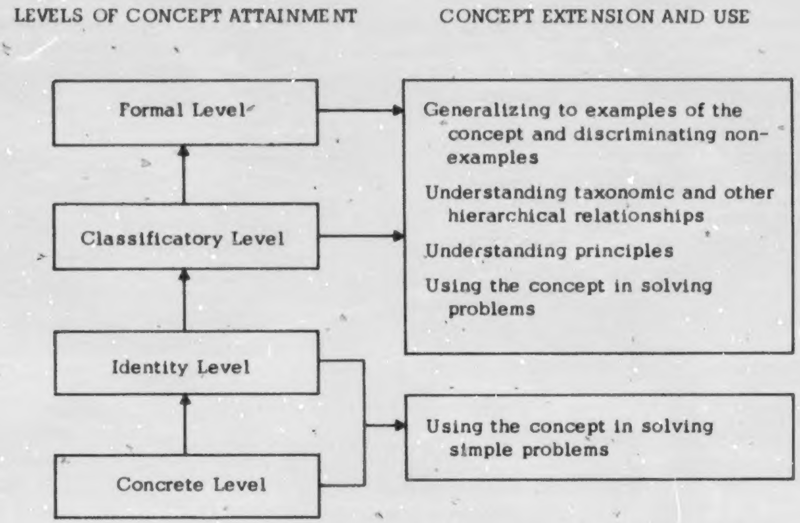

LEVELS OF CONCEPT ATTAINMENT
CONCEPT EXTENSION AND USE
Formal Level
Classificatory Level
Identity Level
Concrete Level
Generalizing to examples of the concept and discriminating non-examples
Understanding taxonomic and other hierarchical relationships
Understanding principles
Using the concept in solving problems
Using the concept in solving simple problems

Figure 1-4. Levels of Concept Attainment and Use

to the next. Principles are assumed to transfer according to the generalization theory. According to transposition theory the broader patterns of means-ends relationships between situations provide the basis for transfer. A still more comprehensive model (Ferguson, 1956; Klausmeier & Goodwin, 1975) specifies that the abilities and information transfer from one situation to the next. The essential elements and relationships of this information-abilities model are shown in Figure 1-5. The model is intended to be applicable to all products of learning in the cognitive domain, in the psychomotor domain, and also in the affective domain. Similarly, it accounts for both vertical transfer and horizontal or lateral transfer.

The information and abilities aspects of vertical transfer may be illustrated in connection with an individual having attained the identity level of a concept and then learning the classificatory level. Assume that an individual has attained the concept of *tree* at the identity level and is now involved in attaining the classificatory level. The individual's task is to learn that two different trees belong to the same class. We may speculate that, when learning the identity level, the individual acquired perceptual information about each tree related to its form, size, and color. This information acquired earlier when learning to recognize each of two trees as the same ones at the identity level transfers to this new task.

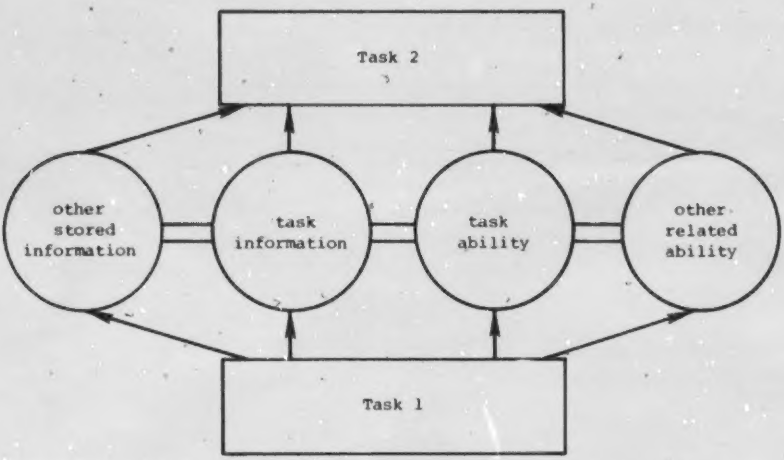


Figure 1-5. Information and Ability Bases of Transfer

When attaining the identity level of each of the two particular trees, the student demonstrated the ability to attend to and discriminate one tree as different from other treelike objects in the environment and also generalized that each of the two particular trees, when experienced later, was the same one that had been experienced earlier from a different spatio-temporal orientation. These attending, discriminating, and generalizing operations are also presumed to transfer when learning the concept at the classificatory level.

Having the task information and being capable of the attending, discriminating, and generalizing operations necessary for the identity level are necessary but not sufficient for attaining the classificatory level. Generalizing that two or more examples are equivalent, in this example that the two trees attained at the identity level are equivalent, is the new and higher level operation required. Attending to and discriminating more information related to two trees, rather than to one, and also more complex information (assume the trees vary in size and are an elm and a maple) are required.

Other information may also transfer from learning the identity level to learning the classificatory level. For example, some of the contextual information (the relative position of each tree in the yard) when learning the identity level may also transfer. In addition to the prior specific abilities, generalized conceptualizing abilities involving objects other than trees may also transfer to learning the classificatory level if the individual has also attained other concepts at the identity level and classificatory level. Similarly, strategies for learning and remembering may also transfer.

Lateral or horizontal transfer may be illustrated by showing the relationship between attainment of the formal level of a concept and acquiring a taxonomy. Figure 3-7, which appears in Chapter 3, shows a taxonomy of behavior management procedures. In the taxonomy, *behavior management procedures* is the generic concept; all examples of the subclasses are also examples of behavior management procedures. Two coordinate concepts that are subordinate to the generic concept are *procedures for increasing behavior* and *procedures for decreasing behavior*. Each of these coordinate classes is defined by a set of mutually exclusive critical attributes combined with the noncritical defining attributes of the generic concept. A procedure for increasing behavior cannot also be a procedure for decreasing behavior, but both are behavior management procedures.

Let us assume that the student has attained the concept *procedures for increasing behavior* at the formal level. How do the information and abilities required for attaining the formal level of this concept transfer to learning other concepts of the taxonomy and their relationships? We may deal with the information aspect first. Knowledge of the noncritical defining attributes of *procedures for increasing behavior* presumably will transfer to learning the generic concept *behavior management procedures* and also to learning the two coordinate concepts, *positive reinforcement procedures* and *negative reinforcement procedures*. These two coordinate concepts are subordinate to *procedures for increasing behavior*. Knowledge of the critical attributes of *procedures for increasing behavior* should transfer to discriminating the examples of the coordinate concept *procedures for decreasing behavior*. Let us assume now that a meaningful reception approach was employed when learning the concept *procedures for increasing behavior*. The mental operations that were used when attaining *procedures for increasing behavior*, as well as the strategies for learning and remembering, should also transfer in part or totally to learning the other concepts mentioned if they too are attained by means of a meaningful reception approach.

There are many relationships involved in the behavior management taxonomy, including those between the supraordinate and subordinate classes and between the different sets of coordinate classes. For example, all examples of subordinate classes are examples of the next higher supraordinate class. In addition, no example of any coordinate class is a member of another coordinate class. There are many other relationships embedded in the taxonomy. Also other information and abilities transfer but these mentioned here are sufficient to illustrate application of the information and ability model to horizontal transfer. As indicated earlier in the chapter, having all of the concepts at the formal level is necessary but not sufficient for understanding the taxonomic relations. Learning the many relations among the concepts is also required in order to fully understand a taxonomy. Similarly, having the concepts involved in a principle is necessary but not sufficient for understanding a principle, just as having the concepts and principles are necessary but not sufficient for solving problems.

We may relate the preceding analysis of transfer regarding *tree* to short-term learning situations and also to long-term development. During a short period of time, perhaps in less than a minute, the individual after having attained the identity level of two trees and having become capable of generalizing that they were equivalent, moved from the identity level of *tree* to the classificatory level. But this was the very earliest rudimentary classificatory level. That is, only two particular trees were categorized as being equivalent. As more trees and other green things, including shrubs, are experienced during the next months and years the person will continue to learn to discriminate between trees and other plants and will also learn to discriminate among different classes of trees, such as coniferous and deciduous. The final mature classificatory level of *tree* will not be attained on the basis of merely adding together the short-term, discrete experiences with individual trees. Instead, as transfer of learning continues to occur the individual will also continuously organize and reorganize the products of these short-term learning experiences into a more highly differentiated and inclusive concept of *tree*. Furthermore, other concepts, including *shrub* and *herb*, will be learned, and the entire set of concepts will be organized and combined into an increasingly inclusive but differentiated conceptual core. One may speculate that this kind of reciprocal interweaving of the products of short-term learning with an ever-increasing information base and more advanced abilities characterizes the relationship between learning across short time intervals and development across longer periods of time.

It may be added that during the developmental years from infancy into adolescence, individuals vary in the mental operations they are capable of performing because of the interaction of maturational factors and learning opportunities. With maturation and practice, they become increasingly capable of the higher mental operations that are necessary for attaining the successively higher levels of a concept and for forming more complete and inclusive conceptual cores. This does not imply that the emergence and refinement of the higher mental operations ceases or that the ability to employ the operations on more content and more complex content ceases when physical maturation is achieved. On the contrary, the producers of new knowledge typically make their significant discoveries long after achieving full physical maturation.

To this point the successive levels of concept attainment, the mental operations involved in attaining each level, and the uses and extensions of concepts have been indicated as providing a framework for describing and possibly explaining conceptual development across long time periods. An information and ability model of transfer has been presented to indicate how learning may proceed across short and extended time intervals. This very brief introduction is intended to be sufficiently complete so that the construction of tests based on CLD theory and the design of the longitudinal study presented in Chapter 2 may be understood. Although not dealt with in later chapters of this book, ideas regarding motivation are now presented in order to provide a more complete account of CLD theory.

Motivation

In explaining human cognitive behavior, Miller and Johnson-Laird (1976) employ the theoretical construct, executive control, to account for the self-regulation of all behavior, including mental and motor activity. The construct, intend, is employed to indicate the operation concerned specifically with the motivational aspects of behavior; the term, plan, is used to indicate any set of internal hierarchical processes that control the order in which a sequence of goal-directed instrumental activities is performed. A person's intending to carry out a plan to attain a goal initiates and directs a sequence of instrumental activities. The intention arises from a perceived incongruence or inconsistency between a desired or expected state of affairs or conditions and a current state of affairs or conditions.

This very brief account of the self-regulatory and the motivational bases of both mental and motor activity is taken as the starting point for the treatment of motivation that follows. A more complete discussion of executive control and intend may be found in Miller and Johnson-Laird (1976) and of plan in Miller, Galanter, and Pribram (1960).

The human being is an information-processing, stimulus-seeking organism whose thought influences action. Thoughts, particularly in the form of intentions, have a powerful effect on the first three of four phases of a behavior sequence: its activation or instigation, its direction, its persistence until goal achievement, and finally, the consequences following achievement or nonachievement of the goal. Although thought influences action and thinking is involved throughout an action sequence, not all behavior is consciously intended. Reflex actions and firmly established habitual patterns of behavior are carried out without conscious intent.

The relationships between a goal-directed sequence of observable or inferrable instrumental activities and cognitive motivational activities are indicated in Figure 1-6. A sequence of activities may be completed during a very short time interval or it may be carried out over a period of years. The infant's behavior sequences are necessarily of short duration, but those of adults range from very short duration to extending across years. We may give further consideration to the relationships between the instrumental and motivational aspects of the four phases of a behavioral sequence.

1. Let us assume that a person planning a nature walk finds out that there is some poison ivy in the trail area. Being aware of this the person experiences an incongruency or inconsistency between taking the walk and possibly being poisoned. This cognitive incongruence is

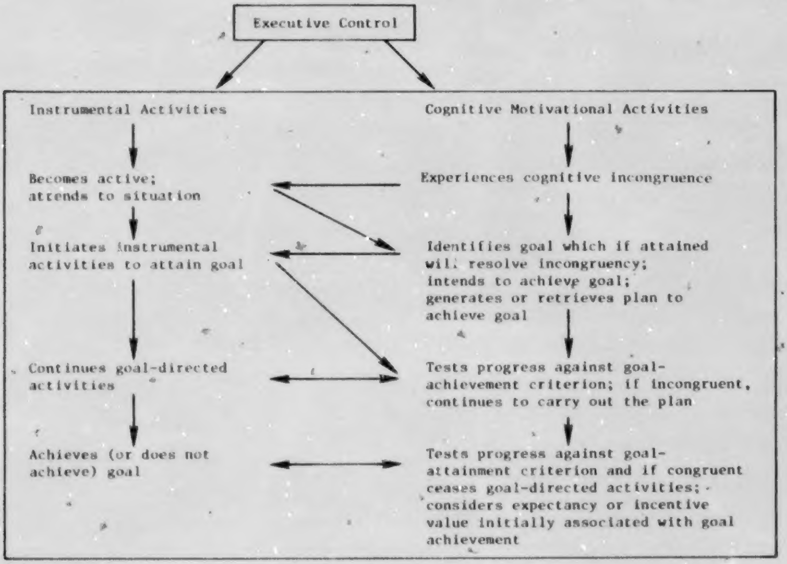

Executive Control
Instrumental Activities
Cognitive Motivational Activities
Becomes active; attends to situation
Experiences cognitive incongruence
Initiates instrumental activities to attain goal
Identifies goal which if attained will resolve incongruency; intends to achieve goal; generates or retrieves plan to achieve goal
Continues goal-directed activities
Tests progress against goal-achievement criterion; if incongruent, continues to carry out the plan
Achieves (or does not achieve) goal
Tests progress against goal-attainment criterion and if congruent ceases goal-directed activities; considers expectancy or incentive value initially associated with goal achievement

Figure 1-6. Parallel Sequences of Instrumental Goal-Directed Activities and Related Cognitive Motivational Activities

accompanied with mental activation, including attention to this particular situation.

2. The cognitive incongruence is followed with two closely related activities—formulating a goal and intending to achieve it. The goal is to learn to identify poison ivy plants.

Goals may take the form of an expectancy, that is, an expected condition or state of affairs to be achieved; a desired incentive, such as a feeling of success to accrue from attainment of a goal; or a combination of the two. Both function in connection with many goals.

Having formulated a goal and intending to attain it, the individual either generates a plan for achieving it or retrieves from memory a plan that has been carried out earlier. Intending to carry out the plan is followed with an executive control instruction to start carrying out the instrumental activities to achieve the goal. Let us assume that in our example the plan calls for studying a particular book to identify poison ivy. The person gets the book and starts studying it.

3. The plan for attaining the goal—study the book until poison ivy can be identified—is used to test progress toward the goal. The plan serves to direct and sustain the goal-related activities in that as long as the plan is not completed and the intention remains, the individual continues the instrumental activities. During this phase of the behavioral sequence, the individual tests the progress being made against the goal achievement criterion. If the goal is not attained and the individual still intends to attain it, the initial plan or a refined plan will serve to continue to direct the instrumental activities. In our example, the person studies the book until the criterion of being able to recognize poison ivy is met.

4. Assume that the goal is achieved, that is, the individual has learned to identify poison ivy plants. Further thought may result and influence instrumental actions. For example, if the individual feels confident of being able to recognize poison ivy, no ointment or other remedy may be purchased. If the individual is not certain, another plan may be generated that starts another behavioral sequence, namely, to purchase the ointment.

We may assume that individuals differ in their motivational activities and states as they do in other areas of cognitive functioning. Recent approaches to understanding motivation from a cognitive frame of reference, including attribution theory and locus of control, hold promise for understanding the differences (Wittrock & Lumsdaine, 1977).

Attribution theory (Heider, 1958; Jones, Kanouse, Kelley, Nisbett, Valins, & Weiner, 1972; Rotter, 1966; Weiner, 1975, 1977) is concerned with the individuals' perception of causality. One dimension of attribution is ascribing causes to events that occur in terms of the stability or instability of the causes, and another is the locus of control of the causes of behavior.

Assume that a student fails an examination. The student's perception of the causes of the failure influences the student's affective reactions to the situation and also expectancy regarding similar situations that may be experienced later. The four main perceived causes of success and failure are ability, effort, task difficulty, and luck (Weiner, 1975, 1977). Expectancy regarding future success and failure is related to the stability of these causes. Success that is attributed to good ability, which is regarded as relatively stable cause, does not increase motivation for similar tasks, whereas failure attributed to lack of ability reduces motivation. On the other hand, success attributed to task difficulty and luck, which are changeable and unstable factors, tends to increase motivation for performing similar tasks, while failure ascribed to these unstable factors does not reduce motivation. Prior experiences in which success or failure have been experienced and are attributed to stable or unstable causes markedly influence expectancies of success and failure in similar tasks that are encountered later.

The two polar dimensions of locus of control are ascribing the causes of events to oneself or to external agents and events (Rotter, 1966). In this approach to motivation, stable and unstable factors are given less emphasis than is the internal or external locus of control. The hypothesis is that persons who ascribe causes of success and failure to themselves (internal control) achieve higher than do those who ascribe the causes to external agents and events (external control). Substantial support is found for this hypothesis; for example, Grade 6 students instructed in assuming internal control subsequently showed higher achievement motivation and also performed higher on educational achievement tests and ability tests (deCharms, 1972).

deCharms (1968) developed a model of instrinsic and extrinsic motivation related to locus of control. Perceiving oneself as causing one's own behavior is regarded as intrinsic motivation and perceiving forces external to oneself as the cause is extrinsic. According to this model, giving rewards to persons for performance is hypothesized to decrease intrinsic motivation. The majority of studies support the hypothesis.

Based on the available research regarding both the stability of ascribed causes and locus of control, Wittrock and Lumsdaine (1977) conclude that profound changes should be made in education so as to develop persons with an internal locus of control.

We now seem to agree that earlier stimulus-response (S–R) formulations regarding reinforcement as automatically strengthening responses and as reliably and automatically serving as incentives for initiating and directing instrumental activities are inadequate bases for understanding human motivation. The fact that receiving reinforcements tends to strengthen behavior and to serve as an incentive for engaging in the same behaviors should be interpreted on the basis of informational feedback as well as the incentive value that the reinforcer may provide. Receiving the reinforcement apparently provides information that is analyzed by the recipient; and this analysis, along with other cognitive variables, determines the effects of the reinforcer.

OVERVIEW OF THREE INTERRELATED STUDIES AND PLAN OF THIS BOOK

The preceding overview of the Piagetian and CLD theories suggests that both should contribute to describing and explaining cognitive development during the school years. Accordingly, a longitudinal study involving three interrelated studies was carried out to describe the course of cognitive development during the school years, Grades 1–12, using tests based on CLD theory; to describe cognitive development during the concrete operations stage, primarily kindergarten–Grade 6, using tasks based on Piagetian theory; and to identify similarities and differences in the patterns of development of children and youth Grade 1 through Grade 9, as derived from various analyses of the two sets of measurements based on the two theories. The sampling design of the studies and a full account of the construction of tests is given in Chapter 2.

The same four longitudinal groups of students participated in the interrelated studies and were tested at four 12-month intervals; however, not all groups were administered all the tests as is made explicit in Chapter 2. The four longitudinal groups were in kindergarten, Grade 3, Grade 6, and Grade 9 in the first year of the study and continued into Grades 3, 6, 9, and 12 in the last year.

Four batteries of tests based on CLD theory were administered at one-year intervals to all the students of the four groups when they were successively enrolled in Grades 1-2-3, 4-5-6, 7-8-9, and 10-11-12. (The CLD batteries were group tests and were not individually administered.) The results from these tests were used in charting the course of cognitive development of children and youth during the school years and in empirically testing the CLD principles of cognitive development. The complete description of cognitive development of children and youth during the school years based on this part of the longitudinal study is reported in Klausmeier and Allen (1978). The main principles of development are summarized in Chapter 3 of this book; however, the changes in the structure of the abilities that were found to occur from Grade 1 through grade 12 are reported for the first time in Chapter 4 of this book.

Twelve tasks frequently used in research to measure performances associated with Piaget's stage of concrete operations were identified, modified, and refined (Hooper, Brainerd, & Sipple, 1975), and one task, combinatorial reasoning, was identified for the formal operations stage. These 13 tests were administered individually to 60 percent of the same two younger longitudinal groups who received the CLD tests when they were in Grades Kg.-1-2-3 and in Grades 3-4-5-6, respectively. (Since much time was used and some disruption of teaching occurred in order to administer the tests individually, each sample of boys and girls was reduced from 50 to 30.) Two of the tests were administered to a random sample of 60 percent of the Grade 6-7-8-9 longitudinal group.

Brainerd (1972b) constructed eight tests to measure the groupement structures formulated by Piaget. Piaget uses the groupement structures to explain the concrete operations stage. Accordingly, certain predictable relationships should be found within the groupement structures themselves and also between measures of the structures and measures of the performances of individuals based on Piagetian tests when they are presumably in the stage of concrete operations. The groupement tests were administered in the first year of the study and annually thereafter to the same three samples of students that received the Piagetian tests starting in kindergarten, Grade 3, and Grade 6, respectively.

Results from the preceding Piagetian tests, groupement tests, a test of arithmetic, and three memory tests administered to the two younger groups when in kindergarten-Grade 6, respectively, were analyzed to get a better understanding of children's cognitive development during the stage of concrete operations as defined by Piaget. Hooper, Swinton, and Sipple report these results in Chapter 5 of this book and give their views of the nature of Piaget's concrete operations stage.

The scores of the same students of the longitudinal groups who took the Piagetian tests, the groupement tests, and the CLD tests were correlated to identify relationships among the three sets of measurements. Other statistical analyses were performed on the test scores, and these analyses were related to the constructs and propositions of the two theories that are presented in Chapters 1, 3, 4, and

5. Based on the various analyses, attributes of development were identified that were found to be common to the three data bases—CLD, Piagetian, and groupement. These results are presented in Chapter 6. The intent in Chapter 6 is to provide the reader sufficient empirical information to draw inferences independently regarding the key Piagetian constructs of stages and groupement structures, and the corresponding but very different CLD constructs of mental operations, levels of concept attainment, and the vertical and horizontal transfer of concepts.

In Chapter 7, I comment briefly on antecedent theoretical views and my earlier research leading up to the present studies. This is followed with a synthesis of developmental progressions based on the empirical information given in the earlier chapters. The progressions are accounted for from the CLD perspective.

In Chapter 8, Brainerd analyzes the relative importance given to developmental factors and to learning in the Piagetian and CLD approaches. It is argued that whereas development and learning are separable processes in Piagetian theory, they are not in CLD. Brainerd also reports some experiments concerned with whether children's ability to learn conservation and perspective-taking concepts is constrained by what they already know about these concepts.

In Chapter 9, Murray presents his analysis of the relationship between psychological theories in general and their possible applications to educational practice. Drawing upon his expertise in Piagetian theory and education, he evaluates the claims made about the application of the theory to both curriculum and instruction. It is noted in passing that a theory of instruction based on CLD theory is given in Klausmeier and Allen (1978).

Chapter 2

Design and Methods

Thomas S. Sipple and Herbert J. Klausmeier

Over the past two decades a great deal of research on cognition and cognitive growth has accumulated, encompassing a full range of research methodologies—from normative, descriptive, and correlational studies to rigorously controlled laboratory experiments. An exhaustive and detailed survey of the many research strategies that have been used to study cognitive development is beyond the scope of this chapter. For an in-depth discussion of research methods, the interested reader is referred to Kessen (1960), Nesselroade and Reese (1973), and Wohlwill (1973).

This chapter focuses on the sampling design and the tests used in each of the three studies of cognitive development that were carried out simultaneously using data from the same four longitudinal groups of students. Special attention is given to the measurement tools and procedures since the results of the studies are directly related to the reliability and validity of the measures.

The first study is based on CLD theory (Klausmeier, 1971; Klausmeier, Ghatala, & Frayer, 1974; Klausmeier & Allen, 1978) and utilizes data from all the students of four longitudinal groups of students when they were enrolled in Grades 1-12. The second study is based on Piagetian theory and deals with the stage of concrete operations. Here most of the data were collected on stratified random samples of the two preceding younger longitudinal groups when they were enrolled in kindergarten-Grade 6. The third study relates students' performances on measures based on the two theories and draws from the data of the stratified random samples of the first three longitudinal groups when they were enrolled in Grades 1-9. The methodology of each of these three studies combines features of both cross-sectional and longitudinal designs.[1]

COMBINED LONGITUDINAL AND CROSS-SECTIONAL METHODOLOGY

A study that combines the best features of the cross-sectional and the longitudinal method enables the researcher to study cognitive development over a relatively broad age span in a fashion that is both economical and methodologically and conceptually sound, if attention is given to potential biases and if assumptions regarding the operation and interpretation of extraneous variables are made explicit.

In discussing mixed designs, Nunnally (1973) has noted:

> . . . it would be very useful to employ designs that combine some of the economy of the cross-sectional design with the conceptual advantages of the longitudinal design. . . . such mixed designs potentially can supply some useful forms of information that cannot be supplied either by cross-sectional or longitudinal designs separately. One purpose of such research strategies would be to estimate truly longitudinal curves from partly cross-sectional data (Nunnally, 1973, p. 93).

The design of each of the three present studies was cross-sectional in that students of different age groups participated. The design was longitudinal in that the data were gathered on each age group at one-year intervals. Combining cross-sectional and longitudinal measurement in this manner permitted the study of cognitive development, including individual differences in growth patterns, throughout the school years.

DESIGN AND METHODS OF THE CLD STUDY

Four longitudinal samples, each consisting of 50 boys and 50 girls, participated in the study in the first year, 1973 (Table 2-1). The four samples were drawn from kindergarten, Grade 3, Grade 6, and Grade 9, respectively, and were designated Longitudinal Groups, or Blocks, A, B, C, and D. In the first year, these 400 students were administered one battery of seven CLD tests designed to measure the attainment

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[1] A more complete discussion of this methodology is given in Klausmeier, H.J. and Allen, P.S. (1978), pp. 29-58.

Table 2-1. Sampling Design for the CLD Study

Grade in School Each Year of Measurement								
1973		*1974*		*1975*		*1976*	N in *1976*[b]	*Group Designation*
Kg[a]	→	1	→	2	→	3	62	*Longitudinal Block A*
				1	→	2	23-26	Cohort Control 1
						1	33	Cohort Control 2
				2			34-40	Retest Control 1
						3	32-40	Retest Control 2
3[a]	→	4	→	5	→	6	77	*Longitudinal Block B*
				4	→	5	36-37	Cohort Control 1
						4	34-36	Cohort Control 2
				5			34-40	Retest Control 1
						6	37-40	Retest Control 2
6[a]	→	7	→	8	→	9	80	*Longitudinal Block C*
				7	→	8	32	Cohort Control 1
						7	35	Cohort Control 2
				8			33-40	Retest Control 1
						9	37-40	Retest Control 2
9[a]	→	10	→	11	→	12	73	*Longitudinal Block D*
				10	→	11	32-33	Cohort Control 1
						10	38	Cohort Control 2
				11			33-40	Retest Control 1
						12	34-40	Retest Control 2

[a] One battery, *equilateral triangle*, was administered in 1973; the complete set of four batteries was administered in 1974, 1975, and 1976 to the four longitudinal blocks of students. The cohort and retest control groups that received all four batteries starting in 1975 are shown above; the 1974 cohort and retest control groups for *equilateral triangle* are not shown.

[b] Unequal *N*s for the four longitudinal groups and the various control groups and ranges in *N*s within some of the control groups resulted mainly from differing incidences of attrition among the various groups and from absenteeism for one or more tests within the same control group.

of the four levels and three uses of the concept, *equilateral triangle.* Three batteries for *cutting tool*, *tree*, and *noun* were constructed and tried out with a different sample of students in 1973. All four batteries were administered at one-year intervals to the students of each longitudinal group in 1974, 1975, and 1976. (Results of the 1973 assessment are reported as a cross-sectional study (Klausmeier, Sipple, & Allen, 1974b). Two hundred ninety-two of the 400 children of 1973 remained in the final year of assessment, 1976. These 292 children were distributed fairly evenly across the four grade groups and also by sex—Group A: 32 boys, 30 girls; Group B: 40 boys, 37 girls; Group C: 39 boys, 41 girls; and Group D: 36 boys, 37 girls. Of the 108 students who did not complete the longitudinal study, 65 moved out of the city, 22 entered parochial schools, and 21 left for other reasons. Statistical analyses of the test scores of students who remained in the study and of those who left showed no significant differences.

Control groups were incorporated in the design to permit evaluations of possible cohort differences (i.e., differences among children born in different years) and possible effects of repeated testing on the longitudinal groups.

There were two cohort control groups for each of the four longitudinal groups. The first set of cohort groups consisted of individuals of the same age and grade as those of each of the longitudinal groups, but they were born one year later. This set of four cohort groups, one for each longitudinal group, was administered the four CLD batteries in 1975 and again in 1976. The second set of four cohort groups consisted of individuals at the same age and grade as those of each of the longitudinal blocks, but they were born two years later. This second set of four cohort groups was administered the four CLD batteries only in 1976.

For each longitudinal group there were also two control groups to evaluate the effects of retesting. Each retest control group was of the same age and grade as the longitudinal group and was tested only once. The first set of four retest control groups was administered the four CLD batteries in 1975 in order to compare their performances with those of the longitudinal groups that had been tested twice. The second set of retest control groups was administered all four batteries in 1976 in order to compare their performances with those of the longitudinal groups that had been tested three times.

The children in the cohort and retest control groups were randomly drawn from the same schools and classrooms from which the children in the four longitudinal groups had been drawn in 1973. Each control group initially contained 20 boys and 20 girls. The same conditions accounting for the loss of some students of the longitudinal groups also accounted for the loss of some control group students. The greatest loss occurred with Longitudinal Group A and the cohort control groups for this block. Many of these original children entered parochial schools upon completing kindergarten.

Locale

To permit generalizability of the results, a small industrial midwestern city was selected in which the public school population reflects the racial mix, distribution of socioeconomic levels, and achievement levels of the student population of the United States. This manufacturing and industrial city had a 1972 population of approximately 36,000, of which 7 percent was black. The economic level was slightly above the average for midwestern industrial cities. The public school system, which enrolled about 10,000 students at the time this study was begun, had 14 elementary schools, three junior high schools, and one senior high school. The four longitudinal samples of 50 boys and 50 girls were drawn randomly from four elementary schools, one junior high school, and the senior high school, all of which were representative of the socioeconomic, ethnic, and general demographic classifications found in the city.

Development of CLD Assessment Batteries

One objective of the CLD study was to chart the course of cognitive development from Grade 1 through Grade 12. CLD theory as explained in Chapter 1 provided the substantive framework for constructing the four batteries that were used. Four tests of each battery were constructed to measure the students' attainment of each of the four levels of the same concept—*cutting tool*, *tree*, *equilateral triangle*, and *noun*. One test was constructed to measure understanding of related principles, one to measure understanding of taxonomic relations, and one to measure the ability to solve problems that involved use of the concept and principles.

Criteria for Selecting Concepts. The CLD theory attempts to account for concept attainment from the earliest discrimination of an object, event, quality, or relation as one of permanence different from other things through a high level of attainment, such as by a high school or college student who can readily classify many examples and nonexamples and can also indicate the defining attributes of the concept. Measurement to chart change of this magnitude across the years requires careful selection of the concepts, guided by explicitly formulated criteria.

First, the concept should be of the kind that potentially can be attained at each of the four levels and used in the three ways specified by CLD theory; furthermore, it should be such that attainment of the four levels and three uses can be measured reliably. This criterion calls for concepts that have examples that are experienced by individuals in concrete form, pictorially, or verbally, starting during the early years of life. The examples should be entities of permanence such that they can be presented to the individual, removed, and then presented again. This is necessary for determining attainment of the concrete, identity, and classificatory levels. The concept should also have a societally accepted definition stated in terms of its defining attributes. The attributes may be stated as properties intrinsic to the examples of the concept, as functional properties, that is, as indicators of what can be done with the examples of the concept or what they can do, or as dependency relations between the given examples and other examples in a hierarchy. Having a societally accepted definition is essential for measuring attainment at the formal level.

Although many concepts satisfy this criterion, others do not. For example, such important concepts as *love*, *fear*, *time*, *space*, and *death* have societally accepted definitions, but examples and nonexamples of them cannot be depicted readily at the concrete and identity levels so that young children's attainment of those levels can be measured reliably. Disagreement among experts concerning the meaning of other powerful concepts such as *intelligence*, *democracy*, *justice*, and *morality* does not permit the development of reliable measures of their attainment at the formal level (or any other level).

The second criterion requires that the operations, the levels, and the uses of the selected concepts should be generalizable to a larger body of concepts, such as found in taxonomies and other hierarchical structures of knowledge. Harris and Harris (1973), in factor analytic studies involving children completing the fourth grade, identified a separate factor associated with the achievement of concepts in each subject field: language arts, mathematics, science, and to a lesser degree, social studies. They also found that tests measuring achievement of many different concepts within each subject field loaded on the one subject field factor. To assure generalizability within a subject field and also therefore the possibility of making comparisons across subject fields, *equilateral triangle* was selected from mathematics, *noun* from English, and *tree* from science. The fourth concept, *cutting tool*, may be more closely related to science than to the other subject areas; however, it was selected because it is not typically taught at any time during the school years. Therefore, growth in attaining this kind of concept can be related to that of the other three concepts that are included in school curricula.

The third criterion involves generalizability to concepts that vary on a concrete-abstract dimension. Examples of cutting tools and trees are experienced as concrete three-dimensional objects. Examples of equilateral triangles are experienced typically as drawings, edges of surfaces, or boundaries of surface areas. Nouns are represented symbolically by single words, phrases, or clauses that in turn have observable referents and that also have clearly defined functions in a sentence. Thus, the concepts used in the present study have three categories of examples: those that can be experienced by seeing and touching, those that have pictorial representations that can be seen, and those that can be experienced only symbolically in oral or written form.

Concepts from music, such as of *harmony* and *rhythm*, might have been used. These and other concepts involving the auditory modality were considered. They were not selected because of the difficulties involved in constructing reliable measuring instruments and the costs of administering them.

Criteria for Test Construction. In addition to the usual criteria of reliability and usability, tests for each of the four concepts were constructed in accord with the constructs and propositions of CLD theory.

First, the test items for a particular level of concept attainment should measure the mental operations, or processes, specified for attainment of the level, that is, attending, discriminating, generalizing, and so on. Here the attempt was made to measure the final operation for the level rather than the operations thought to be prerequisite to the final one. At the concrete level discriminating the target example from the nonexamples was measured rather than attending to the target example. At the identity level, generalizing across the different orientations or other situational contexts in which the same target example was presented was measured. At the classificatory level, generalizing across two or more different examples was measured. At the formal level the two prerequisites of being able to discriminate the defining attributes and having the concept name and the names of the attributes were measured separately. Recognition of the definition of the concept and evaluation of examples and nonexamples in terms of the presence or absence of the defining attributes were also measured separately. These four measures were combined to get a single score indicating attainment of the formal level. Early attempts to construct test items to measure the formulation of hypotheses, testing of hypotheses, and inferring the concept proved to be very time-consuming. Although many experiments have been done which indicate that hypothesizing and inferring are the key operations in inductive concept learning (Bourne, Ekstrand, & Dominowski, 1971; Klausmeier, Ghatala, & Frayer, 1974), reliable measurement of these operations has proved to be elusive. For example, Marliave (1976) used a lengthy oral questioning period to infer differences among older students in their ability to generate and test hypotheses.

The second criterion for test construction was that the items for the four successively higher levels should measure the individual's ability to perform the CLD operations of attending, discriminating, remembering, and generalizing on an increasing amount of material and on increasingly complex material. The amount of material within each level of each test was controlled by varying the number of nonexamples included in the items of the test. Complexity was controlled by varying the number of defining attributes and variable attributes shared by the examples and nonexamples. As more nonexamples were used and as they became more like the target example in both the defining and variable attributes, greater demands were placed on the individual's ability to attend, discriminate, remember, and generalize.

A third criterion was that tests should measure the three uses of concepts. This called for a test to be constructed to measure the understanding of principles in which the particular concept was embedded, understanding the taxonomic relationships of which the particular concept was a part, and solving problems involving use of the particular concept or a related principle or both. Taxonomic relations involving supraordinate-subordinate-coordinate relations were chosen for this study, rather than other hierarchical relations, because of the large amount of research already completed and also because of the importance of taxonomies as organizing frameworks for many areas of human knowledge. At the time of constructing the three tests measuring uses of the concepts, no evidence was available as to which would be mastered earlier or later, and no order of mastery is assumed by CLD theory.

Test Construction and Validation. The four CLD test batteries were constructed and validated during a three-year period. Students of a school district not in the present study participated in the test validation.

The first pilot study involving *equilateral triangle* and *cutting tool* was carried out with 50 children—10 each of nursery school, kindergarten, third- , sixth- , and eighth-grade age (Frayer, Klausmeier, & Nelson, 1973). For *equilateral triangle*, 36 three-dimensional blocks of three shapes, three colors, two sizes (surface area), and three thicknesses were used. For *cutting tool*, 36 drawings on 3" × 5" cards were used; the drawings depicted five kinds of cutting tools and four kinds of noncutting tools. The test administrations were individual and oral. Problems were identified with some of the questions and materials used in the first tryout and the attempt was made to correct them.

The next pilot test for *equilateral triangle* (Klausmeier, Sipple, & Frayer, 1973) involved 280 students, 40 each of seven grade groups: preschool, kindergarten, second, fourth, sixth, eighth, and tenth. (Pilot studies of this magnitude were conducted for every battery.) The administration was also individual, and three-dimensional blocks were used.

The technical characteristics of the tests of the four levels and three uses were such that it appeared feasible to undertake the present longitudinal/cross-sectional study. However, three major problems remained. The three-dimensional blocks worked reasonably well at the concrete, identity, and classificatory levels with younger children but not at the formal level with older students. The older students know that *equilateral triangles* are figures that are plane simple, and closed. Three-dimensional blocks are not plane nor simple. In addition, some test items called for the student to use terms such as *equilateral triangle*, *equal angles*, and *plane* when answering orally. The examiner, despite repeated questioning to elicit acceptable verbal responses, frequently had difficulty in deciding whether to score answers as correct or incorrect. The individual administrations were also very time-consuming. To eliminate these three problems and to maintain consistency across all four batteries, the decision was made to use a paper-pencil format in which the correct responses had to be recognized by the student rather than supplied, to use line drawings instead of three-dimensional objects, and to administer the tests to groups, except in the case of young children who might need clarification of the instructions individually. The instructions were read to all students who participated in the study. Test items were also read to the younger children if they were judged not to have some of the words of the test items in their reading vocabularies.

Further tryouts of the *cutting tool* and *equilateral triangle* batteries and tryouts for *tree* and *noun* consisted of administering them in paper-pencil format in schools of another district under the identical conditions as in the district in which the longitudinal/cross-sectional data were gathered. As noted earlier, only the *equilateral triangle* battery was ready for administration to the longitudinal samples of kindergarten, third- , sixth- , and ninth-grade students in the first year of the study.

A battery of seven tests was constructed for each of four concepts: *equilateral triangle* (Klausmeier, Ingison, Sipple, & Katzenmeyer, 1973a), *noun* (Klausmeier, Ingison, Sipple, & Katzenmeyer, 1973b), *tree* (Klausmeier, Marliave, Katzenmeyer, & Sipple, 1974), and *cutting tool* (Klausmeier, Bernard, Katzenmeyer, & Sipple, 1973).

Table 2-2 presents the number of items in each test of each CLD battery. Also given is the number of items correct that was set and subsequently used in evaluating the student's mastery of the test.

A technical manual was developed for each battery (DiLuzio, Katzenmeyer, and Klausmeier, 1975a, 1975b, 1975c, 1975d). The manuals give the technical characteristics of the tests.

Sample items drawn from three of the batteries to measure the four levels follow. Each set of items, except at the formal level, is preceded by the cognitive operations that are presumed to be necessary for attaining the level. These are followed by other sample items assessing the three uses of attained concepts.

Table 2-2. Number of Items and Criteria for Mastery of Each of the 28 CLD Tests

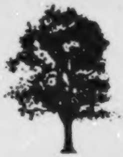
Battery, Test, and Abbreviation	*Number of Items*	*Criteria for Mastery*
Cutting tool, concrete (CTCO)	8	7
Cutting tool, identity (CTID)	8	7
Cutting tool, classificatory (CTCL)	8	7
Cutting tool, formal (CTFO)	18	15
Cutting tool, principle (CTPR)	5 pairs	4 pairs
Cutting tool, taxonomic relations (CTTAX)	4 pairs	3 pairs
Cutting tool, problem solving (CTPS)	5	4
Tree, concrete (TRCO)	8	7
Tree, identity (TRID)	8	7
Tree, classificatory (TRCL)	8	7
Tree, formal (TRFO)	35	28
Tree, principle (TRPR)	10 pairs	8 pairs
Tree, taxonomic relations (TRTAX)	4 pairs	3 pairs
Tree, problem solving (TRPS)	10	8
Equilateral triangle, concrete (ETCO)	8	7
Equilateral triangle, identity (ETID)	8	7
Equilateral triangle, classificatory (ETCL)	3	3
Equilateral triangle, formal (ETFO)	18	15
Equilateral triangle, principle (ETPR)	5 pairs	4 pairs
Equilateral triangle, taxonomic relations (ETTAX)	4 pairs	3 pairs
Equilateral triangle, problem solving (ETPS)	6	5
Noun, concrete (NNCO)	8	7
Noun, identity (NNID)	8	7
Noun, classificatory (NNCL)	8	7
Noun, formal (NNFO)	25	20
Noun, principle (NNPR)	5 pairs	4 pairs
Noun, taxonomic relations (NNTAX)	4 pairs	3 pairs
Noun, problem solving (NNPS)	5	4

Operations at concrete level:

Attending to perceptible features of an object
Discriminating the object from other objects
Remembering the discriminated object (internally representing, storing, and retrieving the concrete-level representation)

Item for concrete level of *tree:*

"Look carefully at the drawing."



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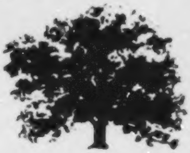
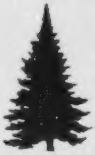
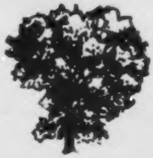
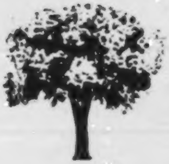
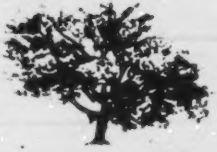
"Put an X on the drawing that looks exactly the same as the one you just saw."

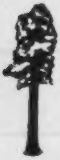


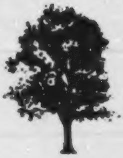












Operations at identity level:

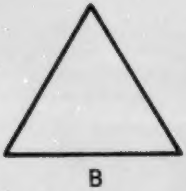
Attending to perceptible features of an object
Discriminating the object from other objects
Remembering the discriminated object
*Generalizing that the object when experienced in different contexts or modalities is the same object

*New operation that is essential for attaining the identity level.

Remembering the generalization (internally representing, storing, and retrieving the identity-level representation)

Item for identity level of *equilateral triangle:*

"Look carefully at the drawing."



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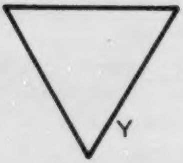
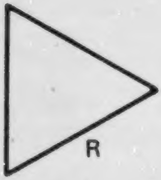
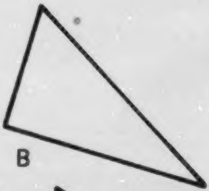
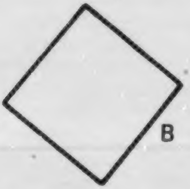
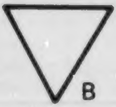
"Put an X on the drawing that looks exactly the same as the one you just saw."

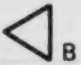


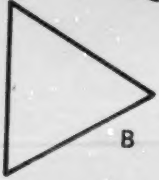












Key: Y = Yellow
R = Red
B = Blue

Operations at classificatory level:

Attending to the less obvious attributes of at least two examples of the class of objects
Discriminating each example from nonexamples
Remembering the discriminated examples

Generalizing that each example when experienced in different contexts or modalities is the same example

*Generalizing that two examples are equivalent (belong to the same class)

Remembering the generalization (internally representing, storing, and retrieving the classificatory level representation.)

Item for classificatory level of *noun:*

Put an X on the things below that name one or more persons, places, or things.

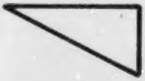
tent go rob Dick when

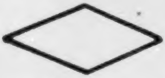
Operations at formal level:

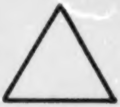
See Figure 1-3 of Chapter 1 for the operations of this level.

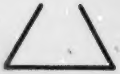
Item for discriminating the attributes of the concept *equilateral triangle:*

Below are four drawings. Put an X on the one that has three equal angles.









Item for acquisition of the name of the concept *noun:*

Fred mercy children's
Group 1

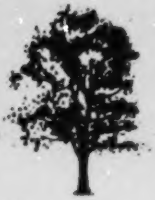
green together listen
Group 2

What one name *best* fits all of the words in Group 1 but does not fit all of the words in Group 2?

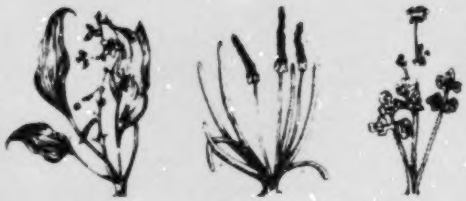
a. adverbs
b. adjectives
c. verbs
d. nouns
e. I don't know

*New operation that is essential for attaining the classificatory level.

Item for evaluating examples and nonexamples of the concept *tree* in terms of its defining attributes:



Plant X



Plant X is different from *all* of the other plants in some way. How is it different?

a. Only Plant X has a stem.
b. Only Plant X has roots.
c. Only Plant X has many main stems.
d. Only Plant X has a single main stem.
e. I don't know.

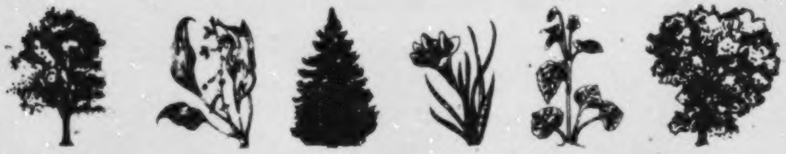
Item for acquisition of the definition of the concept *equilateral triangle:*

What is the definition of equilateral triangle?

a. A plane, complex, closed figure with three equal sides.
b. A plane, simple, open figure with three equal sides and three equal angles.
c. A plane, simple, closed figure with three equal sides and three equal angles.
d. A plane, complex, open figure with three equal angles.
e. I don't know.

Items of concept extensions and uses:
Item for transfer of understanding the concept *tree* to understanding a taxonomic relationship involving *tree:*

Are all of the plants below trees?



a. Yes, all of them are trees.
b. No, only some of them are trees.
c. No, none of them are trees.
d. I don't know.

Item for transfer of understanding the concept *equilateral triangle* to understanding a related principle:

"If the angles of a triangle are equal in the number of degrees, ___."

a. one side of the triangle is longer than the other two sides.
b. one side of the triangle is shorter than the other two sides.
c. the sides of the triangle are not equal in length.
d. the sides of the triangle are of equal length.
e. I don't know.

Item for transfer of understanding the concept *noun* to solving a problem involving *noun:*

See the sentence below.
"A ___ ran down the street."
What should be added to complete the sentence above?
a. the word *horse* used as the subject
b. the word *horse* used as a predicate
c. the word *horse* used as an object of a verb
d. the word *horse* used as a predicate nominative
e. I don't know

Data Collection

The CLD tests were administered to groups of 20 to 30 children with the exception that first-grade children were tested in small groups of five to eight children in order to reduce distractibility and, in general, to enable the test administrator to supervise the test-taking situation more closely.

Not all tests were administered to every longitudinal group of children. Tests responded to on a chance basis in kindergarten, Grade 1, or Grade 2 because of their difficulty were not administered in those grades; also tests that near 100 percent of the students at Grade 6 or Grade 9 got correct were not administered at Grade 7 or Grade 10, respectively. These decisions were based on results of administering the tests to similar samples of students in another school district. Table A2-1 in the appendix indicates the grades at which tests were and were not administered to the four longitudinal groups of students of the present study.

During the first year of data collection two test administrators were responsible for testing. Thereafter, the same person administered all tests. The same two administrators also scored the tests and coded the test information for subsequent data analyses. Following each yearly administration of the tests, the data were analyzed cross-sectionally (e.g., Grades 1, 4, 7, 10; Grades 2, 5, 8, 11) for sex differences in performance (Klausmeier, Sipple, & Allen, 1974b; Klausmeier, Allen, Sipple, & White, 1976a; Klausmeier, Allen, Sipple, & White, 1976b). No systematic sex differences were found. Therefore the data to be presented subsequently in Chapters 3 and 4 were not analyzed separately for boys and for girls.

Reliability of the Tests

The reliability of the tests for each of the three years they were administered to the longitudinal groups was assessed with Hoyt reliability estimates based on the tests administered each year. The number of items per test in each battery, the Hoyt reliability estimates, and the numbers of students on which each estimate is based are given in Table 2-3.

In order to eliminate spurious ceiling or floor effects on the statistical reliabilities of the various tests, a procedure was established for determining which grades to include in deriving each coefficient. It was required that the mean number of items correct on a subtest be within the range of 5 to 95 percent of the subtest items. These cutoff points were set in order that grade groups would not be included in which there was very little variance because the test was either very easy or very hard for all the students. All grade groups that met the 5 to 95 percent criterion for any year of the study were then combined for deriving the reliability coefficient for a particular test, the combined levels tests, and the combined uses tests. Carrying out these procedures resulted in the unequal *N*s given in Table 2-3 and also in the lack of coefficients for 22 tests at the concrete, identity, and classificatory levels.

Tests from the *cutting tool* battery follow as examples of applying the procedure. All four grade groups, that is, A 1-3, B 4-6, C 7-9, D 10-12, in all years were administered the classificatory level test, but the means for the students of Grades 3, 6, 9, and 12 who were administered the test in 1976 exceeded 7.6, or 95 percent. These four grade groups were eliminated because of the ceiling effect, and consequently, a Hoyt reliability coefficient was not calculated for the classificatory test of *cutting tool* in 1976. All grade groups in each year, $N = 292$, were also administered the formal test. The mean score of each grade was within the range of 5 to 95 percent; therefore, all grades were included in computing this coefficient. Related to the coefficients for the four combined levels of *cutting tool*, 139 first- and fourth-grade students in 1974 and the same 139 students in 1975 as second-graders and fifth-graders took all four tests; their mean scores fell within the 5 to 95 percent range, and they were included in computing the coefficient. Students of Grades 7 to 12 were not administered the concrete and identity level tests so they were not included.

Unavoidable limitations of this procedure are apparent. First, since there are no estimates for 22 tests, their reliabilities may be inferred only when included as part of a combined test. Second, when some of the mean scores were close to the cutoff point, the variance generally was small and the derived reliability coefficient was also quite low. Examples of low coefficients related to easy tests or small *N*s are as follows: *Noun*, concrete level in 1974, $N = 62$, coefficient = .39; *cutting tool*, combined levels test in 1976, $N = 62$, coefficient = .49.

The effects on reliability of a range of scores on a test is indicated in Table 2-4, drawn from an experiment carried out by Garguilo (1974) in which the *equilateral triangle* battery was used. The four levels tests were administered to 80 educable mentally retarded children whose MAs ranged from 5.0 to 10.0 and 80 normally developing younger children of comparable mental age. The Hoyt coefficients obtained were higher for the concrete and identity levels, which were of appropriate difficulty for these children, than for the classificatory and formal levels, which were more difficult.

Hoyt reliability coefficients were also obtained in 1974 when the tests were first administered to the longitudinal blocks of students (DiLuzio, Katzenmeyer, & Klausmeier, 1975a, 1975b, 1975c, 1975d). They are reported for the combined grades in Table A2-2 in the appendix. These coefficients were based on larger numbers of students (including those who dropped out later) and did not take into account the ceiling and floor effects with the 95 and 5 percent criterion. For these reasons the coefficients in Table A2-2 in the appendix differ from those reported in Table 2-3.

One may infer from the reliability coefficients based on the four longitudinal groups and the Garguilo experiment that the tests are reasonably reliable and generally acceptable for comparing the mean performances of groups. However, any individual's pattern of growth related to any area measured by these tests must be interpreted with caution, particularly when the percent the individual gets correct is quite high or quite low.

Table 2-3. Hoyt Reliability Coefficients and Standard Errors of Measurement for Separate Tests and Combined Tests[a]

	Year								
	1, 1974			2, 1975			3, 1976		
	Grades 1, 4, 7, 10			Grades 2, 5, 8, 11			Grades 3, 6, 9, 12		
Test and Battery	*N*	*Hoyt*	SE_M	*N*	*Hoyt*	SE_M	*N*	*Hoyt*	SE_M
Concrete									
Cutting tool (8 items)	*			*			*		
Tree (8 items)	*			*			*		
Equilateral triangle (8 items)	*			*			*		
Noun (8 items)	62	.39	.79	*			*		
Identity									
Cutting tool (8 items)	*			*			*		
Tree (8 items)	*			*			*		
Equilateral triangle (8 items)	*			*			*		
Noun (8 items)	62	.67	1.01	62	.70	.59	62	.67	.49
Classificatory									
Cutting tool (8 items)	219	.72	.50	142	.63	.46	*		
Tree (8 items)	219	.82	.68	139	.66	.57	62	.90	.39
Equilateral triangle (3 items)	219	.46	.34	139	.48	.24	*		
Noun (8 items)	230	.76	.90	292	.76	.89	219	.77	.80
Formal									
Cutting tool (18 items)	292	.77	1.38	292	.74	1.34	292	.81	1.08
Tree (35 items)	292	.91	2.09	292	.86	2.08	292	.92	2.07
Equilateral triangle (18 items)	292	.84	1.62	292	.82	1.60	292	.83	1.34
Noun (25 items)	230	.85	2.07	292	.92	1.91	292	.87	2.06
Combined levels									
Cutting tool (42 items)	139	.73	1.70	139	.66	1.61	62	.49	1.61
Tree (59 items)	139	.86	2.54	139	.82	2.52	139	.88	2.56
Equilateral triangle (37 items)	139	.70	2.02	139	.71	1.92	139	.76	1.64
Noun (49 items)	157	.82	2.44	139	.90	2.25	139	.79	2.47

Principle									
Cutting tool (10 items)	292	.72	1.26	292	.76	1.26	292	.75	1.10
Tree (20 items)	230	.85	1.79	292	.92	1.68	292	.91	1.72
Equilateral triangle (11 items)	230	.78	1.36	230	.73	1.34	292	.76	1.25
Noun (10 items)	230	.70	1.30	230	.69	1.29	292	.77	1.29
Taxonomic relations									
Cutting tool (8 items)	292	.58	.89	292	.58	.90	292	.70	.75
Tree (8 items)	292	.72	1.01	292	.65	1.03	292	.76	.95
Equilateral triangle (8 items)	292	.67	1.10	292	.69	1.07	292	.79	.95
Noun (8 items)	230	.62	1.08	230	.61	1.05	292	.64	1.13
Problem solving									
Cutting tool (5 items)	292	.59	.80	292	.55	.81	292	.43	.69
Tree (10 items)	292	.68	1.24	292	.67	1.28	292	.75	1.21
Equilateral triangle (6 items)	230	.66	.91	230	.65	.89	292	.66	.82
Noun (5 items)	230	.52	.84	230	.41	.87	292	.60	.86
Combined uses									
Cutting tool (23 items)	292	.81	1.86	292	.83	1.87	292	.81	1.63
Tree (38 items)	230	.89	2.51	292	.93	2.47	292	.93	2.42
Equilateral triangle (25 items)*	230	.85	2.09	230	.84	2.04	292	.86	1.92
Noun (23 items)	230	.83	1.99	230	.81	1.97	292	.86	2.02

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[a] An asterisk indicates that mean items correct for each block equalled or exceeded 95% of subtest items.

Table 2-4. Hoyt Reliability Coefficients for the Levels Tests of *Equilateral Triangle*[a]

	Combined Group (N = 160)	*Normal (N = 80)*	*EMR (N = 80)*
Concrete level	.82	.80	.83
Identity level	.72	.79	.67
Classificatory level	.65	.55	.76
Formal level	.26	.27	.25

[a] Based on Gargiulo, 1974.

Empirical Curve Fitting

One purpose of the study was to chart the course of cognitive growth throughout the school years. The means of portraying normative growth was to construct curves that show the form and rate of growth, Grades 1-12. Representative illustrative curves are presented in Chapter 3.

Some combined effects of retesting and cohort differences in the third year were found, which in most cases resulted in higher means for the students who had experienced retesting when in Grades 3, 6, and 9 than for those who were tested for the first time when in Grades 4, 7, and 10. These effects tended to distort curves based on these observed means for these grades. Empirical curve fitting, rather than simply connecting the obtained means, was employed to eliminate these distortions.

The empirical curve-fitting process employed in the study followed a two-step sequence. First, the means obtained on each test administered in Grades 1-12 were graphed. The means for each test were connected, as shown in Figure 2-1 for three tests. The resulting curves for each test, one for each of the four longitudinal blocks of students, were analyzed and compared. For a test where the mean of the students in Grades 1 or 2 was below 40 percent correct and where the mean at Grade 6 or later approached 95 percent, the curve across blocks, when connected, was of the form of a quadratic. For a test where the mean was near or below 40 percent correct in Grades 1 or 2 and where the mean in Grade 12 was near 85 percent or less, the curve across blocks, when connected, was of the form of a natural log curve. For a test where the mean in Grade 1 was near 70 percent or higher but a level of 95 percent or higher was not reached until Grade 6 or thereafter, the curve across blocks, when connected, was of the form of a straight line. A straight line also best fits a set of means that show the same amount of increase across each grade within each longitudinal block.

Second, based on these analyses of the observed means, empirical curve fitting was performed in the following manner. The obtained means from each test, changed to percentages, were used in deriving a smoothed curve. A computer program involving regression analysis was employed that empirically determined whether the curve based on a set of obtained means was more nearly of the form of a straight line, a quadratic curve, or natural log curve.[2] The best fitting curve of the three was identified by determining the least difference between the sum of the squares of the deviations of the predicted values from the observed values. In turn, the predicted values were plotted and the smoothed curves were drawn by connecting the predicted values.

Figure 2-1 shows the curves that connect only the three observed means for each longitudinal block and the smoothed curves that connect all grades. In this figure and in all figures in which smoothed curves are presented in this book, the obtained means, having been changed to percentages, permit all "*y*" axes to be of identical length. All "*x*" axes showing the twelve grades are also of identical length. This procedure was followed to facilitate direct comparison of various sets of curves.

Earlier in this section, possible sources of distortion of the true developmental curves were stated. In Figure 2-1 distortions due primarily to the effects of repeated testing, being born later, or both, are reflected in the *observed* means for *equilateral triangle* and *noun* between Grades 3 and 4, 6 and 7, and 9 and 10. Distortions primarily due to error of measurement are illustrated in the observed means for *cutting tool* between Grades 1, 2, and 3 and between Grades 4 and 5. The effects of repeated testing, being born later, or both, as well as error of measurement are reflected in the higher mean for *cutting tool* in Grade 6 than in Grade 7.

The three smoothed curves shown in Figure 2-1 and those that will be presented in Chapter 3 are hypothesized to reflect reliably developmental trends for children and youth currently enrolled in American schools, Grades 1 through 12, where the dropout rate does not exceed 5 to 10 percent annually, where mean school achievements are near the median on national norms, and where the population of the school reflects that of the nation in racial mix and socioeconomic status.

It may be well to recall that all the information presented in this section deals only with the CLD study and that the related results

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[2] The REGAN 3 computer program used was part of the STATJOB package, 1977, Madison Academic Computing Center, University of Wisconsin, Madison, Wisconsin.

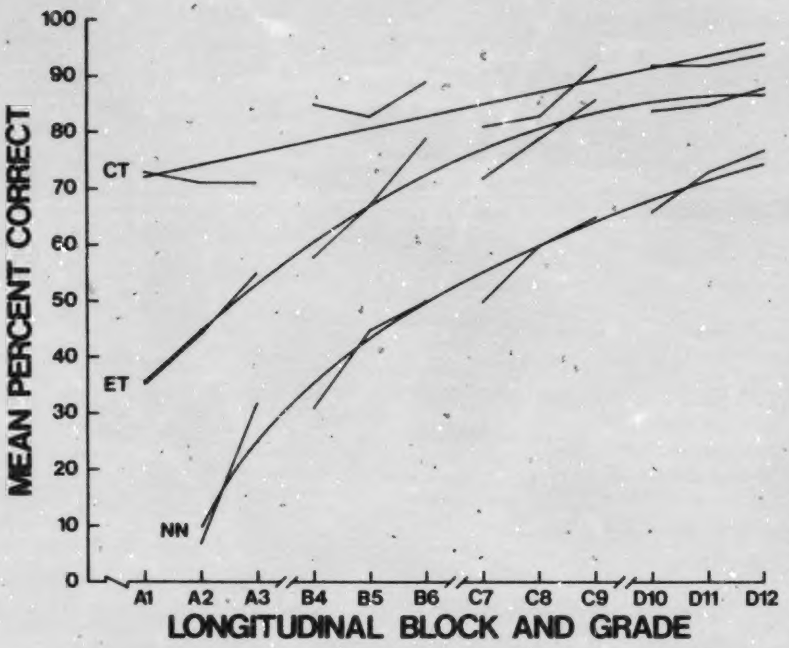

Figure 2-1. Observed Means and Smoothed Curves for *Cutting Tool,* Taxonomic Relations (Straight Line); *Equilateral Triangle,* Formal (Quadratic Curve); and *Noun,* Formal (Natural Log Curve) (Klausmeier & Allen, Chapter 2, Figure 2-1, 1978)

are presented in Chapters 3 and 4. As will be explained in the next section, smaller groups of students that were samples of the larger numbers in this CLD study were administered the Piagetian tests. The relationships between the CLD and Piagetian dimensions of development presented in Chapter 6 are based on the smaller samples.

DESIGN AND METHODS OF THE PIAGETIAN STUDY

The primary objective of the combined longitudinal and cross-sectional study based on Piagetian theory is the normative assessment and critical analysis of logical concept development in children during middle childhood. This period of development is designated by Piaget as the concrete operations stage.

In the first year of the study, 30 boys and 30 girls each from kindergarten, Grade 3, and Grade 6 were drawn randomly from the 50 boys and 50 girls who composed the total longitudinal groups reported earlier in this chapter. They were assessed for the first time in 1973 with a series of logical concept tasks, three tests of short-term memory, and two arithmetic computation tests. The students who remained and were available for testing in each of the following three years were administered the same tests they received in the first year. In the fourth and final year of the study, 134 of the original 180 children remained in the study.

Control groups for evaluating the effects of repeated testing were introduced in 1974 and 1975. There were no control groups to assess possible cohort biases. The sampling design of the Piagetian study is given in Table 2-5. The measures used in the study included a battery of 13 familiar Piagetian-based tests, 16 tests developed by Brainerd to assess Piaget's groupement structure of classes and relations (Brainerd, 1972b), three tests of short-term memory, and two arithmetic tests.

Familiar Piagetian Tasks

Concrete operations tasks were adapted from the procedures and instruments used in previous investigations based on Piagetian theory. These include dichotomous sorting, some-all relationships, class inclusion, cardinality, seriation, transitivity of length, transitivity of weight, identity conservation of length, equivalence conservation of length, identity conservation of weight, equivalence

Table 2-5. Sampling Design for the Piagetian Study

Cohort	*Time of Measurement*[a]						
	1973		*1974*		*1975*		*1976*
1967	6 (*N* = 60) [Kindergarten]	→	7 (*N* = 48)	→	8 (*N* = 46)	→	9 (*N* = 41)
1967			7 (*N* = 24)				
1967					8 (*N* = 20)		
1964	9 (*N* = 60) [Third Grade]	→	10 (*N* = 54)	→	11 (*N* = 50)	→	12 (*N* = 47)
1964			10 (*N* = 24)				
1964					11 (*N* = 22)		
1961	12 (*N* = 60) [Sixth Grade]	→	13 (*N* = 54)	→	14 (*N* = 52)	→	15 (*N* = 46)
1961			13 (*N* = 24)				
1961					14 (*N* = 24)		

[a] Underlined entries are approximate mean ages in years, and arrows represent continuity of subgroups.

conservation of weight, and conservation of number. A test of combinatorial reasoning, associated with the formal operations period, was also adapted. (An arithmetic computation test and three memory tests not based on Piagetian theory were also administered: auditory memory span (word series; Wepman & Morency, 1973a), auditory sequential memory (digit span; Wepman & Morency, 1973b), and visual orientation memory (Swinton, 1973).) Descriptions of the various Piagetian tests and their sources follow.

Classification Tasks

1. Dichotomous Sorting (Score Range = 0–3)

The *S* is asked to sort 22 3" × 5" cards into two groups, according to one of three dimensions, regroup the cards on another of the dimensions, and finally regroup them on the third dimension. Each card contains a picture of either one or two (number dimension) squares or circles (shape dimension). The squares and circles on each card are either red or blue (color dimension). 3 items. Adapted from Kamii and Peper (1969).

2. Some-All Relationships (Score Range = 0–4)

In reference to an array of nine pictorial stimuli (three red and two blue triangles and four blue circles), the *S* is asked questions representing each of the following two relationships: (1) *All* members of a given subclass are members of the superordinate class (e.g., "Are all the red figures triangles?"). 2 items. (2) *Some* but not all members of a superordinate class are members of a given subclass (e.g., "Are all the blue figures circles?") 2 items. Adapted from Kofsky (1966).

3. Class Inclusion (Score Range = 0–5)

a. In reference to an array of five pictorial stimuli (three red and two blue triangles), the *S* is asked whether there are more members of the superordinate class than members of the nested subclass (e.g., "Are there more triangles or more red figures?"). (The procedure is repeated in another item using circles.) 2 items.

b. In reference to an array of nine pictorial stimuli (three red and two blue triangles and four blue circles), the *S* is asked whether there are more members of the superordinate class than members of the nested subclass (e.g., "Are there more blue figures or more circles?"). (The procedure is repeated in another item with the same stimuli using triangles as the superordinate class and red figures as the nested subclass.) 2 items.

c. In reference to the same stimuli used in (b), the *S* is asked which of two intersecting classes has more members ("Are there more blue figures or more triangles?"). 1 item. 5 total class inclusion items. Adapted from Kofsky (1966).

4. Combinatorial Reasoning (Score Range = 0-28)

The *S* is asked to form all possible (28) pairs of eight differently colored chips without repeating pairs. Adapted from Goodnow (1962).

5. Cardinality (Score Range = 0-5)

The *S* is asked to judge the equivalence or nonequivalence of number of (a) two parallel rows of dots of unequal number dispersed over equal lengths, (b) two parallel rows of dots of unequal number dispersed over unequal lengths, (c) two parallel rows of dots of equal number dispersed over unequal lengths, (d) two parallel rows of dots of equal number dispersed over equal lengths, and (e) two single digit numbers presented orally. 5 judgments. Adapted from Brainerd (1973b) and Gonchar (1975).

Relations Tasks

1. Seriation (Score Range = 0-5)

a. Serial Construction

The *S* is asked to order (1) an array of seven sticks that differ in length (by one centimeter increments), and (2) an array of 10 circles that differ in diameter (by three centimeter increments). Adapted from Elkind (1964).

b. Additive Seriation

The *S* is asked to insert three additional sticks into their proper place in an existing ordered array of seven sticks. Adapted from Elkind (1964).

c. Serial Correspondence: Matching Case

Given a seriated array of sticks and a seriated array of circles, the *S* is asked to match each member of one seriated array with the corresponding member of the other seriated array. Adapted from Coxford (1964).

d. Serial Correspondence: Extension and Compression Cases

The members of one seriated array are first extended, then compressed, and under each condition the *S* is asked to match three members of the deformed array to the corresponding members of a second, unmanipulated, seriated array. Adapted from Coxford (1964).

e. Serial Correspondence: Scrambled Case

The members of one seriated array are reoriented in a scrambled fashion, and the *S* is asked to match three members of the deformed array to the corresponding members of a second, unmanipulated, seriated array. Adapted from Coxford (1964).

2. Transitivity of Length (Score Range = 0-5)

The *S* compares (1) one stick A to another stick B in terms of their equal lengths, (2) stick A to a third stick C in terms of their unequal lengths, and is asked to judge the inequality of length of sticks B and C. 5 judgments. Adapted from Brainerd (1973c).

3. **Transitivity of Weight (Score Range = 0-5)**

The *S* compares (1) one clay ball A to another clay ball B in terms of their equal weights, (2) clay ball A to a third clay ball C in terms of their unequal weights, and is asked to judge the inequality of weight of clay balls B and C. 5 judgments. Adapted from Brainerd (1973c).

Conservation Tasks

1. **Identity Conservation of Length (Score Range = 0-6)**

The *S* is asked to (1) predict the length of a single string to be the same if the string was perceptually altered (3 predictions, one followed by an explanation inquiry), and (2) judge the length of the string to be the same after being perceptually altered (3 judgments, one followed by an explanation inquiry). 6 items. Adapted from Hooper (1969) and Brainerd (1973c).

2. **Equivalence Conservation of Length (Score Range = 0-6)**

The *S* is asked to (1) predict the length of two perceptually equivalent strings to be equal if one were perceptually altered (3 predictions, one followed by an explanation inquiry), and (2) judge the length of the strings to be equal after one has been perceptually altered (3 judgments, one followed by an explanation inquiry). 6 items. Adapted from Brainerd (1973c).

3. **Identity Conservation of Weight (Score Range = 0-6)**

The *S* is asked to (1) predict the weight of a piece of clay to be the same if the clay was perceptually altered (3 predictions, one followed by an explanation inquiry), and (2) judge the weight of the clay ball to be the same after being perceptually altered (3 judgments, one followed by an explanation inquiry). 6 items. Adapted from Hooper (1969) and Brainerd (1973c).

4. **Equivalence Conservation of Weight (Score Range = 0-6)**

The *S* is asked to (1) predict the weight of two perceptually equivalent clay balls to be equal if one were perceptually altered (3 predictions, one followed by an explanation inquiry), and (2) judge the weight of the clay balls to be equal after one has been perceptually altered (3 judgments, one followed by an explanation inquiry). 6 items. Adapted from Brainerd (1973c)

5. **Conservation of Number (Score Range = 0-7)**

The *S* is asked to (1) predict the number of two sets of chips to be equal if one of the sets were perceptually altered (3 predictions, one followed by an explanation inquiry). (2) judge the number of each of the sets to be equal after one has been perceptually altered (3 judgments, one followed by an explanation inquiry), and (3) judge the number of the sets to be unequal after one chip has been removed from one of the sets. 7 items. Adapted from Brainerd and Brainerd (1972) and Rothenberg (1969).

Brainerd's Groupement Test Battery

A series of tests was designed by Brainerd (1972b) to assess the logical groupements associated with the concrete operations period of middle childhood. Sixty-four binary choice items were constructed; one-half to assess the operation of logical composition and the other half to assess the inverse operation. This results in 16 different four-item tests; composition and inversion of each of eight groupements.

A description of the groupement tasks with statements of the corresponding logical operations are given below. For complete administration procedures and associated scoring protocols see *A Representative Series of Piagetian Concrete Operations Tasks* (Hooper, Brainerd, and Sipple, 1975).

Groupment I: Addition and Subtraction of Primary Classes

Operations: A. Composition (addition) of primary classes A (triangles with yellow) + A′ (triangles without yellow) = B (triangles)

Questions: 1. Are there the same number of triangles with *yellow* on them as there are triangles?

2. Are there *more* triangles than there are triangles with *yellow* on them?

B. Inversion (substraction) of primary classes B (triangles) - A (triangles with yellow) = A′ (triangles without yellow)

Questions: 3. If I took away the triangles with *yellow* on them would there be some triangles left?

4. If I took away the triangles with *yellow* on them would all the triangles be gone?

Stimulus Materials: See Figure 2-2 for stimuli.

Four questions comparable to questions 1 through 4 using circular stimuli are also included.

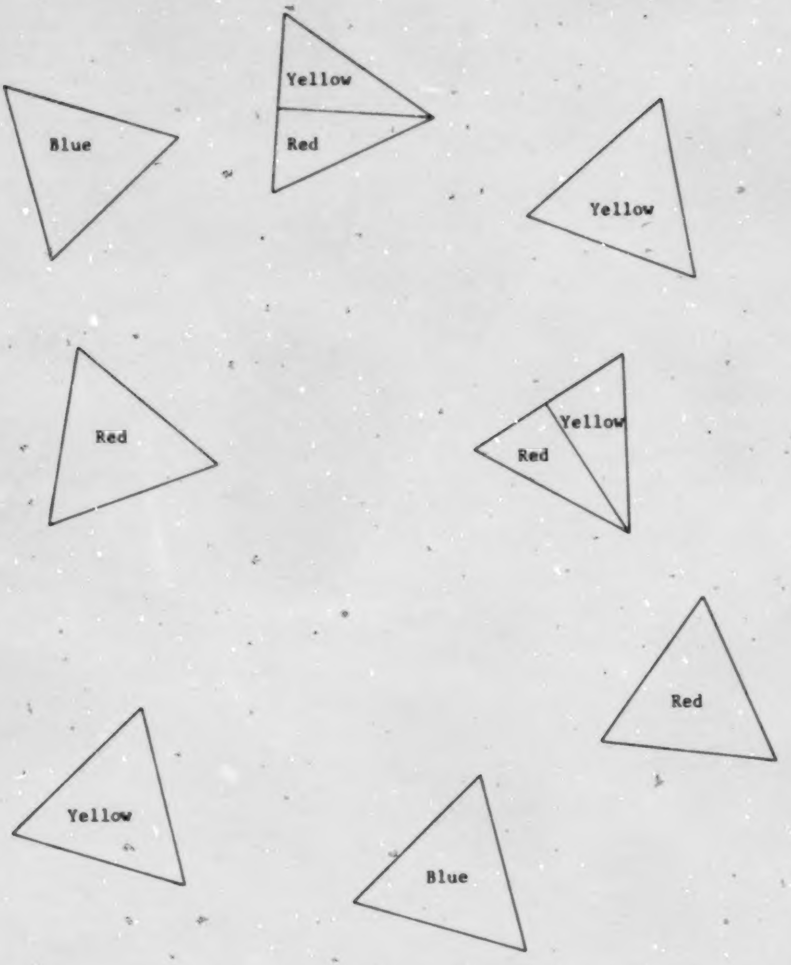
Blue
Yellow
Red
Yellow
Red
Yellow
Red
Red
Yellow
Blue

Figure 2-2. Stimuli for Groupements I, II, and III

Groupement II: Addition and Subtraction of Secondary Classes

Operations: A. Composition (addition) of secondary classes A_2 (triangles with red) + A_2' (triangles without red) = B (triangles)

Questions: 1. Are there the same number of triangles with *red* on them as there are triangles?

2. Are there more triangles than there are triangles with *red* on them?

B. Inversion (subtraction) of secondary classes B (triangles) - A_2 (triangles with red) = A_2' (triangles without red)

Questions: 3. If I took away the triangles with *red* on them, would there be some triangles left?

4. If I took away the triangles with *red* on them, would all the triangles be gone?

Stimulus Materials: Same triangles as used in Groupement I. See Figure 2-2.

Four questions comparable to questions 1 through 4 using circular stimuli are also included.

Groupement III: Biunivocal Multiplication and Division of Classes

Operations: A. One-to-one composition (multiplication) of classes A_1 (triangles with yellow) × A_2 (triangles with red) = $A_1 A_2$ (triangles with both yellow and red)

Questions: 1. Are there the same number of triangles with *yellow* on them as there are triangles with both *red and yellow* on them?

2. Are there more triangles with *yellow* on them than there are triangles with both *red and yellow* on them?

B. Inversion (division) of a product class by one class $A_1 A_2$ (triangles with both yellow and red) ÷ A_1 (triangles with yellow) = A_2 (triangles with red)

Questions: 3. If I took away the triangles with *yellow* on them would there be any triangles with both *red and yellow* on them left?

4. If I took away the triangles with *yellow* on them, would all the triangles with both *red and yellow* on them be gone?

Stimulus Materials: Same triangles as used in Groupements I and

II. See Figure 2-2.

Four questions comparable to questions 1 through 4 using circular stimuli are also included.

Groupement IV: Counivocal Multiplication and Division of Classes

Operations: A. One-to-many composition (multiplication) of classes K_1 (triangles (A_1 = red, B_1 = yellow)) × K_2 (yellow (A_2 = triangles, A_2' = circles)) = $A_1 A_2$ (red triangles) + $B_1 A_2$ (yellow triangles) + $B_1 A_2'$ (yellow circles)

Questions: 1. Are there the same number of *yellow* triangles as *yellow* things?

2. Are there more *yellow* things than *yellow* triangles?

B. Inversion (division) of product classes by one class ($A_1 A_2$ (red triangles) + $B_1 A_2$ (yellow triangles) + $B_1 A_2'$ (yellow circles)) ÷ (B_1 (yellow) + A_2 (triangles) + A_2' (circles)) = A_1 (red).

Questions: 3. If I took away the *yellow* things, would there be any *yellow* triangles left?

4. If I took away the *yellow* things, would all the *yellow* triangles be gone?

Stimulus Materials: See Figure 2-3 for stimuli.

Four questions comparable to questions 1 through 4 using an alternate set of geometric figures are also included.

Groupement V: Addition and Reciprocity of Difference Relations

Operations: A. Composition (addition) of difference relations $((A \leftarrow B) + (B \leftarrow C) = (A \leftarrow C))$

Questions: 1. Are the *blue* and *red* sticks the same length?

2. Is the *blue* stick shorter than the *red* stick?

B. Reciprocity of difference relations $((A \leftarrow B) + (B \leftarrow C) = (A \leftarrow C)) + ((C \rightarrow B) + (B \rightarrow A) = (C \rightarrow A)) = (A = A)$

Questions: 3. Are the *red* and *blue* sticks the same length?

4. Is the *red* stick longer than the blue stick?

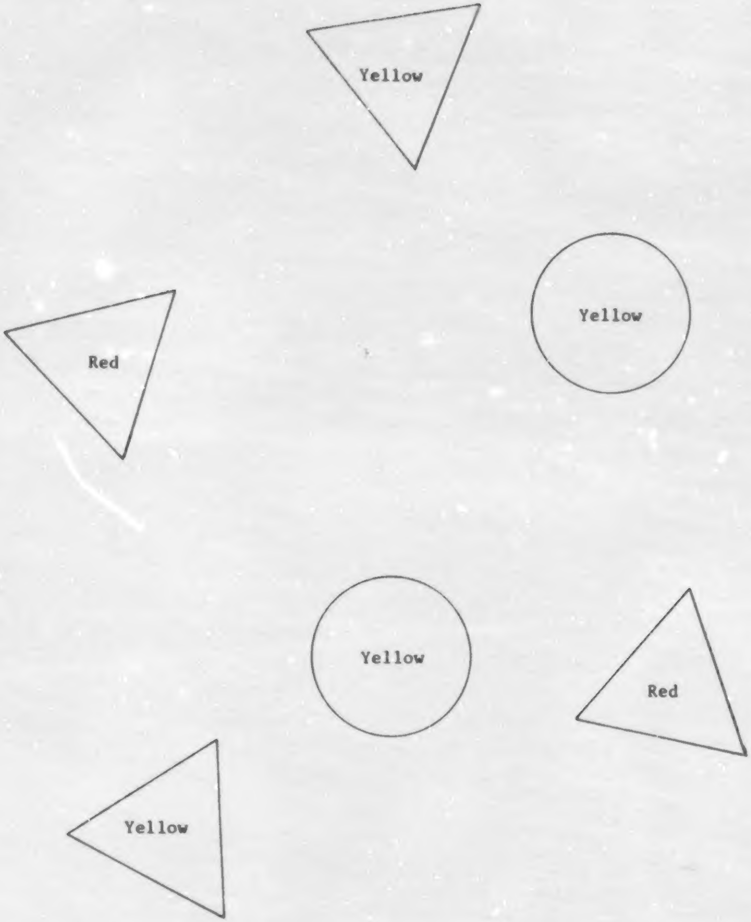

Figure 2-3. Stimuli for Groupement IV

Stimulus Materials: See Figure 2-4 for stimuli.

Four questions comparable to questions 1 through 4 are asked of the weights of the same sticks.

Groupement VI: Addition and Reciprocity of Equivalence Relations

Operations: A. Composition (addition) of equivalence relations

$(A \leftrightarrow B) + (B \leftrightarrow C) = (A \leftrightarrow C)$

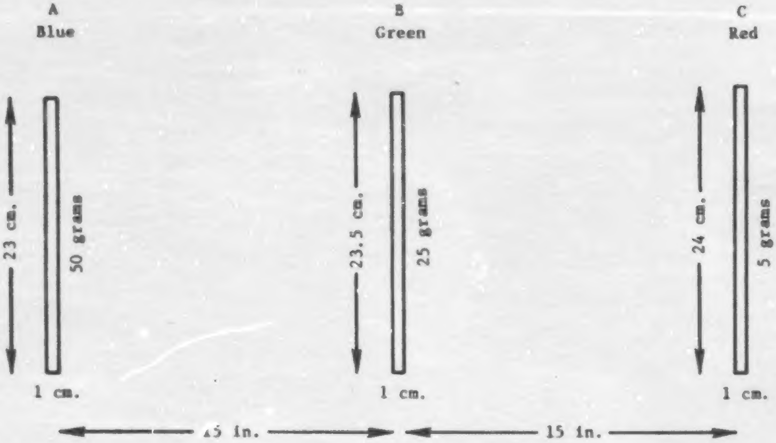

Figure 2-4. Drawing of Stimuli for Groupement V

Questions: 1. Are the *blue* and *red* sticks the same length?

2. Is the *blue* stick shorter than the *red* stick?

B. Reciprocity of equivalence relations $((A \leftrightarrow B) + (B \leftrightarrow C) = (A \leftrightarrow C)) + (C \leftrightarrow B) + (B \leftrightarrow A) = (C \leftrightarrow A)) = (A \leftrightarrow A)$

Questions: 3. Are the *red* and *blue* sticks the same length?

4. Is the *red* stick longer than the *blue* stick?

Stimulus Materials: See Figure 2-5 for stimuli.

Four questions comparable to questions 1 through 4 are asked of the weights of the same sticks.

Groupement VII: Biunivocal Multiplication and Reciprocity of Relations

Operations: A. Composition (multiplication) of difference relations $(A \leftarrow\downarrow B) \times (B \leftarrow\downarrow C) = (A \leftarrow\downarrow C)$

Questions: 1. Is the *red* stick shorter than the *green* stick?

2. Is the *red* stick lighter than the *green* stick?

B. Reciprocity of difference relations $((C \rightarrow\uparrow B) \times (B \rightarrow\uparrow A)) \div (C \rightarrow\uparrow A) = A \overset{\circ}{\rightarrow}\uparrow^{\circ} A$

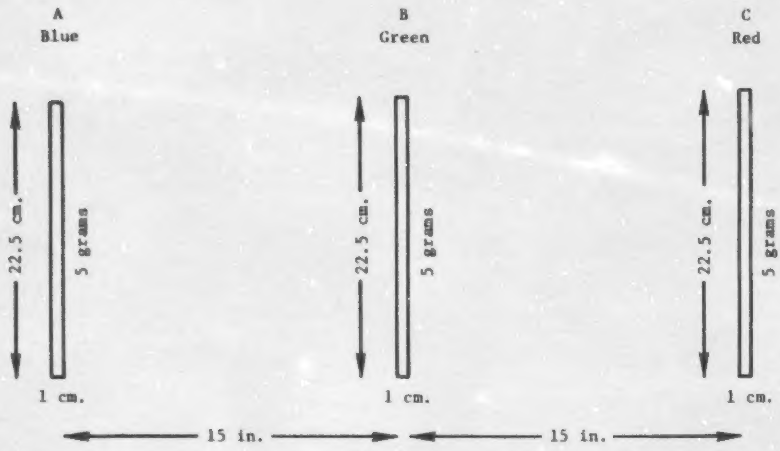

Figure 2-5. Drawing of Stimuli for Groupement VI

Questions: 3. Is the *green* stick longer than the *red* stick?

4. Is the *green* stick heavier than the *red* stick?

Stimulus Materials: See Figure 2-6 for stimuli.

Four supplementary questions are also included.

Groupement VIII: Counivocal Multiplication and Reciprocity of Relations

Operations: A. Composition (multiplication) of difference relations by equivalence relations $(A \leftrightarrow\downarrow B) \times (B \leftrightarrow\downarrow C) = (A \leftrightarrow\downarrow C)$

Questions: 1. Do the *green* and *blue* sticks weigh the same?

2. Is the *green* stick shorter than the *blue* stick?

B. Reciprocity of difference relations by equivalence relations $((C \leftrightarrow\uparrow B) \times (B \leftrightarrow\uparrow A)) \div (C \leftrightarrow\uparrow A) = (A \overset{0}{\leftrightarrow}\uparrow_{\circ} A)$

Questions: 3. Do the *blue* and *green* sticks weigh the same?

4. Is the *blue* stick longer than the *green* stick?

Stimulus Materials: See Figure 2-7 for stimuli.

Four supplementary questions are also included.

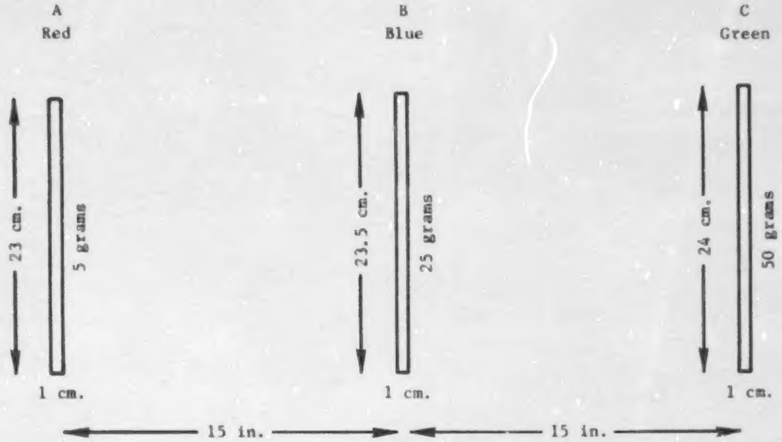
A
Red
23 cm.
5 grams
1 cm.
B
Blue
23.5 cm.
25 grams
1 cm.
C
Green
24 cm.
50 grams
1 cm.
15 in.
15 in.

Figure 2-6. Drawing of Stimuli for Groupement VII

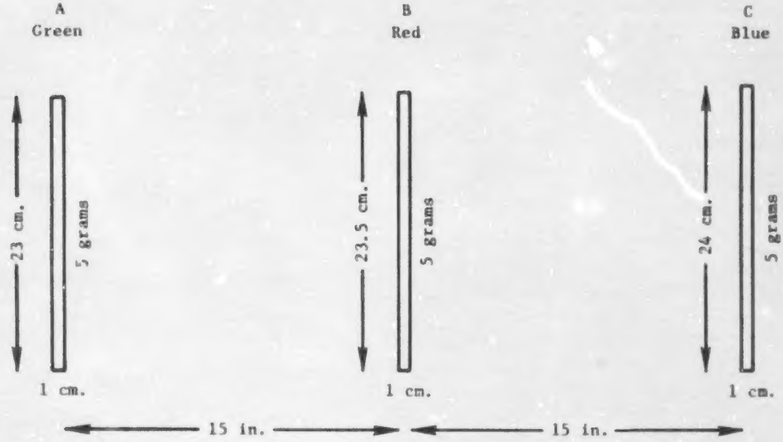
A
Green
23 cm.
5 grams
1 cm.
B
Red
23.5 cm.
5 grams
1 cm.
C
Blue
24 cm.
5 grams
1 cm.
15 in.
15 in.

Figure 2-7. Drawing of Stimuli for Groupement VIII

Data Collection and Treatment

The familiar Piagetian tasks, the tasks developed by Brainerd, and the arithmetic and short-term memory tests were administered individually to the students in the study. The only familiar Piagetian tasks administered to the children designated longitudinal Group C were class inclusion and combinatorial reasoning. Children of all three groups received Brainerd's groupement tests and the short-term memory tests.

Data were quantified to compute means and standard deviations on each test for each year of the study. Analyses of variance were run to determine potential effects of selective survival, repeated testing, and sex differences. Analyses of the four years of longitudinal data were also conducted to examine changes in the performances of the longitudinal sample with increasing age. Finally, trend analyses and cross-lagged panel analyses were conducted. Selective summaries of these analyses are reported in Chapter 5.

Administration of the Tasks. In contrast to the CLD assessment procedures, all the Piagetian-based measures were administered individually and an attempt was made to retain the desirable flexibility features of the *méthode clinique.* Thus, following the establishment of essential rapport with the subject, the tasks were administered in a relaxed, gamelike atmosphere. No explicit corrective reinforcement was introduced except in the warm-up sessions. Questions were restated if the child's responses were ambiguous or if an obvious misunderstanding of the criterial question was evident. The tasks were presented in three sessions[3] as follows:

Session One: (A) Seriation, (B) conservation of number, (C) arithmetic addition, (D) cardinality, and (E) dichotomous sorting.[4]

Session Two: (A) Memory tests (visual orientation memory, auditory memory span (word series), and auditory sequential memory (digital span), (B) groupement tasks, (C) combinatorial reasoning, and (D) some-all relationships.

Session Three: (A) conservation and transitivity tasks,[5] and (B) class inclusion.

Reliability of the Tests

The reliability of the familiar Piagetian tasks for the first year of testing, 1973, and the fourth and final year of testing, 1976, was assessed by means of Hoyt reliability coefficients including only the subjects who participated in all four annual assessments. The number of items per test, the reliability coefficients, and the standard errors of measurement are given in Table 2-6. The Hoyt reliability values for the familiar Piagetian tasks in 1973 ranged from .62 to .95 in all cases except some-all relationships, which was .27. The values for 1976 ranged from .56 to .98, except for seriation (.25), some-all relationships (.31), and cardinality (.36). Reliability coefficients were not obtained for dichotomous sorting and combinatorial reasoning because of the scoring methods employed.

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[3] The battery of tests given to the Group C students, e.g., the memory tests, the groupement tests, combinatorial reasoning, and class inclusion, was administered in a single session.

[4] The tasks in the initial session were presented in one of the following four orders of presentation: E, D, A, B, C; A, C, D, B, E; E, B, D, C, A; C, B, A, D, E.

[5] The identity conservation, equivalence conservation, and transitivity tasks were presented in one of six counterbalanced orders of presentation. The length cases always preceded the weight cases.

Table 2-7 presents the reliability coefficients for each of Brainerd's eight groupement tasks. The Hoyt values ranged from .59 to .81 based on 1973 data and .64 to .87 using 1976 data.

TESTS USED AND STUDENTS PARTICIPATING IN THE RELATIONSHIP STUDY

In order to identify the relationships among dimensions of growth as represented by the two theories of cognitive development, results of the CLD, Piagetian, and groupement tests that were administered to the same students were used. It will be recalled that only samples of students of three of the longitudinal blocks received both the CLD and groupement batteries. These same samples received the Piagetian

Table 2-6. Hoyt Reliability Coefficients and Standard Errors of Measurement for Familiar Piagetian Tests for Year 1 and Year 4 of the Study Based on Scores of the Students Tested All Four Years[a]

N	*Subscale*	*No. of Items*	*Year 1*		*Year 4*	
			Hoyt	SE_M	*Hoyt*	SE_M
88	Seriation	5	.62	.70	.25	.61
88	Transitivity of Length	5	.94	.39	.94	.23
88	Transitivity of Weight	5	.86	.33	.96	.16
88	Some-All	4	.27	.41	.31	.28
88	Cardinality	5	.72	.72	.36	.69
88	Conservation of Number	7	.86	.74	.98	.16
88	Identity Conservation of Length	6	.84	.68	.85	.40
88	Equivalence Conservation of Length	6	.94	.58	.92	.33
88	Identity Conservation of Weight	6	.82	.75	.88	.41
88	Equivalence Conservation of Weight	6	.95	.54	.91	.44
134	Class Inclusion	5	.65	.75	.56	.52

[a] Reliability coefficients could not be obtained for the dichotomous sorting and combinatorial reasoning tests because of the scoring methods used.

Table 2-7. Hoyt Reliability Coefficients and Standard Errors of Measurement for Eight Groupement Tests for Year 1 and Year 4 of the Study Based on Scores of Students Tested All Four Years (*N* = 134)

Test	*No. of Items*	*Year 1*		*Year 4*	
		Hoyt	SE_M	*Hoyt*	SE_M
Groupement I: Addition and Subtraction of Primary Classes	8	.59	.92	.73	.80
Groupement II: Addition and Subtraction of Secondary Classes	8	.66	.92	.77	.76
Groupement III: Biunivocal Multiplication and Division of Classes	8	.70	1.13	.76	1.04
Groupement IV: Counivocal Multiplication and Division of Classes	8	.76	1.02	.87	.61
Groupement V: Addition and Reciprocity of Difference Relations	8	.70	.78	.64	.52
Groupement VI: Addition and Reciprocity of Equivalence Relations	8	.81	.63	.68	.20
Groupement VII: Biunivocal Multiplication and Reciprocity of Relations	8	.76	.95	.77	.58
Groupement VIII: Counivocal Multiplication and Reciprocity of Relations	8	.63	1.01	.75	.69

tests, except that the Group C students (i.e., Grades 6-7-8-9) received only two of the Piagetian measures. It will also be recalled that only one of the four CLD batteries was administered the first year of the study, whereas the complete sets of Piagetian and groupement tests were administered. For these reasons, the number of students and the number of tests that were involved in any kind of data analysis to identify relationships among three test domains and between any two test domains varied considerably.

Achieving certain purposes of the relationship study called for tests to be administered after the first year that the students of a longitudinal group scored near 100 percent correct. This was essential for determining the final year that a cognitive ability or dimension was achieved and also for ascertaining the extent to which individuals maintained mastery of a dimension after first having achieved it. The high scores could be used properly in deriving means, but as the mean percentages correct approached 95 percent correct, the test reliability coefficients became erratic and usually decreased. Tests with reliability coefficients below .20 were regarded as too low for use in computing correlations. Because of this the number of students and the number of tests varied somewhat depending upon the particular analysis conducted.

In accordance with the preceding conditions, from 20 to 28 CLD tests were used in all analyses, except correlational, involving the students of Grades 1-9 but only four from the equilateral triangle battery were used involving kindergarten children. Thirteen familiar Piagetian tests were used in all analyses, except correlational, kindergarten through Grade 6, but only two were used for Grades 6-7-8-9. The 16 groupement tests were used in all analyses, except correlational, kindergarten through Grade 9.

The scores of a total of 126 of the original 180 students comprising the three first year samples were included as follows. There were 37 from the original Grade Kg.-1-2-3 sample of 60, 45 from the Grade 3-4-5-6 sample, and 44 from the Grade 6-7-8-9 sample. The exact number of students and tests used in the specific relationship analyses are given in the tables in Chapter 6.

❋ *Chapter 3*

Principles of Cognitive Development: CLD Theory

Herbert J. Klausmeier

Attaining concepts at increasingly higher levels and gaining a more complete understanding of the principles that express relationships among concepts enable the individual to interpret the physical and social world with greater economy, accuracy, and elegance. Continuously organizing sets of related concepts and principles into more comprehensive conceptual cores permits persons to acquire, organize, store, and retrieve greater amounts of information. Being able to solve problems makes it possible to learn independently without direction from others.

In this chapter the development of these basic intellectual capabilities during the school years is summarized in principles. The principles are based on a large amount of controlled experimentation on concept and principle learning that is given in Klausmeier, Ghatala, and Frayer (1974) and on the results of a longitudinal study (Klausmeier & Allen, 1978) of which the studies reported in this book are integral parts. Throughout this chapter, only illustrative empirical information on which each principle is based is given. Complete information is presented in Chapters 6, 7, 8, 9, and 10 of Klausmeier and Allen (1978). The statements of principles follow in the form of headings of the successive parts of the chapter. Major points of difference between the CLD and Piagetians perspectives are noted.

FOUR SUCCESSIVELY HIGHER LEVELS OF THE SAME CONCEPT ARE ATTAINED IN AN INVARIANT SEQUENCE

Orderliness and invariance are key terms used in describing and explaining sequences of development. Concerning orderliness, several theorists employ constructs that represent development as a progression through a series of qualitatively different and increasingly complex steps, levels, or phases. One such theoretical construct is that of developmental stage. Stages indicate developmental progressions in which each stage emerges sequentially over time. Piaget's six substages of the sensorimotor period are one example of an orderly progression in early cognitive development. Other examples of stage sequential approaches are Kohlberg's (1969) six stages of moral development and Loevinger's and Wessel's (1970) stages of personality development. Stage theories have many other formal properties in addition to that of developmental progressions or sequencing. A comprehensive discussion of stage theories is beyond the scope and intent of this chapter. Extensive discussions of stage theories can be found in various sources, including Brainerd (1978a), Flavell (1971, 1972, 1977), Flavell and Wohlwill (1969), Van den Daele (1969), and Wohlwill (1973).

Many theorists do not accept the demands imposed by a "true" stage theory. For example, McNeill (1970) describes sequences in language development and Bruner and others (1966) describe the enactive-ikonic-symbolic progression in the internal representation of experience. These two developmental progressions simply are sequences for describing the initial emergence, extension, and refinement of a cognitive process or structure rather than true stages.

Invariance implies that the successive parts of a complete sequence appear in a fixed, constant progression. For example, if A, B, and C are identified as three hypothetical cognitive acquisitions in an invariant developmental series, then A must appear before B and B must occur before C. The sequence in attaining A, B, and C does not vary among individuals; it is invariant. However, the time at which A, B, and C emerge initially and the rate at which each reaches full functional maturity may vary among individuals. While the beginning of a particular item of the sequence may appear earlier or later among individuals, the invariant order itself is identical for all individuals.

Flavell (1972), on logical grounds, identified three possible types of explanation for invariant ordering. One is the structure of the organism; that is, the invariant sequence is determined by maturation (e.g., the development of locomotion). Another is environmental; that is, the sequence is determined by the regularities and universal conditions in the environmental conditions that are typically experienced by individuals (e.g., the development of role-taking skills). A final explanation for invariance may be found in the structure of the cognitive acquisitions comprising the sequence. The nature of the acquisitions A and B may be such that A preceding B is logically (and psychologically) necessary, but the possibility of B preceding A must be eliminated purely on logical grounds. As an example, the concrete operations in Piaget's system must logically appear before the formal operations because the latter are defined by cognitive acts upon the former and the reverse sequence is logically impossible. Brainerd (1978c) has provided several illustrations of concept sequences in Piaget's theory that appear to be logical in origin. (Although logically invariant, the sequences may prove not to be verifiable empirically.)

It is possible that no one of these three types of explanation independently may account for all the invariance in cognitive development. A fourth source of invariance in a developmental progression may be found in the interaction of biological maturation with the regularities that individuals experience in their physical and social environments. This point is stressed in CLD theory, as was indicated earlier in Chapter 1. The new mental operations or processes that make possible the attainment of the same concept at the four successively higher levels emerge through the interaction of biological maturation with experience and practice in informal and formal learning environments.

Identifying the pattern by which students attained the four successive levels of *cutting tool*, *tree*, *equilateral triangle*, and *noun* was done as follows. The performance of each student on each test of each level of each concept each year was evaluated to ascertain which of the four levels the student had and had not mastered. Each protocol was then examined to determine if it conformed to the invariant sequence. The total incidences of protocols conforming and not conforming were then identified as shown in Table 3-1. Some explanation of this tabular information is required to clarify the procedures.

In the three left columns of Table 3-1 are given the patterns of mastering the successive levels of *equilateral triangle* for the year 1, 2, and 3 assessments that are consistent with the principle of invariance. These are called conforming patterns of attainment. The percentage of students of each longitudinal block who had a conforming pattern for the three years of measurement is shown in the appro-

Table 3-1. Percentages of Students Whose Patterns of Attaining the Levels of *Equilateral Triangle* conformed to an Invariant Sequence

Conforming Patterns of Attainment			*Longitudinal Block and Grade*				*Combined Blocks*
Yr 1	*Yr 2*	*Yr 3*	*A1-3*	*B4-6*	*C7-9*	*D10-12*	
Fo	Fo	Fo	—	3	31	56	23
Cl	Fo	Fo	—	13	23	15	13
Cl	Cl	Fo	3	22	11	3	10
Id	Fo*	Fo	—	4	4	—	2
Id	Cl	Fo	—	4	3	—	2
Id	Id	Fo*	—	1	—	—	<1
Co	Cl*	Fo	—	1	—	—	<1
Cl	Cl	Cl	60	27	13	7	25
Id	Cl	Cl	24	4	5	3	8
Id	Id	Cl	—	4	—	1	1
Co	Cl*	Cl	2	—	—	—	<1
	Cl*	Cl	2	—	—	—	<1
	Id*	Cl	2	—	—	—	<1
Id	Id	Id	2	—	—	—	<1
Percent Conforming			94	83	89	85	87
N Conforming			58	64	71	62	255

*The preceding lower level or two lower levels were also mastered for the first time in this year. Years 1, 2, 3 indicate the years that all four batteries were administered.

priate columns for each longitudinal block separately, and the percentages of all the students with conforming patterns are given in the right columns. The conforming patterns are presented for the four levels of attainment. In the top rows are the seven patterns that conclude with full attainment of the formal level in year 3, the final year of assessment, next the six patterns that end with full attainment of the classificatory level in year 3, and in the bottom row the one pattern ending with the identity level. To illustrate a conforming pattern, an individual whose highest attainment in year 3 was found to be the formal level might have attained it via a number of different patterns, all consistent with the principle of invariance. For example, formal→formal→formal, that is, attaining the formal level in the first year of measurement, or having already attained it, and maintaining it in year 2 and in year 3; classificatory→formal→formal, that is, attaining the classificatory level in the first year of measurement or having already attained it, attaining the formal level in the second year, and maintaining the formal level in year 3; or identity→classificatory→formal. There are 15 possible year 1→2→3 patterns involving full attainment of the formal level in year 3 that are in accord with an invariant progression; there are 10 possible year 1→2→3 patterns for the classificatory level, 6 for the identity level, and 3 for concrete. Only those conforming patterns observed in students' protocols related to *equilateral triangle* are shown in Table 3-1.

Eighty-seven percent of the students, 255 of the total of 292, attained the successive levels of *equilateral triangle* in an invariant sequence. The percentages of each longitudinal group conforming to the invariant sequence ranged from 83 for the Grade 4-5-6 block to 94 for the Grade 1-2-3 block.

Thirty-seven of the 292 protocols did not conform. Each of these was examined (Klausmeier & Allen, 1978). It was found that many students who mastered a particular level for the first time in the first or second year of measurement did not do so the following year. Based on this examination it appeared that the majority of the exceptions, and possibly all of them, resulted from a combination of any one of the following: inevitable error of measurement, the criteria of mastery established for each test, and the tendency of scores to regress toward the mean. The possibility also remains that a few of the exceptions represent a true exception to an invariant sequence. The biological inheritance of individuals and their environmental interactions vary so widely that we should probably not expect every individual to conform to any invariant sequence.

How does CLD theory account for the invariant sequence? The invariant progression through the successive levels is accounted for in terms of internal and external conditions. The necessary and possibly sufficient internal conditions of learning and development are: (1) mastery of the prior level, (2) functioning of at least one new mental operation in combination with the continuing operations, (3) functioning of the operations of the prior level on information of increasing quantity and complexity experienced in a greater variety of contexts, (4) intending to attain the next higher level, and (5) persisting until it is attained. In addition, being able to discriminate the defining attributes of the concept and to name the concept and the defining attributes is prerequisite and necessary for carrying out the new operations at the formal level. The necessary external conditions of learning include experience and practice with examples of the concept and examples of related concepts. The amount and kind of experience and practice vary according both to concepts and to the four levels of attainment.

We may consider the internal conditions further in view of the significance of the principle of invariance both to cognitive theory and educational practice. We shall not, however, deal in any detail with the motivational conditions of intending and persisting. As indicated in Chapter 1, it is assumed that intending to learn and persisting are necessary for learning concepts and that the successive levels cannot be attained solely as a product of maturation, independent of intention, experience, and practice. Nor can it be assumed that every individual will master either the classificatory level or the formal level of all concepts. Some Grade 12 students, all of whom were identified as normally developing individuals, did not master the formal level of some of the concepts. It is presumed that they could have had they experienced effectively guided practice and had they intended to learn.

Mastery of the Prior Level

We have seen that mastery of the prior level with few exceptions preceded mastery of the next level. Based on this empirical information, we infer that mastery of the prior level is a necessary condition for attaining mastery of the next higher level.

Logical analysis of the operations required for learning the four levels also suggests the sequential ordering as was indicated in Chapter 2. There the operations specified for the different levels and examples of test items to measure each level are given. The analysis of the levels and the operations involved at each level suggested that a person could not master a later level before mastering the prior one.

Functioning of New Mental Operations

Functioning of a new operation or operations as discussed earlier in Chapter 1 is regarded as a necessary condition for attaining each successively higher level. As was explained in Chapter 2, a separate test was constructed to measure the operations specified at each level of the four concepts: *cutting tool*, *tree*, *equilateral triangle*, and *noun*. Individuals who got test items correct above a chance level were inferred to be performing the operations. It was inferred also that the operations had not yet emerged in students scoring at a chance level and lower; at least they were not functioning with measurable effectiveness.

As was seen in Table 3-1, the students of the longitudinal groups regularly and sequentially mastered the concrete, identity, classificatory, and formal levels of *equilateral triangle*. Based on this empirical information it is inferred that the new operations as defined operationally and measured are necessary for attaining the particular levels. It is assumed further that functioning of the new mental operation or operations that are necessary for attaining each of the successively higher levels is closely and intimately related to the individual's rate of maturation and also to practice and experience.

Continued Functioning of Earlier Operations

Continued functioning of the operations of the prior level on information of increasing quantity and complexity experienced in a greater variety of contexts is another necessary internal condition for learning successively higher levels. The continuing operations to be considered are attending, discriminating, and remembering, which are involved at all four levels of concept attainment, and generalizing, which is required at the last three levels. These continuing operations might be considered as invariant across all the levels at which they function. Furthermore, while the emergence of new operations is necessary for attainment of the successively higher levels, the continued functioning of prior operations accounts for the concurrent development of two or more levels for a period of time.

Attending and Discriminating. The successive test items for measuring attainment of the concrete, identity, and classificatory levels were constructed in such manner that the student was required to attend to an increasing amount of information and also to make finer discriminations. Attending, discriminating, and remembering are necessary both for the concrete and identity levels. The amount of information to attend to in the concrete and identity level tests was controlled by increasing the number of nonexamples in the successive items of each test. The discrimination requirement was controlled by making the nonexamples more like the examples in terms of both the variable attributes and the defining attributes shared by the examples and nonexamples.

Older students at successively higher grade levels got more of the successive items of the concrete and identity level tests correct. Their attending and discriminating improved, but well before they entered the study their ability to attend to and discriminate three-dimensional objects had emerged through the interaction of biological maturation and learning. Thereafter and in accordance with principles of perceptual learning (Gibson & Levin, 1975), we infer that the students, with increasing economy and effectiveness, learned to attend to the defining attributes of the concepts and ignore the variable ones, to discriminate the less obvious defining attributes, and to process the incoming information so as to retain the defining attributes but not the irrelevant contextual information. For the abstract concept, *noun*, the discrimination emerged after age 6.

Generalizing. The process of generalizing is necessary for attaining a concept at the identity, classificatory, and formal levels and greater competence is necessary at the classificatory level of any concept than at the identity level. The items in the four batteries that were used in the longitudinal study to measure attainment at the classificatory level were constructed to measure the student's ability to generalize from a given target example to one or more examples displayed with three or more nonexamples. The successive items in the four batteries were constructed so as to require generalizing to a greater number of examples that differed in values of the same variable attribute, for example, size or color, while discriminating from nonexamples that were increasingly like the target example in both variable attributes and critical attributes.

Competence in generalizing increased across the consecutive grades, as inferred from students' mean performances on the classificatory level tests. The increasing competence appears to be interpretable in much the same manner as that regarding attending and discriminating. The generalizing operation emerges with maturation and learning and becomes functional. Thereafter, with experience and practice, individuals learn to attend with greater selectivity to the defining attributes of the class and increasingly to ignore the variable attributes shared by members of the class and by members of other coordinate and subordinate classes. They learn to carry out these operations more effectively across situations involving more examples that are increasingly alike on a larger number of variable attributes.

Remembering. Remembering is essential for attaining all four levels of a concept, and greater memory requirements in terms of information storage and retrieval are experienced at each successive level. Knowledge concerning developmental progressions of memory processes, structures, capacities, and strategies is meager. However, Miller and Johnson-Laird (1976) have formulated a theory of perception and language in which they relate sensation, perception, concept formation, and remembering.

According to the theory persons gain information and organize and store it in one of five different locations in secondary memory: semantic, episodic, action, geographic, and person. Sets of related concepts are organized as conceptual cores and are stored within each of these locations. These conceptual cores are the individual's internal representations of taxonomies, other hierarchies, theories, part-whole relations, and other structures of knowledge. The important feature of each conceptual core is that it consists of words, and word meanings, including the defining attributes of the various individual concepts and of the relationships among them. According to Miller and Johnson-Laird, these conceptual cores emerge from preconceptual percepts. Percepts are formed of the intrinsic and functional attributes of the objects, events, qualities, relations, and processes that are experienced.

Miller and Johnson-Laird also indicate that words expressing the concepts are learned initially in close temporal context with the percept and later the concept. These separate words are organized into successively more comprehensive semantic fields corresponding to the successively more inclusive conceptual cores.

Following the Miller and Johnson-Laird theory, Klausmeier and Allen (1978) indicated how visual and other perceptual images may be stored initially in secondary memory and then retrieved in working memory to test whether an incoming image is the same as the one previously experienced. This is presumably how memory functions in the attainment of a concept at either the concrete or the identity level. The memory trace of a concept attained at either the classificatory level or the formal level probably includes the word that stands for the concept and also contextual information. It may also include an image, particularly if the examples of the concept are concrete and experienced as three-dimensional objects, pictures, or drawings. Paivio (1974) has formulated a dual coding theory of information processing and memory that incorporates this view. It may also include an emotional component if the concept is essentially evaluative (for example, good-bad) or aesthetic (for example, beauty). In any event the amount of information that is stored and retrieved at the successively higher levels of concept attainment increases. This would seem to call for an increasing memory capacity at the successively higher levels and also for increasingly effective strategies for organizing, storing, and retrieving the information.

The memory capacity of the longitudinal students was found to increase with age. The three tests used to measure memory capacity were auditory memory span, word series (Wepman & Morency, 1973a); auditory sequential memory, digit series (Wepman & Morency, 1973b); and memory of visual orientation (Swinton, 1973). The mean percentages correct on the three tests at Grades 1, 3, 6, and 9 were as follows: 53, 58, 65, and 70; 43, 46, 55, and 71; and 62, 83, 91, and 95.

Along with an increasing memory capacity, the internal representation of experience in the form of images and symbols becomes more economical and flexible as development continues (Bruner, 1964; Bruner et al., 1966). The economy with which information is represented, stored, and retrieved is dependent upon strategies of learning and remembering. With increasing experience and practice, persons develop more sophisticated strategies of learning and of remembering. These more sophisticated strategies are presumed to contribute to the individual's ability to attain the successively higher levels of concepts.

A plan or strategy of learning may be regarded as a hierarchical sequence of acts used in acquiring information, that is, in searching the environment and in relating the product of the search to what is already known. A strategy for remembering is a hierarchical sequence of acts used in either processing information for storage in secondary memory or retrieving it from long-term memory.

Although knowledge about strategies of remembering is accruing slowly, very little is known about when any particular strategy may originate, how it may develop during the developmental years, or when it may function at a mature level. Flavell (1977) identified three categories of memory strategies—rehearsal, organization, and elaboration—and also one general category of retrieval strategies. He also proposed three periods in the development of rehearsal strategies. During the first period the strategy is not available to the individual. During a transitional period there is continuing difficulty in retrieving and using the strategy. In the final period the strategy is retrieved spontaneously and used effectively.

During this final period the effects of using the strategy to store information are positive. There are also positive effects during the transitional period in those situations when difficulty is not experienced in producing the strategy.

Developing and using increasingly effective memory strategies appear to contribute to the invariant progression in attaining concepts at the four successively higher levels. It is also possible that persons who do not attain the formal level of some concepts have not developed either the necessary memory strategies, learning strategies, or both.

The reader will recall from Chapter 1 that in Piagetian theory, stage rather than level is used as the description of development while logico-mathematical structures are used as the explanatory construct rather than mental operations and conceptual cores. Most of the subsequent differences between the two theories are related to these very basic differences.

The theories bear some resemblance in that CLD theory incorporates the invariant progression through the four levels just described and Piagetian theory the invariant progression through the four stages. As we shall see, the idea of concurrent development of the levels is fundamental to the CLD invariant sequence; whereas concurrence of stages is totally incompatible with the construct of stage since individuals cannot be in two stages simultaneously.

ATTAINMENT OF THE FOUR SUCCESSIVE LEVELS OF THE SAME CONCEPT CONTINUES FOR MANY YEARS AND ANY THREE SUCCESSIVE LEVELS DEVELOP CONCURRENTLY FOR ONE OR MORE YEARS

Mussen (1970) in his introduction to *Carmichael's Manual of Child Psychology* called for more accurate and detailed descriptions of many aspects of development, particularly in cognitive functions and in social and emotional behavior. He pointed out that the developmental psychologists whose work was reviewed were less concerned than the prior generation of psychologists both with precise descriptions of children's capabilities at various ages and with the reliable determination of age changes in psychological functions, including problem solving.

Wohlwill (1973) also stressed the need for both descriptive and correlational studies of the many dimensions of human development, studies of individual differences in development, and continuing theory formulation. He outlined procedures for describing the course of developmental change along scalar dimensions. He proposed that any dimension or function of development should be identified and described according to its form, its rate, the values of its maxima, minima, and terminal levels, and the age corresponding to the specified points of the development.

Precise descriptive information is helpful, probably essential, in diagnosing a maturing individual's current status and in understanding the same individual's pattern of growth across longer time intervals. The medical profession relies heavily upon descriptive information in the diagnosis and prognosis of many dimensions of physical growth, vitality, and health of children and adolescents. Unfortunately, psychologists and educators do not have reliable information regarding the rate and form of most dimensions of cognitive development.

Smoothed curves representative of those for all concepts are presented to indicate the empirical basis of the principle regarding the duration of development of the four levels of the same concept and also the amount of concurrent development of successive levels. Inspection of the form of the curves for the four levels of a concept permits one to infer whether the developmental progression of the concept from the concrete level through the formal level is continuous or discontinuous between successive levels. The curves may also be compared to identify the number of years during which two or more levels develop concurrently.

Figure 3-1 contains the developmental curves for the concrete, identity, classificatory, and formal levels of *noun*. As shown in Figure 3-1, the children made much progress during the primary school years toward full attainment of both the concrete and identity levels of *noun*. Functioning near a chance level in Grade 1, they attained the classificatory level of *noun* at a very rapid rate from Grade 1 through Grade 3; a less rapid but continuous rate of increase was shown by the older groups of students until near full attainment was reached in Grade 12. Functioning at essentially a chance level in Grade 2 the children attained the formal level at a slower rate than the classificatory level. The mean percent correct was about 72 in Grade 12. We see in Figure 3-1 that there was concurrent attainment of the concrete, identity, and classificatory levels

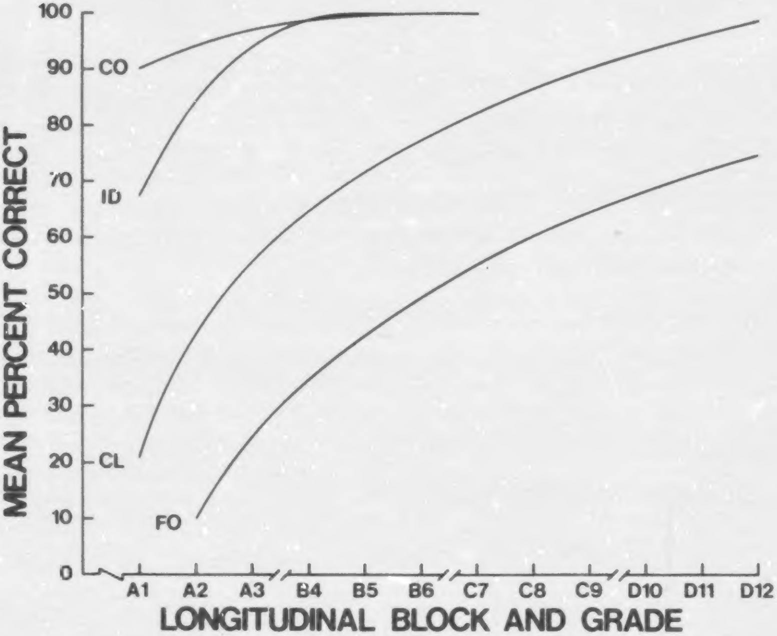

Figure 3-1. Developmental Curves for the Concrete, Identity, Classificatory, and Formal Levels of *Noun*

during Grade 1 through Grade 3 and concurrent attainment of the identity, classificatory, and formal levels, Grade 2 through Grade 4.

The curves for the classificatory level of *tree* and *equilateral triangle* and for the formal level of *cutting tool*, *tree*, and *equilateral triangle* are given in Figure 3-2. Examination of the curves for the classificatory and formal levels of each *tree* and *equilateral triangle* indicates that attainment of one or the other of the two successive levels continued throughout the school years and that there was concurrent development of the two levels across several school years. Inspection of the three curves of the formal level also indicates continuous development from Grade 1 through Grade 12 for these three concepts.

The preceding curves are based on mean scores changed to percents. From these percents, one cannot infer the percent of students who might have mastered any level of a concept. Later the percent of students who mastered the formal level of the four concepts when in Grades 1, 3, 6, 9, and 12 is given (Table 3-3). The percent of

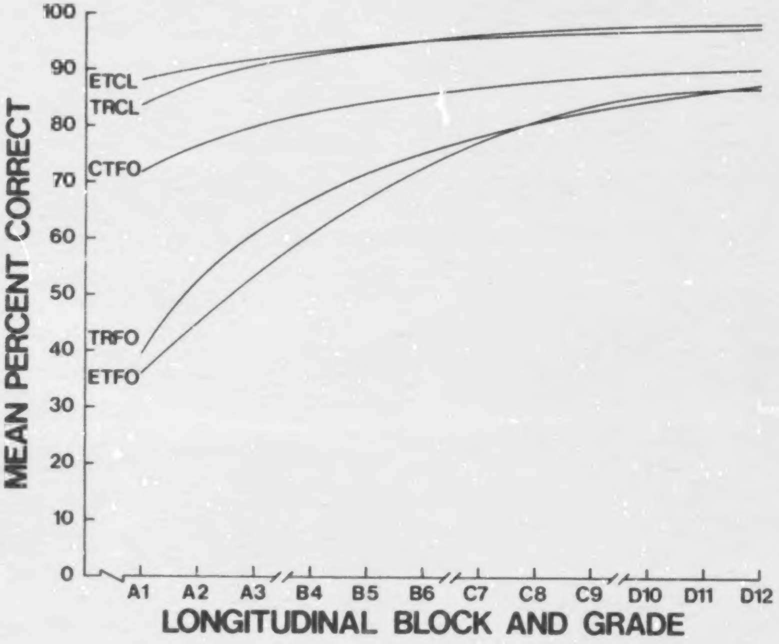

Figure 3-2. Developmental Curves for the Classificatory and Formal Levels of *Tree* and *Equilateral Triangle* and for the Formal Level of *Cutting Tool*

mastery by students when in Grade 3, 6, 9, and 12 was 55, 83, 89, and 95 for *cutting tool;* 8, 38, 78, and 86, *tree;* 3, 52, 75, and 79, *equilateral triangle;* and 0, 1, 35, and 63, *noun.* These mastery scores suggest that although development of the formal level proceeded across most of the school years for some students of the four longitudinal blocks of students, it did not for all individuals. For example and as noted above, when in Grade 6 the percent of students who had already mastered the formal level was 83 *cutting tool*, 38 *tree*, 52 *equilateral triangle*, and 1 *noun.*

Earlier it was indicated that discriminating the defining attributes of the concept and acquiring the name of the concept and the names of its defining attributes are necessary and prerequisite for carrying out the operations at the formal level. The operations include evaluating examples and nonexamples of the concept, based on the presence or absence of the defining attributes, and either inferring the concept or a definition of the concept. The discrimination of the attributes and acquisition of the names may commence at the same time during which any of the first three levels of attaining the same concept occurs. Curves depicting development of these four items based on the students' combined test scores for the four concepts are shown in Figure 3-3. The temporal order of the emergence of these four items from first to last is in the same sequence as given above. Inspection of the curves indicates concurrent development of all four operations from Grade 2 through Grade 12.

We may conclude that the progression from one level of concept attainment to the next is continuous rather than discontinuous. Any three levels develop concurrently for a short period of time while the classificatory and formal level develop concurrently for a longer period. By definition, the curves for items that develop in synchrony must be congruent or nearly so; the items also must develop concurrently, starting to function initially at the same time and later reaching full functional maturity at the same time. On the other hand, items that develop concurrently for a period of time may or may not develop in synchrony.

PROGRESSION IN UNDERSTANDING PRINCIPLES AND UNDERSTANDING TAXONOMIC RELATIONS AND IN SOLVING PROBLEMS CONTINUES FOR MANY YEARS IN A NONSYNCHRONOUS CONCURRENT PATTERN

The developmental curves for understanding principles, understanding taxonomic relations, and problem solving are given in Figure

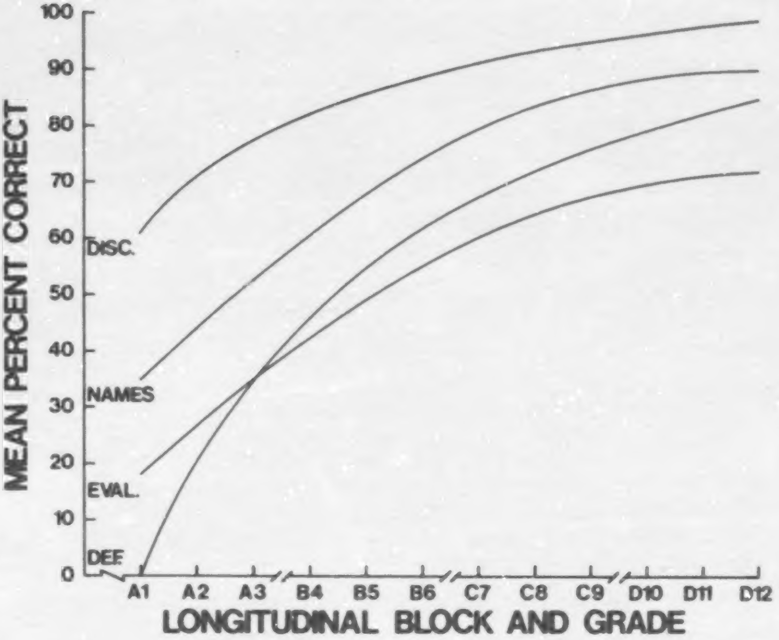

Figure 3-3. Developmental Curves for Discriminating the Defining Attributes, Acquiring the Concept Name, Acquiring the Names of the Defining Attributes, and Evaluating Examples and Nonexamples of the Concept

3-4 a, b, c. Little understanding of principles occurs during the primary school years, but it increases consistently and at about the same rate during the intermediate, junior, and senior high school years. From Grade 11 to Grade 12, the increase is still occurring and understanding is still relatively low in Grade 12, somewhat lower than for attainment of the formal level of the same concepts (see Figure 3-2).

The understanding of taxonomic relations increases rapidly during the primary and the intermediate school years (Figure 3-4b). The rate of increase is quite slow during the junior and senior high school years. Many students go through school and graduate from high school without understanding these relatively simple but important supraordinate-subordinate-coordinate relationships found among concepts organized in taxonomies.

Children have developed considerable proficiency in problem solving by the end of the primary school years. Problem-solving skills continue to develop at a relatively slow rate during the inter-

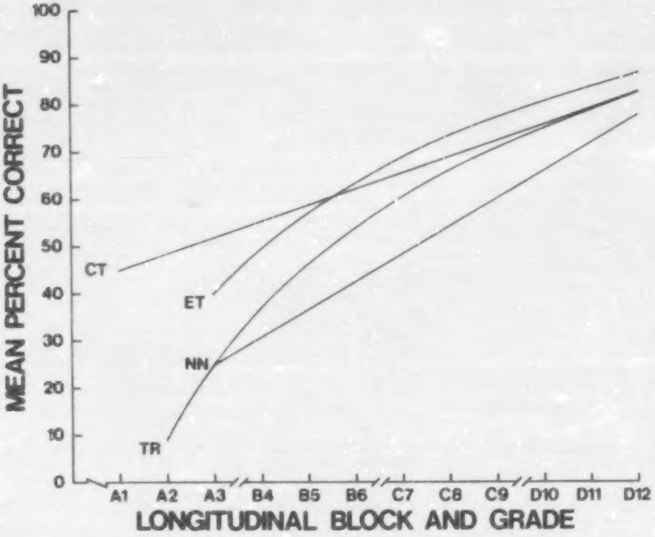

Figure 3-4a. Normative Development Related to Understanding Principles

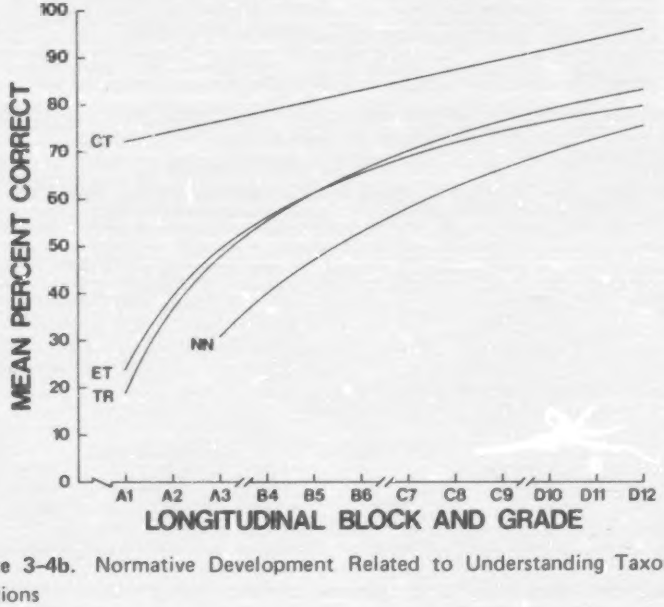

Figure 3-4b. Normative Development Related to Understanding Taxonomic Relations

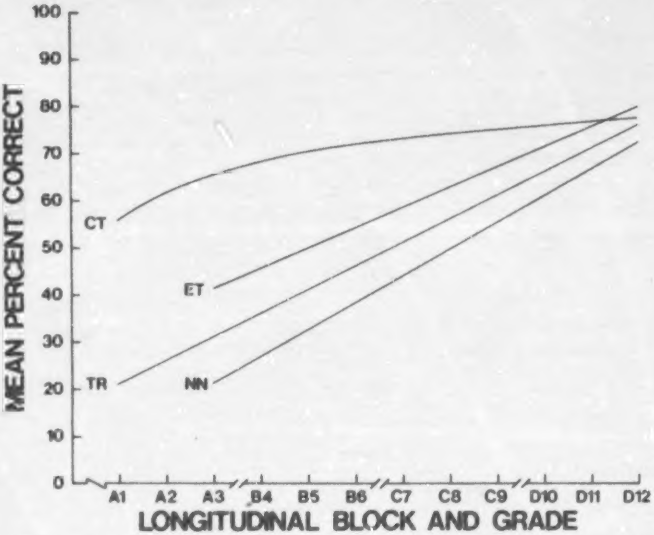

Figure 3-4c. Normative Development Related to Problem Solving

mediate, junior, and senior high school years. The level of performance throughout the school years is quite low, including Grade 12.

Inspection of the developmental curves indicates that growth in the understanding of principles, the understanding of taxonomic relations, and in problem solving continues from the primary school years through Grade 12, and there is concurrent development of these items of development from Grade 3 through Grade 12. The continuous nature of this development throughout the school years may be accounted for primarily by the students' attaining the concepts that are involved to successively higher levels; their ability to attend to, discriminate, and remember more information and also more complex information; their increasing language competence; and their more effective strategies for securing, processing, and remembering information. The relatively low performances in Grade 12 imply that some students are deficient in one or more of these capabilities. The deficiency that was verified empirically in the present study was failure to attain concepts at the formal level.

THE SAME LEVELS OF DIFFERENT CONCEPTS ARE MASTERED AT DIFFERENT AGES

The percentage of the students of each grade who mastered the classificatory level and the formal level of each of the four different concepts varied greatly (Figure 3-5). At each Grade 1, 3, 6, 9, and 12 the percentage of students who mastered the formal level was highest for *cutting tool* and lowest for *noun*. In these same grades the percentages that mastered the classificatory level and the formal level of *tree* and *equilateral triangle* were between those for *cutting tool* and *noun*. The large difference in attaining the same level of different concepts is illustrated by the fact that there were more Grade 1

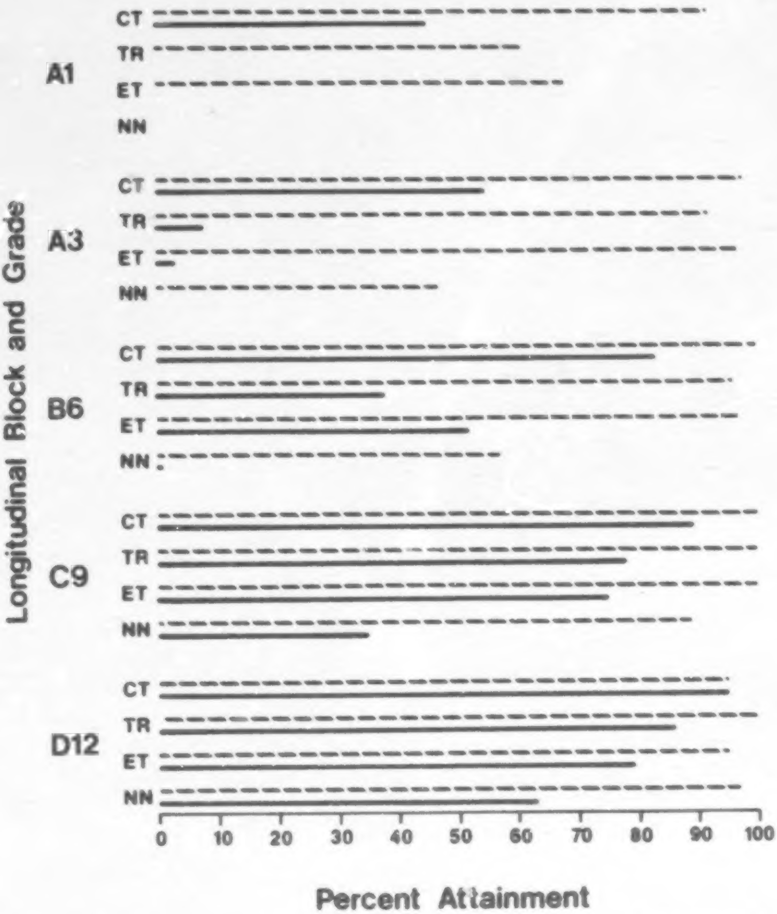

Figure 3-5. Percentage of Students in Grades 1, 3, 6, 9, and 12 Who Mastered Each Concept at the Classificatory Level (---) and the Formal Level (——)

students who mastered the formal level of *cutting tool* than Grade 9 students who mastered the formal level of *noun*.

We see that the age at which individuals become capable of performing the operations and meeting the other conditions necessary for mastering the classificatory and formal levels is related to the content domain of the concept and the abstractness of the examples of the concept. The operations are performed years earlier on concrete examples that can be experienced through the sense organs than on examples represented by symbols only.

That the necessary operations do not emerge at the same time for all kinds of concepts is of major theoretical and practical significance. The most important point for the theorist is that such fundamental operations as discriminating, generalizing, hypothesizing, and inferring emerge and become functional at a much earlier age for concrete, three-dimensional examples of concepts than for symbolically expressed examples. The practicing educator can be confident that maturing children of a certain age will be able to carry out the operations on certain kinds of examples and content but not on others. This points up an interesting difference between the CLD levels and Piaget's stages. In Piaget's theory, children are "in" a particular stage at a given time, except perhaps for the short and unstable transition phases. Hence, the fact that different concepts belonging to the same stage emerge at different times is attributed to *performance* variables. The same-stage operations themselves do not emerge sequentially.

ATTAINING THE SUCCESSIVELY HIGHER LEVELS OF CONCEPTS IS ACCOMPANIED BY INCREASED UNDERSTANDING OF PRINCIPLES AND TAXONOMIC RELATIONS AND BY MORE ADVANCED PROBLEM-SOLVING SKILLS

Feldman (1974) and Katz (1976) found that understanding the concepts embedded in particular principles facilitated understanding of the principles but that it was not sufficient. The relationship among the concepts expressed in the principle also must be understood. Gagné (1977) proposed a linear relationship, indicating that concepts are necessary for learning rules or principles, and that rule using is necessary for solving problems. These relationships deal with concepts attained at the classificatory level, not with concepts attained to successively higher levels.

Three predictions may be derived from CLD theory regarding concepts attained to successively higher levels. Mastery of the concrete and identity levels will be accompanied only with mastery of simple perceptual problems, not with understanding of principles or taxonomic relations; mastery of the classificatory level will be accompanied with some mastery of understanding principles, understanding taxonomic relations, and problem solving; and mastery of the formal level will be accompanied with the greatest mastery of principles, taxonomic relations, and problem solving.

In accordance with the first prediction, students who mastered the concrete or identity levels of different concepts as their highest performance when in Grade 1, 3, 6, 9, or 12 rarely achieved mastery of the related principles and taxonomic relations, or the problem-solving exercises (Table 3-2). Although not shown in the table, most of the instances of the uses mastered were simple problems involving *cutting tool* or *equilateral triangle* in which the correct answer was probably identified through perceptual rather than conceptual processes. The proportion of the total instances of mastery of these uses of the four concepts to the number of students who mastered either the concrete or the identity level as their highest level was .08. The highest proportion possible was 3.0, since each student who mastered a level of a particular concept could also have mastered its three uses dealing with principles, taxonomic relations, and problem solving.

Substantial numbers of students mastered the classificatory level of one or more concepts as their highest achievement. The instances

Table 3-2. Total Number of Students Fully Attaining Each Concept Level and Total Instances of Mastery of Understanding Principles, Understanding Taxonomic Relations, and Problem Solving

Block and Grade		*Concrete and Identity Combined*	*Classificatory*	*Formal*
A1	*N* attained level	90	112	28
	Instances of mastery	2	27	29
A3	*N* attained level	38	167	41
	Instances of mastery	0	60	47
B6	*N* attained level	38	136	134
	Instances of mastery	7	76	254
C9	*N* attained level	9	90	221
	Instances of mastery	4	71	473
D12	*N* attained level	1	55	236
	Instances of mastery	1	43	539
Total	*N* attained level	176	560	660
	Instances of mastery	14	277	1342

of the uses mastered in relation to the numbers of students mastering the classificatory level increased at the successively higher grades. The proportion of the total instances of mastery of the uses to the total number of students who mastered the classificatory level was found to be .49.

Unlike mastery of the other levels, mastery of the formal level was accompanied with far greater mastery of principles, taxonomic relations, and problem-solving skills. The proportion of the total instances of mastery of these uses to the number of students who mastered the formal level of one or more of the four concepts was found to be 2.03.

Persons who accept the validity of these findings should be aware of their significance for education. Instruction on concepts that stops with students' attainment of the classificatory level will not be very helpful to students in learning principles, acquiring structures of knowledge, or developing problem-solving skills. On the other hand, proceeding to the formal level will enable students to move ahead with these very important capabilities.

INDIVIDUALS OF THE SAME AGE VARY GREATLY IN THEIR LEVELS OF CONCEPT ATTAINMENT AND IN THEIR MASTERY OF PRINCIPLES, TAXONOMIC RELATIONS, AND PROBLEM-SOLVING SKILLS

Knowing the extent of the differences in the performances of normally developing individuals of the same age is of much theoretical and practical importance. One or more chapters dealing with individual differences in the cognitive domain may be found in Buss and Poley (1976), Messick and Associates (1976), and Tyler (1974, 1978). Cronbach and Snow (1977) related current knowledge concerning individual differences in abilities and intellectual processing skills to alternative educational methods. Relatively few of these reports deal with individual differences according to the developmental levels of individuals.

Differences among individuals of the same age are very great as can be seen by examining performances related to the formal level of attaining concepts and the understanding of principles. These two areas were selected for illustrating the extent of interindividual differences because their development progresses throughout the school years.

A substantial percentage of the students were found to have mastered the formal level of one or more concepts and the understanding of principles in each Grade 3, 6, 9, and 12 while others had not (Table 3-3). For example, the percentages of the students when in Grade 3 who had and had not mastered the formal level of the various concepts were: 55-45 *cutting tool*, 8-92 *tree*, 3-97 *equilateral triangle*, and 0-100 *noun*. The percentages of the students when in Grade 12 who had and had not mastered the principles related to the various concepts were: 60-40 *cutting tool*, 47-53 *tree*, 73-27 *equilateral triangle*, and 49-51 *noun*.

The magnitude of the differences between the grades in the percentages of students who had mastered and who had not mastered the formal level or the principles was very great. For example, 3 percent of the Grade 3 children had mastered the formal level of *equilateral triangle* whereas 21 percent of the Grade 12 students had not. Thirty-eight percent of the Grade 6 students had mastered understanding of principles related to *equilateral triangle* but 27 percent of the twelfth-graders failed to reach the mastery criterion. Similar differences were found in understanding taxonomic relations and problem solving.

What do these findings imply? Stage theorists make no attempt to account for interindividual differences of this magnitude. Minor variation of two or three years among individuals in either the beginning functioning or the full functioning of all the items of a stage are recognized. However, differences of nine or more years between normally developing individuals in the full functioning of one item of a larger sequence of items are not accounted for.

The implications for educational practice are also important. It is difficult to visualize what would be involved in attempting to arrange instruction that would take these differences into account. To illustrate, although 52 percent of the Grade 6 students had attained the formal level of *equilateral triangle*, among the 48 percent who had not, one would expect 21 percent still not to have done so in their last year of high school.

INTRAINDIVIDUAL VARIABILITY BECOMES INCREASINGLY DIFFERENTIATED ACCORDING TO CONTENT FIELDS DURING THE ELEMENTARY SCHOOL YEARS

This principle is inferred from the results of factor analyses. These analyses for Grades 1-12 are reported in detail in Chapter 4 of this book. Here the results from Grade 6 only are considered in order to

Table 3-3. Percent of Students Who Did and Did Not Fully Attain the Formal Level of Each Concept and Who Did and Did Not Master the Related Principles

Block and Grade	*Formal Level*								*Principle*							
	CT		*TR*		*ET*		*NN*		*CT*		*TR*		*ET*		*NN*	
	A	*NA*	*A*	*NA*	*A*	*NA*	*A*	*NA*	*A*	*NA*	*A*	*NA*	*A*	*NA*	*A*	*NA*
A1	45	55	0	100	0	100			2	98						
A3	55	45	8	92	3	97	0	100	3	97	2	98	0	100	2	98
B6	83	17	38	62	52	48	1	99	49	51	6	94	38	62	1	99
C9	89	11	78	22	75	25	35	65	50	50	31	69	51	49	25	75
D12	95	5	86	14	79	21	63	37	60	40	47	53	73	27	49	51

illustrate the kind of empirical information on which this key principle of cognitive development is based.

In Grade 6, a separate factor was identified for each of the four concepts: *cutting tool*, *tree*, *equilateral triangle*, and *noun*. These were the only factors identified. Moreover, in each grade from 6 through 12 only four or five factors were identified, and generally the same four factors that were identified in Grade 6, but with somewhat different loadings of the various tests.

From the factor analytic results for Grade 6 and the later grades, we may infer that cognitive development from about age 12 onward is sharply and clearly differentiated according to content fields. This differentiation emerges during the intermediate school years as we shall see in Chapter 4. Clearly then, cognitive functioning after age 10 is not unitary or global, characterized by equally high or low performance across all concept domains. Instead, individual students typically vary within themselves in their performances related to their attainment of the different concepts and the related principles, taxonomic relations, and problem-solving skills. Other investigators have found intraindividual variability to be closely related to the subject fields of English, mathematics, science, and social studies (Harris & Harris, 1973; Shaycoft, 1967).

ATTAINMENT OF THE SUCCESSIVELY HIGHER LEVELS OF THE SAME CONCEPT AND OF THE MEANING OF THE WORD EXPRESSING THE CONCEPT PROCEED SIMULTANEOUSLY

Natural languages have two key aspects: structure and meaning. The structural aspect may be divided into the sound system, or phonology; the rules for the formation of words from sounds, or morphology; and the rules for combining words into phrases and sentences, referred to as syntax, or grammar. The semantic aspect of language deals with the use of words to represent the meanings associated with the individual members and classes of objects, events, and processes and the qualities and relations thereof. Accounts of various aspects of language development can be found in Beilin (1975), Brown (1973), Carroll and Freedle (1972), Dale (1976), and Miller and Johnson-Laird (1976).

Semantics play a very important role in communication. Just as concept attainment requires empirical knowledge of the physical and social world, so also do forming and comprehending spoken and written sentences demand it. Thus, it is important to try to determine the relationship between the semantics of language development and the four levels of concept attainment.

As explained in Klausmeier and Allen (1978), the acquisition of the meaning of a word from a meaningless puff of air or a batch of ink on paper to a societally accepted word meaning proceeds simultaneously with the development of the four levels of the concept, from the concrete level through the final mastery of the formal level. Similarly, the development of a conceptual core as a meaningful structure of knowledge proceeds simultaneously from first attaining two concepts at the classificatory level and understanding their relationship through mastery of a more complete structure of knowledge, including all the concepts, principles, and their relationships comprising the structure.

It is not necessary to acquire the name of the concept until the formal level (Figure 3-6); however, the name may be acquired at any of the lower levels. To clarify the relationship among concepts, words, and word meanings, let us assume that an individual acquires the name of a concept at about the same time as attaining the concrete level and then attains the three successively higher levels of

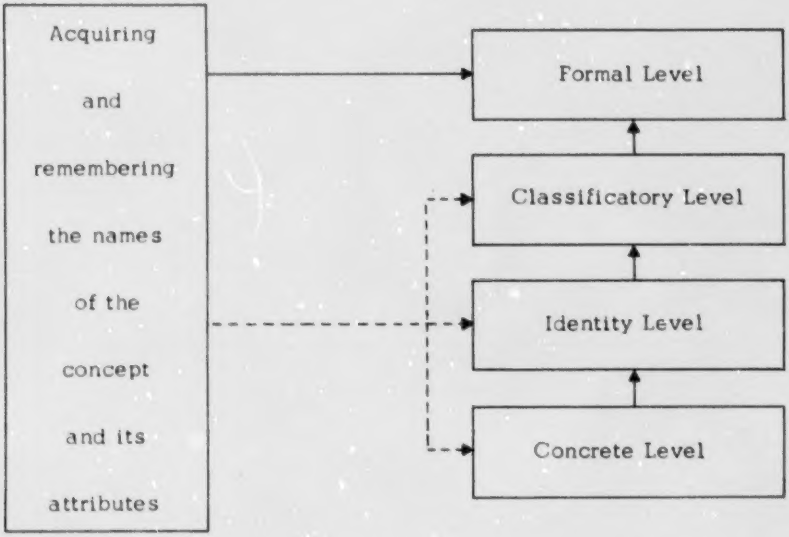
Acquiring and remembering the names of the concept and its attributes
Formal Level
Classificatory Level
Identity Level
Concrete Level

Figure 3-6. Relationship of Acquiring Names to the Levels of Concept Attainment (—— necessary to attain level, --- not necessary)

the same concept. The individual's meaning of the word that expresses the concept is the same as the individual's mental construct at each level. At the formal level of attainment, the individual has not only the name of the concept but also the names of the defining attributes. The word meaning at the formal level is the societally accepted meaning of the word and is designated a lexical concept.

According to Miller and Johnson-Laird (1976), having the meaning of a word as a lexical concept enables the individual to ascertain what is and what is not an entity that can or cannot be labeled by the word. Having the word meaning also enables the individual to indicate the function or purpose of the entity that the word labels. Finally, having the meaning can help the individual retrieve from secondary memory everything that the individual knows about the particular entity. A group of related lexical concepts forms a semantic field.

Just as having a lexical concept requires knowing the defining attributes of the particular concept, so also does a semantic field require having formed a conceptual core. The conceptual core representing a taxonomy includes the defining attributes of the generic concept and the critical attributes of the coordinate concepts. In a taxonomy, the attributes of the generic class are common to all members of the taxonomy while there are critical attributes that are unique to the members of each coordinate class. Knowledge of the critical attributes makes it possible to discriminate between members of the different coordinate subclasses, each of which is represented by a contrastive word.

We may illustrate these relationships among the kinds of attributes and other relations. Figure 3-7 is a taxonomy formulated by Bernard (1975) in which *behavior management procedures* is the generic concept. There are pairs of coordinate concepts, each of which is subordinate to a concept successively subordinate to a concept higher in the taxonomy, for example, *procedures for increasing behavior* and *procedures for decreasing behavior* are coordinate concepts that are subordinate to *behavior management procedures.* Each coordinate concept has critical attributes, but to define any particular coordinate concept, the generic attributes are also necessary. A complete conceptual core for this taxonomy includes the societally accepted meaning of each of the words expressing the concepts (lexical concepts) and also the inclusive-exclusive relationship as well as other relationships among the 11 concepts. An individual's conceptual core of the taxonomy, or internal structure of knowledge, is the same as the semantic field, provided the person

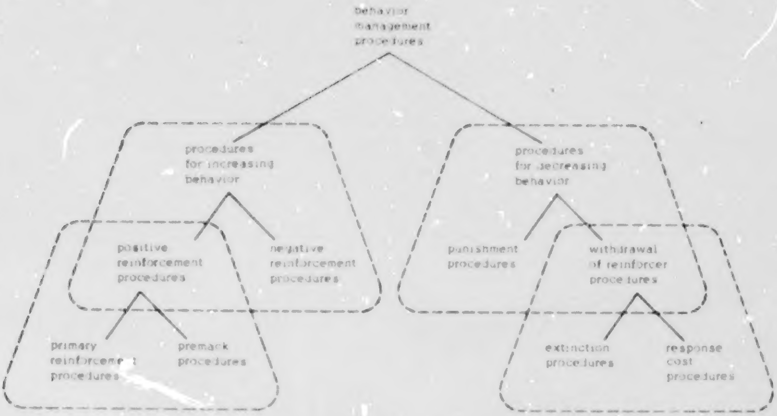
behavior management procedures
procedures for increasing behavior
procedures for decreasing behavior
positive reinforcement procedures
negative reinforcement procedures
punishment procedures
withdrawal of reinforcer procedures
primary reinforcement procedures
premack procedures
extinction procedures
response cost procedures

Figure 3-7. Taxonomy of Behavior Management Concepts

has acquired the names of the concepts, their meanings, and the relationships among them.

The relationship between a concept and the meaning of the word that expresses the concept and between conceptual cores as internal structures of knowledge that correspond to structures of knowledge organized by experts is not well understood, but it is of greatest significance in the study of cognitive learning and development. Continuing study of these critical areas of human cognitive functioning is urgently needed.

HAVING THE NAME OF THE CONCEPT AND THE NAMES OF DEFINING ATTRIBUTES FACILITATES ATTAINMENT OF THE LEVELS

Pointing and naming are widely used in directing the attention of the young child to an object, person, process, quality, or relation. This directing of attention facilitates attainment of the concept at the concrete and identity levels. Language is also used to point to an attribute of a generic class or to the critical attribute of a coordinate class. This facilitates attending and discriminating at both the classificatory and formal levels. The facilitative effects of being instructed about the names of the concepts and the attributes have been interpreted as due to mediating between the environmental input and the individual's responding (Kendler, Glasman, & Ward, 1972), enhancing the distinctiveness of the concept examples (Cantor, 1965; Miller & Dollard, 1941), and enhancing the individual's sensitivity to the defining attributes of the concept (Tighe & Tighe, 1968). While agreement does not exist among theorists about the internal mechanisms that might explain the facilitative effects, it is clear that instruction that gives the name of the concept is helpful in increasing intraclass similarity and decreasing interclass similarity (Carey & Goss, 1957; Goss & Moylan, 1958; and Dietze, 1955). This, in turn, enables individuals to classify members of coordinate concepts that have the same generic attributes and are much alike in variable attributes.

Increasing competence in language is also associated with more sophisticated strategies for learning and for remembering information in that employing a strategy to learn or to remember is accomplished by using language rather than images. Increasing competence in language and more effective rehearsal, elaborative, and organizing strategies appear to develop concurrently. The increasing competence of the individual in using the strategies contributes to the gradual and continuous development of concepts to successively higher levels and also to more mature understanding of principles and taxonomies.

We may pause to point out a sharp difference between the CLD and Piagetian perspectives. CLD theory indicates that language facilitates concept learning starting prior to the school years. Furthermore, having the name of the concept and the names of its defining attributes are prerequisite for attaining the formal level. Piaget's antipathy to any language-based explanation of cognitive development is among the best known features of his theory (for example, Flavell, 1963).

QUALITY OF EDUCATION AND THE RATE OF COGNITIVE DEVELOPMENT

Piaget (1964) indicated that social transmission, which includes formal education, is one of four determinants of cognitive development; the other three are maturation, experience, and equilibration. Piaget ascribes a greater role to maturation in cognitive development than to formal education. Bruner (1973) gives formal education a far more prominent role. After reviewing cross-cultural studies, including those he conducted involving Wolof children of Senegal, Africa, Bruner concluded as follows:

> It is always the schooling variable that makes qualitiative differences in directions of growth. Wolof children who have been to school are more

> different intellectually from unschooled children living in the same bush village than they are from city children in the same country or from Mexico City, Anchorage, Alaska, or Brookline, Massachusetts. Similar results demonstrating the huge impact of school have emerged from the Belgian Congo and South Africa.

We cannot infer from the principles of development given earlier in this chapter how the quality of education may influence the rate of development or the eventual maximum achievement of the students of the longitudinal study. Other research, however, enables us to formulate hypotheses regarding the effects of general educational experiences and of focused learning experiences on cognitive development during the school years (Klausmeier, 1977a).

The conceptualizing skills and geometry achievements of groups of children of high socioeconomic status enrolled in Grades 5, 8, and 11 (Wiviott, 1970) and of children of low socioeconomic status enrolled in the same grades (Nelson & Klausmeier, 1974) were studied. In these studies, each student's task was to indicate how the following geometric forms were alike and different: square, rectangle, rhombus, parallelogram, quadrilateral, triangle, circle, and cube. The students' responses were analyzed and put into four categories: incorrectly classifying by using the nondefining variable attributes; correctly classifying by using the defining attributes; correctly classifying by using defining attributes and also giving the names of the supraordinate-subordinate categories; and fiat, or uninterpretable response patterns. The use of the defining attributes and naming is the most effective conceptualizing skill.

The students were also administered a 56-item test of geometric concepts. The performances of the students of both low and high socioeconomic status were compared (see Table 3-4). At all grade levels the students of low socioeconomic status explained likenesses and differences much more on the basis of nondefining variable attributes and much less on the basis of membership in supraordinate-subordinate classes; they scored much lower on the geometry test. Strikingly, the fifth-grade students of high socioeconomic status scored higher on the geometry test than all three low socioeconomic groups, including the eleventh-graders. The high Grade 5 students used the supraordinate-subordinate classes more and the nondefining variable attributes less than did any low group. Equally unexpected, the eleventh-grade students of low socioeconomic status used more nondefining variable attributes than did the low eighth-grade students, and they also achieved lower on the geometry test than the eighth-graders. The eleventh-grade students did not show the orderly

Table 3-4. Mean Incidences of Using Each Basis of Classifying and Mean Geometry Scores of Students of Low and High Socioeconomic Status

	Low S.E.S.				*High S.E.S.*			
Grade	*Variable Attribute*	*Defining Attribute*	*Supra-Subordinate*	*Geometry Score*	*Variable Attribute*	*Defining Attribute*	*Supra-Subordinate*	*Geometry Score*
5	9.31	5.63	2.72	21.4	5.56	7.38	4.19	33.3
8	7.06	7.16	3.03	28.8	4.00	8.25	5.13	37.9
11	8.03	5.06	3.81	28.4	2.59	9.00	7.09	47.2

progression of categorizing skills found by Oliver (1961), Rigney (1962), and also by Wiviott (1970).

Further study showed that a change in the mathematics curriculum had recently occurred in the school district where the students of low socioeconomic status were enrolled. None of the eleventh-grade students had received prior instruction in geometry during their school years, 41 percent of the eighth-grade students had, and 25 percent of the fifth graders had. The data were then examined in terms of those who had and had not received geometry instruction, and marked differences in categorizing skills and geometry achievements were found. The nondefining variable responses of students of Grades 5 and 8 who had received instruction in geometry concepts were fewer, while the defining attribute responses were greater. Their patterns of categorizing behavior were moving toward those of their high socioeconomic status counterparts.

Apparently a combination of low socioeconomic status and lack of instruction in geometry is associated with immature conceptualizing skills and also with low achievement in geometry. Most important, children of low socioeconomic status who do not receive instruction remain at an immature level of conceptualizing, continuing to improperly classify through use of the nondefining variable attributes rather than classifying properly through use of the defining attributes and the supraordinate-subordinate names. Thus, it appears that the student's curricular experiences are related not only to the level of concept attainment, but also to the very means of conceptualizing that make higher achievements possible.

The preceding results support the hypothesis that general education has a powerful influence on students' cognitive development, including the maximum level eventually attained. It is probable that many students will not master concepts at the formal level, as we observed earlier in this chapter, unless the quality of their education is improved.

Results of the longitudinal study and the comparative study of socioeconomic status permit the indirect inference that some students do not progress toward full conceptual maturity as they might. Other experiments provide evidence about the extent to which specially prepared lessons enable elementary school children to master the formal level of single concepts and high school seniors to master multiple concepts and understanding the taxonomic relationships of which the multiple concepts are a part (see Figure 3-7). The lessons incorporate the knowledge accruing about cognitive development and individual differences. They also take into account a growing body of knowledge concerning the instructional variables that facilitate cognitive learning (Klausmeier, 1976; Markle & Tiemann, 1969; Tennyson, Woolley, & Merrill, 1972), and also the most recent information regarding instructional design (Bloom, Hastings, & Madaus, 1971; Gagné & Briggs. 1974; Glaser & Resnick, 1972; Tiemann & Markle, 1978).

One experiment was conducted with third-grade children in four 50-minute sessions, one session held on each of four consecutive days (Klausmeier, Schilling, & Feldman, 1976). The percentage of experimental children passing the formal level of the concept increased from 3 percent on the pretest to 68 percent on the posttest. There was also a slight increase in the percent of the control children passing the formal level, namely, from 3 to 14.

Another experiment involved the concept *equilateral triangle.* It was conducted in Grade 4 with two experimental groups of 32 and 28 children and two control groups of 30 and 28 (McMurray, Bernard, & Klausmeier, 1974). On the posttest, 60 percent of experimental group 1 and 64 percent of experimental group 2 passed the formal level, whereas 7 percent of control group 1 and 11 percent of control group 2 did.

Through three lessons to teach multiple concepts, high school seniors were brought from functioning at a chance level to functioning at a high level of mastering concepts at the formal level dealing with behavior management procedures, understanding the supraordinate, coordinate, and subordinate relationships involved among the concepts, and placing the names of the concepts properly in the taxonomy (Bernard, 1975).

A comparison of the achievements of the group receiving placebo lessons and of the groups getting other lessons designed to teach mastery of the concepts and understanding of taxonomic relations is most revealing. The mean percents correct for these groups on the three tests were: mastery of the concepts, 30 and 78; understanding taxonomic relations, 26 and 74; and placement of the concept names in an outline of the taxonomy, 33 and 83. The effectiveness of the instructional materials for teaching these multiple concepts, and also the single concepts of the earlier experiments, results from the precision with which the learning process is guided throughout the lessons.

The most important conclusion from these experiments is that focused instruction can be arranged to facilitate the learning of single concepts, multiple concepts, and structures of knowledge. When introduced at the proper time to supplement regular classroom instruction, large gains are made by the students during short time intervals. The lessons are not to be used alone, independent of other excellent instruction. The younger children are and the fewer experiences they may have had with examples of particular concepts, the greater is the need to provide actual examples as well as drawings of the kind used in the lessons. In addition, other procedures may be required to sustain a high level of motivation.

While printed lessons can be prepared to supplement oral interactive instruction, the lessons do not function equally well with all the students when they are instructed as age-graded, classroom groups. The age-graded elementary school in which one teacher instructs 20 to 40 children as a classroom group is no longer effective in nurturing the cognitive development of many students (Klausmeier, 1977a). The familiar age-graded high school in which each teacher instructs four to six classes of 20 to 40 students as classroom groups is failing even more students (Gibbons, 1976). The most powerful theories of development and learning are having very little effect, if any, on nurturing the cognitive development of children and youth in these familar forms of schooling (Rohwer, 1970).

A model of instructional programming for the individual student was developed and tested in many schools of the nation as a means of improving education (Klausmeier, 1976, 1977b). Age-graded, self-contained, and departmentalized schools were changed in many ways to make it possible for teachers to nurture individual student's educational development more effectively. Three main features of this kind of instruction merit careful study but can be mentioned only briefly.

First, the individual student rather than the classroom group serves as the basic unit of instruction. Second, instruction is personalized and interactive rather than impersonal and reactive on the part of the student. Third, instructional decisions are made cooperatively at different levels in the educative process, and the individuals who share in making the decisions are accountable for implementing them. Experiments carried out in this kind of elementary schooling over a period of three years indicated that more children mastered the process concepts of science than did control children instructed as heterogeneous classroom groups (Klausmeier & Allen , 1978).

Chapter 4

Organization of Cognitive Abilities Based on CLD Theory

Herbert J. Klausmeier

In Chapter 3 we saw that individuals of the same age vary widely in their levels of concept attainment, in their understanding of principles and taxonomic relations, and in their problem solving skills. Contributing to these wide differences in performance are the interindividual differences in the age at which the mental operations necessary for attaining the successively higher levels of the concepts emerge and become functional and also interindividual differences in the various information processing operations and strategies for organizing, storing, and retrieving information.

We also saw that the same individual does not attain the various levels of the different concepts at the same age. For example, the classificatory level of *cutting tool* is mastered years earlier than the classificatory level of *noun*. This is attributed in part to the difference among concepts on a concrete-abstract dimension. For example, persons experience many concrete three-dimensional examples of cutting tools, such as knives, starting early in life, and soon thereafter they also acquire the words expressing the concept and its attributes. They also experience examples of nouns quite early in life as verbal symbols when listening and speaking, but they do not recognize them conceptually as examples of nouns until much later when the meaning of noun is learned.

The concrete-abstract dimension of concepts is not the sole determinant of this difference in the age at which the same individuals attain the same level of different concepts. Another appears to be the structure of knowledge of which the particular concept is a part. Structures of knowledge tend to be associated with the substance of separate subject fields such as mathematics, the natural sciences, and English rather than with the substance from two or more diverse subject fields.

In order to gain a better understanding of the preceding inter-individual differences related to the same concepts and also the differences among concepts, the factor structure underlying the performances of the students on the four concept batteries, Grade 1 through Grade 12, was identified. The results are reported in this chapter. The developmental years, corresponding to the grade in school, are identified during which the differentiation and integration of abilities occur and at which the structure eventually stabilizes. In addition, the relationship of the development of these cognitive abilities to the educational development of students when in Grades 6, 9, and 12 is indicated.

PROCEDURES EMPLOYED

Correlations were obtained based on all the tests that were administered *each year at each grade*. To identify the factors for each Grade 1-12, the resulting matrices were analyzed by three methods of factor analysis: the principal components method, the $R-S^2$ method as developed by Harris (1962), and the unrestricted maximum likelihood method (UMLFA) of Joreskog (1967). Both an orthogonal and an oblique rotation were performed on the raw factor matrices, yielding six different factor solutions.

The number of factors identified in the three initial solutions was determined by Guttman's weak lower bound (principal components), Guttman's strong lower bound ($R-S^2$), and Joreskog's sampling theory test (Joreskog, 1967).

The rotated factors of the six solutions were then examined to determine the tests that loaded .30 or higher on each factor, and comparable common factors found in at least four or more of the six solutions were identified. This strategy for identifying only the factors that were well defined in at least four of the six solutions was employed by Harris and Harris (1973). The primary reason for employing it is that neither a single initial factoring method nor a single rotational procedure, in isolation, determines such a factor; instead the comparable common factors span different extraction and rotation methods.

Correlations were also run between the sets of test scores gathered longitudinally on the four groups of students at the two 12-month intervals and the 24-month interval, for example, the tests administered to the younger students when in Grades 1-2, 2-3, and 1-3. An interbattery analysis was made for each of these sets of correlations for each of the four longitudinal blocks of students, a total of 12 analyses. Interbattery factor analysis, a method originally proposed by Tucker (1958), is designed to reveal the factors that two batteries have in common. The method uses only the cross-correlations of one battery with another. For relating the same battery administered on two different occasions, each testing occasion (year) is treated as a battery, and only the intercorrelations of the same tests between one year and another are used. None of the correlations for a single year is included. Harris and Harris (1973) employed interbattery factor analysis in identifying the mental abilities associated with children's concept achievements. One battery of tests measured the mental abilities and the other battery the concept achievements in language arts, mathematics, science, and social studies. In the present study, the interbattery analysis was used mainly to ascertain the factors and the stability of the test loadings for each measurement interval, that is, year 1-2, year 2-3, and year 1-3, for each of the four longitudinal blocks of students.

Two other studies were conducted involving the CLD battery. Some of the students in Grade 6 who received the CLD batteries also were administered 11 educational achievement tests. An interbattery analysis was made of these measures, using the CLD measures as one battery and the educational achievement measures as another battery. Students in Grades 9 and 12 who received the CLD battery were also administered eight selected Project Talent measures. An interbattery analysis was made at each grade of the resulting cross-correlation matrices of the CLD and Project Talent tests.

RESULTS BASED ON EACH ANNUAL MEASUREMENT

From 20 to 28 CLD tests were administered to each of the four longitudinal blocks of students each year. The 12 resulting correlation matrices have from 190 to 378 entries and are presented in Tables TR 4-1 to 4-12 of Technical Report No. 495 (Sipple, Allen & Klausmeier, 1979). The related means and standard deviations for the tests are given in Table A4-1 in the appendix of this book.

A summary and synthesis of these matrices is presented in this chapter by indicating the number of correlation coefficients that fell within four numerical ranges within each battery each year and also across the batteries each year. The matrices were submitted to factor analysis, and the factors for each grade obtained by the strategy involving the three analysis methods and two rotation procedures described earlier are reported.

Correlations Within Batteries and Across Batteries

Before considering the correlation coefficients, we will recall that the design of the longitudinal study included control groups of two kinds: one to evaluate the effects of repeated testing of the students of each longitudinal block and a second to determine the effects of being born one and two years later than the students of each of the longitudinal blocks. The cohort control students were included in the total sample when deriving the correlation coefficients based on the annual measurements in order to maximize the reliability of the test scores and also the stability of the factors obtained. Column 1 of Table A4-2 in the appendix shows that the *N*s at each successive grade within each block of students was substantially lower. The reason is that the *N*s for Grades 1, 4, 7, and 10 include not only the longitudinal students but also the students of their two cohort control groups. The *N*s for Grades 2, 5, 8 and 11 include the longitudinal students and the students of their one cohort control group; while the *N*s for Grades 3, 6, 9, and 12 include only the longitudinal students.

In addition to these differences in *N*s due to the control groups, attrition occurred in both the longitudinal and the control groups between the successive years of measurement. Attrition was primarily due to students leaving the city or entering parochial schools, and the amount of attrition was not the same for the four longitudinal groups. Longitudinal Block A was more affected than Blocks B, C, and D.

In Table A4-2 in the appendix the direction and the size of the rs are indicated in four categories: -.01 to the lowest negative *r* found, .00 to +.19, +.20 to +.39, and +.40 to the highest positive *r* obtained. Why were these categories established? The *N*s were not the same for the four groups or the years of measurement. The smallest *N* for any year was 62. The .01 and .05 levels of significance for an *N* of 62 are .33 and .25, respectively. The similar levels for the largest *N*, 151, are .21 and .16, respectively. Very few negative *r*s were found above -.20 so only one category was set here.

The percentages of the incidences of correlation coefficients (rs) of these sizes are given for each grade and then are summed for all 12 grades in Table A4-2 in the appendix. Similarly, the incidences for each concept battery and all four batteries in totality are provided in the table. Study of Table A4-2 is necessary for identifying any particular relationship for any year of testing or for any concept in which the reader may be interested.

At this time a few conclusions concerning the pattern of correlation coefficients for the tests within each of the four batteries and also for the tests across the four batteries are indicated. A larger percent of higher positive rs and a lower percent of lower and negative rs were found for the tests within each of the four batteries than for the tests across the different batteries. For the 12 grades combined, 48 percent of the within battery *r*s were .20 or higher but only 27 percent of the across battery *r*s were.

The patterns of within and across battery positive and negative rs were similar from Grade 1 through Grade 5; a sharp drop occurred from Grade 5 to Grade 6 in the percent of negative rs. In Grade 7 the percentage of *r*s .20 and higher both within the four batteries and across them was high and it remained high thereafter through Grade 12. From 66 to 88 percent of the within *r*s were .20 or higher in Grade 7 through 12. Unexpectedly and unaccountably, however, there was also an increase in the percentage of negative rs from Grade 6 to Grade 7 but an equal decrease from Grade 7 to Grade 8.

We may now compare the rs for the four concepts. The within battery patterns of rs were very similar for *cutting tool, tree*, and *equilateral triangle*. There was a higher percentage of higher rs and a lower percentage of lower rs for *noun*. The percentages of low to high across battery rs were similar for all four batteries.

We have observed likenesses and differences in the patterns of the rs at the different grades and for the four concepts. How these likenesses and differences are reflected in the factor structure is now indicated.

Factors Identified for Each Grade 1-12

The correlation matrices at each grade were submitted to factor analysis using the procedures reported earlier in this chapter. The loadings and factors for Grades 1-12 are reported in Table A4-3 of the appendix. The loadings and factors for Grade 6 are given in Table 4-1 to illustrate how the factors were identified.

For Grade 6, one factor was identified for each of the four concepts: *cutting tool, tree, equilateral triangle*, and *noun*. As shown in Table 4-1, three tests of the cutting tool battery—understanding taxonomic relations, understanding principles, and problem solving—had loadings of .33 or higher in six, six, and four of the solutions, respectively. This factor may be designated *cutting tool*. The tests of the classificatory level and the formal level of *cutting tool* loaded less than .30 in four of the six solutions.

Five tests of the tree battery—classificatory, formal, and the three

Table 4-1. Observed Loadings Derived by Six Factor Analytic Procedures, Grade 6[a]

	Orthogonal			*Oblique*		
	P	*M*	*H*[b]	*P*	*M*	*H*
Comparable Common Factor 1						
CTtax	57	33	63	62	54	73
CTpr	79	89	35	55	33	37
CTps			73	80	100	85
Comparable Common Factor 2						
TRtax	79	72	68	69	84	74
TRpr	64	60	72	81	75	75
TRps	50	42	61	63	69	72
TRcl	65	51	52	74	53	56
TRfo	71	72	45	45	45	47
Comparable Common Factor 3						
ETtax	73	72	68	78	63	67
ETfo	77	66	63	74	69	67
ETpr	62	45	71	73	72	72
ETps	70	62	45	62	43	44
Comparable Common Factor 4						
NNfo	56	69	64	69	65	66
NNps	61	63	61	43	63	57
NNcl	69	47	56	74	44	55
NNtax	44		67	60	69	65
TRcl	-36			-56	-45	-47

[a] Includes values greater than .30. Decimals have been omitted.

[b] P = Principal Components, M = UMLFA, H = Harris R-S^2.

uses—loaded .42 or higher in all six solutions of the next factor, called *tree*. On factor 3, *equilateral triangle*, the formal level test and the three uses tests of the equilateral triangle battery loaded .43 or higher in all six solutions. The formal level test and the three uses test of the noun battery loaded on the fourth factor, *noun*, and the classificatory level test of the tree battery had negative loadings in four of the six solutions.

I would digress briefly to point out that although both the concrete level and identity level tests of the tree battery were administered in Grade 6, and not in Grade 7 and thereafter, students when in Grade 6 scored near 100 percent correct on these tests. The high percentage correct accounts for the loadings for the concrete and identity level tests of less than .30 on the four factors identified for Grade 6.

Table 4-2 gives the Separate Concept Factors and Combined Factors identified for each Grade 1 through 12. The tests that loaded .30 or higher in four or more of the six solutions are indicated by the abbreviation of the test entered in the appropriate column.

Table 4-2. Summary of Factors for Grade 1 Through Grade 12

	Separate Concept Factors				*Combined Factors*	
Block and Grade	*Cutting Tool (CT)*	*Tree (TR)*	*Equilateral Triangle (ET)*	*Noun (NN)*		*Number of Factors*
	[a]*co id cl fo pr tx ps*	*co id cl fo pr tx ps*	*co id cl fo pr tx ps*	*co id cl fo pr tx pe*		
D12	fo pr tx ps	fo pr tx ps	fo pr tx ps	fo pr tx ps	[ETcl, NNcl]	5
D11	fo tx ps TRcl	fo pr ps	fo pr tx ps	cl fo pr tx ps	[ETcl, TRtx]	5
D10	cl fo pr tx ps	fo pr tx ps	fo pr tx ps	cl fo pr tx ps		4
C9	fo pr tx ps	fo pr tx ps	fo pr tx ps	cl fo pr tx ps		4
C8	cl fo pr tx ps ETtx	fo pr tx ps	fo pr ps TRcl	cl fo pr tx ps		4
C7					[CTfo, pr, tx, ps, ETfo, pr, tx, ps]	
		cl fo pr tx ps		cl fo pr tx ps	[CTcl, TRcl, ETcl]	4
B6	pr tx ps	cl fo pr tx ps	fo pr tx ps	cl fo tx ps		4
B5	co id cl					
	fo ps ETcl	pr tx ps	fo pr tx ps	fo pr tx ps	[TRid, NNcl]	6
B4		co cl ETco		co id TRid		
	cl fo pr tx ps	pr tx ps		cl fo pr tx ps	[ETid, cl, fo, pr, tx, ps, TRfo, CTid]	6
A3				co id ETpr		
	pr tx ps	cl pr ps ETcl	fo tx ps	cl fo pr tx ps	[CTcl, TRid] [CTfo, TRco]	7
A2	co id	id cl	id cl	co id cl		
	pr ps (-)ETfo	pr tx ps CTtx			(-)[CTcl, ETtx]	7
A1	co id[b]	co id	co id cl	co id		
	[b] fo pr tx ps			cl tx	[TRtx, ETfo]	7

[a] Names of tests: co = concrete, id = identity, cl = classificatory, fo = formal, pr = principle, tx = taxonomic relations, ps = problem solving.

[b] Two rows indicate two different factors for the particular battery of tests.

Two combinations of test loadings are treated in the ensuing discussion, and in Table 4-2, as a separate concept factor. A separate concept factor is one on which only tests of the particular concept battery loaded, or on which two or more tests of the particular battery loaded and not more than one test from another battery. Factors based on any other combination of test loadings appear in the right column of Table 4-2 as "Combined Factors."

It may be convenient to consider the separate concept factors and the combined factors that were identified for each of four longitudinal blocks of students. A total of seven factors was identified for each Grade 1, 2, and 3. In Grades 1 and 2 a separate factor was identified for each of the four concepts—*cutting tool*, *tree*, *equilateral triangle*, and *noun*—involving some combination of the concrete, identity, and classificatory levels of attainment. In Grade 3 a similar factor was found only for the concrete and identity level of *noun*. A separate concept factor involving some combination of the formal level, understanding principles, understanding taxonomic relations, and problem solving, was found for *cutting tool* in each Grade 1, 2, and 3; for *tree* in Grades 2 and 3, and for *equilateral triangle* and *noun* in Grade 3. A test from a second concept battery loaded on four of the 17 separate concept factors identified for Grades 1, 2, and 3. Three of these tests were from the equilateral triangle battery and the other from the cutting tool battery. No logical or theoretical basis was identified to account for these equilateral triangle tests loading on these factors rather than on the separate equilateral triangle factor. Apparently the abilities of children of this age associated with *equilateral triangle* are somewhat different from those for the other three concepts.

Four combined factors that include loadings from some combination of tests from two or more batteries but without a clear majority from any one battery were identified: one for each Grade 1 and 2 and two for Grade 3. That such factors were identified unrelated to any one of the four concepts and also to any combination of the levels or uses of the concepts probably is due to the undifferentiated nature of the cognitive development of the boys and girls at this time in their lives.

A total of six factors was found for each Grade 4 and 5 and four for Grade 6. One or more separate concept factors were found in each grade, except in Grade 4 where six equilateral triangle tests and two tests from other batteries loaded on a combined factor. Factors related to the levels of concept attainment continued to appear as separate concept factors in Grades 4 and 5, two in Grade 4, and one in Grade 5. As noted earlier in this chapter, one separate concept factor was found for each of the four concepts in Grade 6. Based on these results in Grades 4, 5, and 6 we may infer that from Grades 4 to 6 the abilities associated with cognitive development as measured by these batteries became more integrated and also more highly differentiated according to the contents, or substance, of the four concepts.

A total of four factors was identified for each Grade 7, 8, and 9. In Grade 7 two separate concept factors were identified, one for *tree* and one for *noun*. Two combined factors were identified, one of which included four tests of *equilateral triangle* and four of *cutting tool*. The other combined factor had loadings from the classificatory level test of *cutting tool*, *tree*, and *equilateral triangle*. Despite this unexpected factor pattern for Grade 7, one separate factor for each of the four concepts was identified for Grade 8 and for Grade 9. As for Grade 6, no combined factor was identified in Grades 8 and 9.

Why does the factor structure for Grade 7 differ from that for Grades 6, 8, and 9, which are very similar? The students of Longitudinal Block C attended the same junior high school when in Grades 7 and 8, and also received the identical tests each year. The Grade 7 students had previously been enrolled in Grade 6 in the same elementary schools as the Grade 6 students were. The tests in Grades 6, 7, and 8, were administered by the same administrator. Based on these considerations we may conjecture that unidentifiable testing conditions involving the *cutting tool* and *equilateral triangle* batteries were associated with the changed structure, or that a temporary but significant change occurred in the cognitive functioning of students between Grades 6 and 7 and again between Grades 7 and 8. The first conjecture would appear to be more probable than the second.

A total of four factors was identified for Grade 10 and five for each Grade 11 and Grade 12. In Grades 10, 11, and 12, a separate factor was identified for each of the four concepts. A combined factor was found in Grade 11 based on the classificatory test of *equilateral triangle* and the taxonomic relations test of *tree*. A combined factor for Grade 12 involved the classificatory level test of *equilateral triangle* and the classificatory level test of *noun*.

Before proceeding to the final conclusions based on the factor patterns for Grades 1-12, we should note that the classificatory level test of one or more of the four batteries loaded on one of the combined factors at each Grade 2, 3, 4, 5, 7, 11, and 12. Also in Grades 3, 5, 8, and 11 a classificatory test loaded on one of the otherwise separate concept factors. This indicates that the ability to classify is somewhat different from the abilities associated with attaining the other levels of the concepts and with understanding principles, understanding taxonomic relations, and problem solving.

Another interesting finding is related to differences in the factors identified for the four batteries. The tests of the *noun* battery most consistently loaded on a separate noun factor in each Grade 1 through 12 while the tests of *equilateral triangle* least consistently loaded on a separate equilateral triangle factor.

Returning to the factors identified for Grades 1 through 12, we conclude that cognitive development from Grades 1 through 6 is characterized by increasingly stable performance from one year to the next, greater integration of the abilities underlying the performances on each test battery, and increasing differentiation according to the four content domains measured by the four batteries. Harris and Harris (1973) earlier found that concept achievement among fifth-grade boys and girls was differentiated clearly according to the subject matters of language arts, mathematics, science, and social studies. Shaycoft (1967) identified a major group factor at each Grade 9 and Grade 12 for English, mathematics, spatial relations, and technical, the latter based mainly on science tests. These three studies strongly support the idea that the organization of students' cognitive abilities from Grade 5 onward is closely related to the content of the subject matters taught in school.

Factors Identified by Analysis of Grade 1 and Grade 3 Levels of Concept Attainment Measures

We observed in the preceding section that in Grades 1 and 2 at least one separate factor was found for each concept on which some combination of the tests of the concrete, identity, and classificatory levels of attainment loaded. In Grades 3 and 4, a concrete identity factor was found for *noun.* To arrive at a more complete picture of the nature of the development of the levels of concept attainment from Grade 1 through Grade 3, a factor analysis was performed based on combined level scores. The combined level scores for Grade 1 were obtained by adding the student's scores on the four concrete level tests, that is, one concrete level test for each of the four concepts, the four identity level tests, the four classificatory level tests, and the four formal level tests. The same procedure was followed to get the student's four combined level scores when in Grade 3. Thus, each student had eight combined level scores. Correlations among these combined level scores were obtained for each Grade 1 and Grade 3 and also between Grades 1 and 3. The resulting correlation

Table 4-3. Correlation Matrix for Combined Level of Attainment Tests for the Grade 1-3 Interval

Test and Grade	*CO1*	*ID1*	*CL1*	*FO1*	*CO3*	*ID3*	*CL3*	*FO3*
CO1								
ID1	.53							
CL1	.48	.33						
FO1	.16	.16	.27					
CO3	.30	.22	.33	-.09				
ID3	.35	.26	.42	.13	.63			
CL3	.35	.18	.48	.16	.35	.61		
FO3	.08	.11	.14	.37	-.02	.11	.33	

matrix appears in Table 4-3. Visual inspection of the table shows the highest positive *r*s between the concrete and identity levels of Grade 1 (.53), the concrete and identity levels of Grade 3 (.63), the concrete and classificatory levels of Grade 1 (.48), and the identity and classificatory levels of Grade 3 (.61). The *r* between the classificatory level tests administered in Grades 1 and 3 was .48 while the *r* between the formal level tests for Grades 1 and 3 was .37.

In Table 4-4 the Grade 1-3 factors identified by the Principal Components method using an orthogonal rotation are shown. Four factors were identified; only loadings above .30 were used in interpreting the factors.

Inspection of Table 4-4 indicates that the Grade 3 concrete, identity, and classificatory tests had the high loadings on factor 1, and the concrete, identity, and classificatory tests for Grade 1 had the high loadings on factor 2. Clearly then these three lowest levels of attainment combine for a factor at both Grade 1 and Grade 3. Furthermore, and in accord with the idea that attainment of the classificatory level follows attainment of the identity level, the loading for the classificatory test for Grade 3 was .74 on factor 1 but only .35 on factor 2 for Grade 1. The third factor was a classificatory and formal level factor for Grade 1 while the fourth was a classificatory and formal level factor for Grade 3. The loading of the classificatory test was lower for the Grade 3 factor than for the Grade 1 factor, whereas the loading for the formal level was higher for Grade 3 than Grade 1. These changes in loadings are in accord with the idea that attainment of the formal level follows attainment of the classificatory level.

The results of this factor analysis support the interpretation of the levels factors for Grades 1 through 3 reported in the prior section of this chapter and also the principle stated in Chapter 3 that successive

Table 4-4. Level of Attainment Factors Identified for the Grade 1-3 Interval[a]

Level and Grade	*Factor*			
	1	*2*	*3*	*4*
CO1	29	77	22	-06
ID1	07	91	01	12
CL1	50	35	58	-13
FO1	-07	07	84	29
CO3	80	20	-20	-11
ID3	87	14	10	06
CL3	74	08	24	33
FO3	08	05	17	94
Percent of total factor variance	37	26	20	18
Percent of total variance	29	20	16	14

[a] Decimals have been omitted.

levels of the same concept are attained in an invariant sequence. As indicated in Chapter 3, the large majority of students in the longitudinal study mastered the successive levels of the four concepts in an invariant sequence.

To complete this discussion of the levels of attainment, we note that nearly all the students when in Grade 1 had already mastered the concrete and identity levels of *cutting tool*, *tree*, and *equilateral triangle*. Therefore, we could not expect to get a separate factor for these earliest levels of attainment. Study of younger children of preschool age is required to identify the beginnings of these levels of concept attainment.

RESULTS BASED ON CLD LONGITUDINAL MEASUREMENTS

In the previous section we presented the factors identified for each Grade 1 through 12 based on the measurements made in each grade. We are now ready to examine the rs based on the measurements taken at the two one-year intervals and the one two-year interval for each of the four longitudinal groups of students.

Correlations Between Measures Taken at One- and Two-Year Intervals

The CLD batteries were administered to each of the four longitudinal blocks of students at 12-month intervals in 1973-1974, 1974-1975, and 1975-1976. The means and standard deviations based on these measurements are presented in Table A4-4 in the appendix. The related intercorrelations are given in Table 4-13 of Technical Report 495 (Sipple, Allen & Klausmeier, 1979).

Table A4-5 in the appendix contains a summary of the rs for the tests within each of the four batteries and also for the tests across the four batteries for the 12- and 24-month intervals of testing. The entries are summarized for the two 12-month intervals and for the 24-month interval between test administrations. The interested reader may refer to the table to identify specific relationships of interest. A few general conclusions follow.

At the two 12-month intervals of testing, the rs among the seven tests within each battery were generally higher than the rs across the tests of the four batteries. The within battery rs were highest for *noun*, second highest for *tree*, and about the same for *equilateral triangle* and *cutting tool*. The rs across batteries were fairly consistent for all four concepts.

The percentage of negative rs decreased from the lower to the higher grades, while the percentage of higher and positive rs increased. The sharpest increase in the percentage of higher rs occurred between the Grade 5-6 and the Grade 7-8 intervals. For the Grade 7-8 interval, 69 percent of the within *r*s were +.20 or higher and 60 percent of the across-battery *r*s were also +.20 or higher. This pattern of relatively high *r*s was also found for the remaining one-year intervals through Grade 12.

As might be expected, the percentage of negative and low positive rs was somewhat higher for the 24-month interval than for the 12-month intervals, whereas the percentage of positive and high rs was somewhat lower for the 24-month interval. To illustrate, for the two 12-month intervals, 59 percent of the within-battery *r*s were +.19 or lower, or negative, and for the 24-month interval, 68 percent were.

As indicated earlier for the 12-month intervals of testing, the percentage of *r*s +.20 and higher for the tests administered at 24-month interval was somewhat higher for *noun* than for the other concepts. In the discussion of the results of the factor analyses that follows, the relationships among the many tests will be clarified.

Factors Identified by Interbattery Analysis

The main purposes in carrying out the interbattery analyses were to identify the factors and the stability of the loadings of the same tests between the successive years of test administration to each longitudinal block of students, and also to compare the factors and test loadings obtained for the successive longitudinal blocks of students.

Tables A4-6 through 4-9 in the appendix give the results of the interbattery analyses. For the Grade 1-2-3 group of longitudinal children, seven factors were identified for each the Grade 1-2, 2-3, and 1-3 intervals of testing. For the Grade 1-2 interval, only three of the 21 tests loaded .30 or higher both years on factor 1, and only one test for factor 6 (Appendix Table A4-6). No other test loaded as high as .30 for both years. Based on this small number of loadings of .30 or higher and the large number of factors identified, no interpretable factor or pattern of test loadings is identifiable for the Grade 1-2 interval. Similarly, no interpretable pattern is identifiable for the Grade 2-3 and the Grade 1-3 interval.

How may this lack of interpretable factors and consistent test loadings be accounted for? The rate of growth on all the cognitive dimensions measured by the tests was faster during Grade 1-2-3 than for any other grades, that is, 4-5-6, 7-8-9, or 10-11-12, as inferred from increases in the means. The lack of a single interpretable factor for any of the intervals and the related totally inconsistent test loadings from one time of measurement to the next appears to reflect the true nature of cognitive development during the primary school years. The various dimensions, or items, of cognitive development are not yet stable; growth of each item is proceeding at its own differential rate unrelated to the rate of the other items.

Appendix Table A4-7 shows the factors and test loadings of the various tests for the Grade 4-5 interval, 5-6 interval, and 4-6 interval. Eight factors were identified for the Grade 4-5 interval of testing. The fifth factor shows a beginning emergence of tests with somewhat similar loadings for both Grades 4 and 5 in that the formal level test, principle test, and the test of taxonomic relations of *tree* for Grade 4 have loadings somewhat similar to the same tests for Grade 5. Similarly, the formal level, principle, and problem-solving tests of *equilateral triangle* have somewhat similar loadings for both Grades 4 and 5.

Five factors were identified for the Grade 5-6 interval. Here clear factors emerged and also consistent test loadings for three of the five factors. For factor one, four tests of *cutting tool* and four tests of *tree* have loadings above .30 in both Grades 5 and 6. *Equilateral triangle* tests load on the second factor with some consistency. Tests related to *noun* load on the fourth factor with some consistency.

Five factors were identified for the Grade 4-6 interval but there was little consistency in test loadings for any of the factors. Four tests of *noun* load somewhat similarly on factor 2 for both Grades 4 and 6. Similarly, four tests of *tree* have similar loadings on factor 4 for Grades 4 and 6.

From these results we may infer that stability in performance and integration of abilities proceed quite rapidly from Grade 5 to 6. Both the number of factors and the test loadings on the factors have become much more consistent and stable than during the primary school years.

Appendix Table A4-8 shows that one factor was identified for the Grade 7-8 interval, one for the Grade 8-9 interval, and one for the Grade 7-9 interval. For the Grade 7-8 interval, three of five tests of *cutting tool*, four of *tree*, two of *equilateral triangle*, and five of *noun* had loadings of .30 or higher for both Grades 7 and 8. Two tests—the classificatory level and understanding taxonomic relations of *equilateral triangle*—had loadings below .30 for both Grades 7 and 8. For the single factor identified for the Grade 8-9 interval, four of the five tests of *cutting tool*, four of *tree*, four of *equilateral triangle*, and five of *noun* loaded .30 or higher for each Grade 8 and 9. The three tests not loading .30 or higher at each Grade 8 and 9 were the classificatory level of *cutting tool*, *tree*, and *equilateral triangle*. The test loadings on the single factor identified for the Grade 7-9 interval were very much like those of the Grades 8-9 interval.

From the results of these interbattery analyses involving Grades 7-8-9 it is clear that stability of performance and integration of the various cognitive abilities are relatively high throughout the junior high school years.

One factor was identified for the Grade 10-11 interval, one for Grade 11-12, and one for Grade 10-12 (Table A4-9). Four tests of *cutting tool*, four of *tree*, four of *equilateral triangle*, and five of *noun* loaded .30 or higher for both Grades 10 and 11 on the single factor for the Grade 10-11 interval. The two tests loading less than .30 in both Grades 10 and 11 were the classificatory level of *tree* and of *equilateral triangle*. The loadings for the classificatory level of *cutting tool* and *noun* were only .30 and .32, respectively, in Grade 10.

On the single factor identified for the Grade 11-12 interval, four tests of *cutting tool*, four of *tree*, four of *equilateral triangle*, and four of *noun* loaded .30 or higher for both Grades 11 and 12. The three tests loading less than .30 for both grades were the classificatory level of *cutting tool*, *tree*, and *equilateral triangle*.

Four tests each of *cutting tool*, *tree*, *equilateral triangle*, and *noun* had loadings of .30 or higher for each Grade 10 and 12 on the single factor identified for the Grade 10-12 interval. None of the classificatory tests were included among the four.

Clearly, cognitive growth during the senior high school years, as during the junior high school years, is stable; and integration of abilities continues in much the same pattern as during the junior high school years. The fact that the classificatory level did not load from one time of measurement to the next as consistently as did other tests is probably due to the fact that the means for all the classificatory tests were very high. The means in percentages correct for the classificatory level tests in Grades 7, 9, and 12 were as follows: *cutting tool*—95, 100, 97; *tree*—91, 100, 100; *equilateral triangle*—93, 100, 98; and *noun*—82, 95, 98.

RELATIONSHIP BETWEEN CLD MEASURES AND EDUCATIONAL ACHIEVEMENTS

CLD theory predicts that the abilities underlying conceptual learning and development as measured by the CLD tests also underlie performances in subject fields that are heavily conceptually oriented such as mathematics, English, science, and social studies. To test this prediction, the CLD batteries and an educational achievement battery were administered to Grade 6 students. In addition, the CLD batteries and eight tests drawn from the Project Talent battery were administered to Grade 9 and Grade 12 students.

Correlations Between CLD Measures and Educational Achievement Measures in Grade 6

The CLD batteries and the Stanford Achievement tests were administered to 45 students of Longitudinal Block B when they were enrolled in Grade 6. The Grade 6 students received five tests from each of the CLD batteries, namely, the classificatory, formal, understanding principles, understanding taxonomic relations, and problem-solving tests. The separate scores obtained from the Stanford Achievement tests included arithmetic concepts, arithmetic computation, arithmetic application, vocabulary, reading, word meaning, spelling, language, listening comprehension, science, and social studies. The rs obtained among the 20 tests of the CLD battery and the rs among the 11 tests of educational achievement are given in Table A4-10 in the appendix. The cross-correlations between the 20 CLD tests and the 11 educational achievement tests are reported in Table 4-5.

Factors Identified by Interbattery Analysis of CLD Measures and Educational Achievement Measures in Grade 6

The cross-correlation matrix presented in Table 4-5 was submitted to an interbattery factor analysis (Tucker, 1958). Only one

Table 4-5. Correlations Between CLD Tests and Educational Achievement Tests, Grade 6[a]

	Arithmetic										
Test	*Concepts*	*Computation*	*Application*	*Vocabulary*	*Reading*	*Word Meaning*	*Spelling*	*Language*	*Listening Comprehension*	*Science*	*Social Studies*
CTCL	03	-18	-07	-02	01	-09	01	-06	-11	01	06
CTFO	22	35	22	14	18	16	11	16	36	22	25
CTPR	35	35	37	41	31	30	27	31	52	27	37
CTTAX	43	38	40	37	46	38	36	31	42	38	45
CTPS	31	15	33	11	13	22	10	28	19	07	08
TRCL	01	-10	05	22	-02	-12	-21	-02	10	-01	08
TRFO	37	41	38	32	26	21	26	39	52	29	38
TRPR	56	40	43	35	25	39	16	38	49	34	43
TRTAX	41	27	40	31	27	33	19	30	60	25	30
TRPS	35	22	20	21	17	15	03	23	35	15	25
ETCL	11	23	22	18	24	19	22	13	39	15	19
ETFO	36	36	31	21	21	10	11	26	30	18	28
ETPR	12	26	15	26	01	-01	-02	33	31	10	17
ETTAX	34	35	22	15	05	11	14	27	19	12	15
ETPS	04	-05	-06	02	-25	-04	-15	03	07	-11	04
NNCL	52	29	44	33	40	48	45	42	45	45	39
NNFO	68	52	63	57	55	53	53	55	53	59	56
NNPR	46	45	58	41	46	31	40	47	42	48	45
NNTAX	14	32	18	07	07	09	21	20	23	-01	07
NNPS	50	35	59	17	45	35	38	47	43	37	37

r	*N*	*Percent*
+.28 - (-.25)	116	53
+.29 - .36 ($p < .05$)	34	15
+.37 - .68 ($p < .01$)	70	32

---

[a] Decimals have been omitted.

factor was found, and it contributed 94 percent of the covariance of the cross correlations of the two batteries. Table 4-6 shows that three tests from the *cutting tool* battery, four from *tree*, three from *equilateral triangle*, and four from *noun* loaded .30 or higher on this single factor. In general, the CLD tests measuring the formal level, understanding principles, and understanding taxonomic relations from all four batteries loaded somewhat higher than did the other tests of the batteries. The classificatory level tests and problem-solving tests of all batteries loaded quite low except for *noun*, where the loadings were quite high, .65 and .64, respectively.

The loadings of the various educational achievement tests were all quite high, ranging from .51 to .78. The three highest loadings were for listening comprehension, arithmetic concepts, and arithmetic applications, respectively.

It is interesting to note that of the various CLD tests those related to *equilateral triangle* generally had lower loadings than those related to any other concept. The Stanford educational achievement tests in mathematics measured performance related to arithemtic almost exclusively. *Equilateral triangle* is a geometric concept. As shown earlier in Table 4-5, the rs between the *equilateral triangle* tests and the arithmetic achievement tests were statistically significant in a number of cases and were also higher than were those between *equilateral triangle* and the other educational achievement tests, except listening comprehension. They were not as high, however, between *equilateral triangle* and the arithmetic tests as were the rs between other CLD tests and the other areas of educational achievement.

One may infer from the factor analysis results that similar conceptualizing and problem-solving abilities underlie the students' performances on the CLD tests and also their achievements on the Stanford educational achievement tests. The two CLD batteries constructed to deal specifically with concepts in mathematics and English contributed unequally to the single factor identified. The conceptualizing abilities associated with *noun* appear to be related more powerfully to a variety of educational achievements than do those associated with *equilateral triangle*. The ability to attain concepts at the formal level and to understand principles appears to contribute substantially to educational achievement in both the mathematical and verbal domains.

Correlations Between CLD Measures and Selected Project Talent Measures in Grades 9 and 12

Project Talent is a large-scale longitudinal study of more than

Table 4-6. Interbattery Factor Between CLD Tests and Educational Achievement Tests, Grade 6[a]

	Factor
CTCL	-06
CTFO	34
CTPR	55
CTTAX	61
CTPS	29
TRCL	01
TRFO	55
TRPR	60
TRTAX	53
TRPS	34
ETCL	32
ETFO	39
ETPR	25
ETTAX	31
ETPS	-05
NNCL	65
NNFO	88
NNPR	69
NNTAX	23
NNPS	64
Vocabulary	56
Reading	58
Word Meaning	55
Arithmetic Concepts	76
Arithmetic Computation	65
Arithmetic Applications	74
Spelling	51
Language	65
Social Studies	64
Science	58
Listening Comprehension	78
Eigenvalue	20.26
Percent of sum of eigenvalues	94

[a] Decimals have been omitted.

4,000 persons who were enrolled in high school in Grades 9, 10, 11, and 12 in 1960. The persons are being followed at five-year intervals after their graduation from high school. The study is being carried out by the American Institutes for Research.

Shaycoft (1967) reported a study done as part of Project Talent. In this study about 3,500 boys and 3,600 girls who were freshmen in 1960 and seniors in 1963 in 118 different comprehensive and vocational high schools of the country were tested at both times. These students were given 36 information and achievement tests and 14 aptitude tests. Scores from the tests were analyzed to find out more about what the students had learned during their high school years and also about their emerging and possibly changing aptitudes. One result of the study was that there were gains both on the information and achievement tests during the high school years and also on the aptitude tests.

Figure 4-1 shows the one general and the four major group factors that were found to be common to both Grade 9 and Grade 12 boys and girls and also the tests that loaded on each factor. A general verbal ability factor was identified that involved nearly all of the tests used in the study. Four major group factors were identified as follows: mathematics, spatial, English, and technical information. The tests loading on three of these factors were some combination of information and achievement tests and aptitude tests.

Two tests from each of these major group factors were selected for administration to the groups of students in Grade 9 and in Grade 12 who also received CLD batteries. Each selected test had high reliability and also a high loading on the particular group factor. The two tests that met these criteria related to the group factors of mathematics, English, and technical information were information or achievement tests. The two tests related to the spatial factor were aptitude tests. The mathematics tests were called, Introductory High School Mathematics and Mathematics Information. The English tests were Capitalization and English Usage. The technical information tests were Biological Science Information and Physical Science Information. The two aptitude tests for the spatial factor were Visualization in Three Dimensions and Mechanical Reasoning. It is noted that the technical information factor might have been called a science factor inasmuch as the tests that loaded on it are from science rather than other subject fields.

Tables A4-11 and A4-12 in the appendix give the rs among the CLD tests and among the Project Talent tests for Grade 9. The meaning of the relationships among the many correlations will become more clear later in this chapter where the results of the factor analyses are given. Therefore only the primary relationships are summarized in this discussion. The reader may refer to the appendix tables for more information about the relationships between particular variables that may be of interest. Forty-six percent of the *r*s within and across the four CLD batteries were not statistically significant at the .05 level. Twelve percent were statistically significant at the .05 level and 42 percent at the .01 level. All of the *r*s between the eight Project Talent tests in Grade 9 were statistically significant at the .01 level.

Twenty-six percent of the cross-correlations between the CLD tests and the Project Talent tests in Grade 9 were not statistically significant, as is indicated in Table 4-7. Seventeen percent of the rs were statistically significant at the .05 level and 57 percent at the .01

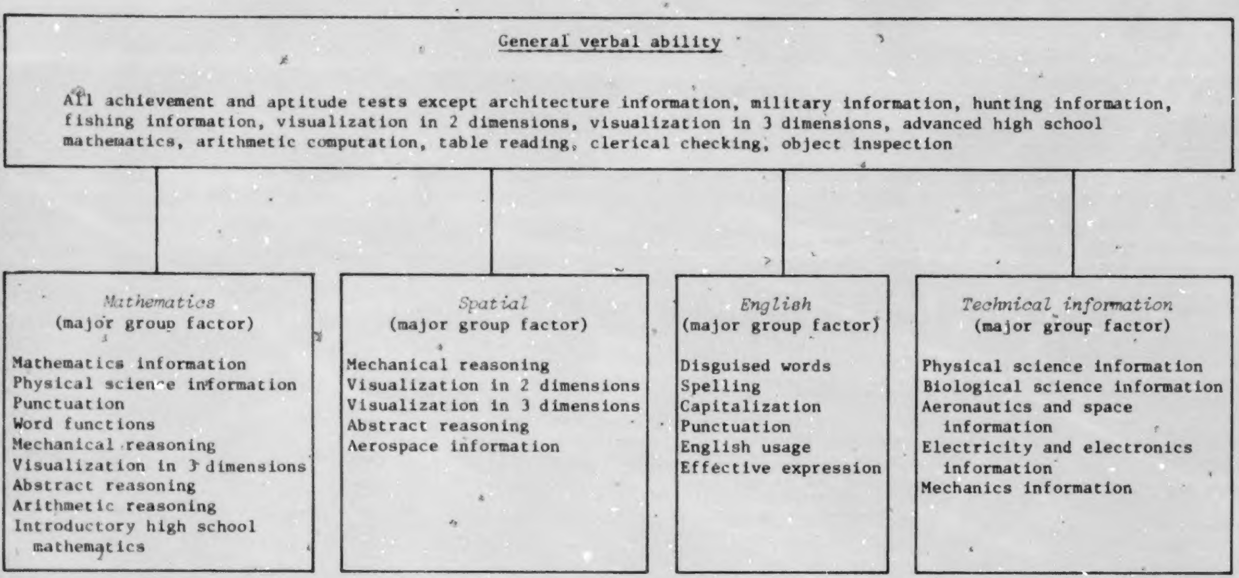
General verbal ability
All achievement and aptitude tests except architecture information, military information, hunting information, fishing information, visualization in 2 dimensions, visualization in 3 dimensions, advanced high school mathematics, arithmetic computation, table reading, clerical checking, object inspection
Mathematics
(major group factor)
Mathematics information
Physical science information
Punctuation
Word functions
Mechanical reasoning
Visualization in 3 dimensions
Abstract reasoning
Arithmetic reasoning
Introductory high school mathematics
Spatial
(major group factor)
Mechanical reasoning
Visualization in 2 dimensions
Visualization in 3 dimensions
Abstract reasoning
Aerospace information
English
(major group factor)
Disguised words
Spelling
Capitalization
Punctuation
English usage
Effective expression
Technical information
(major group factor)
Physical science information
Biological science information
Aeronautics and space information
Electricity and electronics information
Mechanics information

Figure 4-1. General and Major Group Factors Common to Grades 9 and 12 and to Boys and Girls (Identified by Shaycoft, 1967)

Table 4-7. Correlations Between CLD Tests and Project Talent Tests, Grade 9[a]

	Introductory High School Math	*Math Information*	*Mechanical Reasoning*	*Visualization in 3D*	*Capitalization*	*English Usage*	*Biological Science Information*	*Physical Science Information*
CTFO	34	38	31	24	31	35	25	28
CTPR	41	37	40	29	36	40	33	36
CTTAX	32	33	21	16	12	17	06	29
CTPS	37	32	35	13	36	32	37	33
TRFO	44	43	47	34	37	43	42	36
TRPR	40	40	42	21	29	41	39	32
TRTAX	35	30	37	23	29	37	35	31
TRPS	43	45	21	12	31	45	28	24
ETFO	31	44	41	21	33	32	23	30
ETPR	-02	12	18	13	22	19	16	01
ETTAX	22	31	16	24	25	22	17	25
ETPS	22	37	27	42	33	28	13	19
NNCL	19	26	18	25	24	20	23	23
NNFO	38	43	32	17	43	37	42	30
NNPR	33	52	39	27	35	37	37	33
NNTAX	37	51	40	23	47	38	40	43
NNPS	20	30	17	05	19	19	14	18

r	*N*	*Percent*
.22 - (-.02)	36	26
.23 - .29 ($p < .05$)	23	17
.30 - .52 ($p < .01$)	77	57

[a] Decimals have been omitted.

level. Nearly all of the cross-correlations between the CLD tests of the formal level and the understanding principles and Project Talent tests were statistically significant. The tests of understanding taxonomic relationships for the concepts *tree* and *noun* correlated high with the Project Talent tests. The problem-solving tests of *cutting tool* and *tree* also correlated high with the Project Talent tests. The CLD tests correlated lower with visualization in three dimensions than with the other Project Talent tests.

Tables A4-13 and A4-14 in the appendix give the rs among the tests of the CLD battery and among the tests of the Project Talent battery for Grade 12. The within battery *r*s among the CLD tests of the formal level, understanding principles, understanding taxonomic relations, and problem solving were generally statistically significant, often at the .01 level. Related to Project Talent, the *r*s between the two mathematics tests were high, .85, as were also those between the two science tests, .69, and the two spatial relations tests, .66. The *r* between the tests of capitalization and English usage was quite low, .36.

Twenty-seven percent of the cross-correlations at Grade 12 between CLD tests and Project Talent tests were not statistically significant, as is indicated in Table 4-8. Fifteen percent were significant at the .05 level and 58 percent at the .01 level. These percentages are much the same as they were for Grade 9; however, the four CLD batteries have slightly different patterns of cross-correlations with the Project Talent tests for Grades 9 and 12. In Grade 12 the tests of the *equilateral triangle* battery correlate quite high with the Project Talent tests. This was not true for Grade 9. Whereas visualization in three dimensions correlated quite low with the CLD battery tests in Grade 9, the rs were relatively high in Grade 12. Similarly, the rs involving physical science information are somewhat lower in Grade 12 than they were in Grade 9. These changed patterns of rs may be related to the fact that students take many of the same required subjects Grade 1 through Grade 9 but with the many available electives they take different subject matters in Grades 10, 11, and 12.

Factors Identified by Interbattery Analysis of CLD Measures and Project Talent Measures in Grade 9 and 12

Table 4-9 shows one interbattery factor and the loadings of the 17 CLD tests and the eight Project Talent tests administered at each Grade 9 and 12. Ninety-seven percent of the total variance was

Table 4-8. Correlations Between CLD Tests and Project Talent Tests, Grade 12[a]

	Introductory High School Math	*Math Information*	*Mechanical Reasoning*	*Visualization in 3D*	*Capitalization*	*English Usage*	*Biological Science Information*	*Physical Science Information*
CTFO	34	30	31	36	27	38	44	21
CTPR	27	32	30	25	33	46	33	13
CTTAX	02	10	25	14	17	33	07	03
CTPS	31	24	38	44	15	46	31	22
TRFO	18	19	14	25	32	17	09	-06
TRPR	47	48	47	52	30	56	43	27
TRTAX	39	35	19	30	19	29	24	13
TRPS	31	29	36	39	34	37	29	21
ETFO	39	43	20	39	35	49	33	33
ETPR	43	45	24	48	36	49	37	28
ETTAX	41	51	31	37	53	47	32	19
ETPS	67	64	56	48	41	51	50	45
NNCL	29	16	-02	10	-07	18	09	-01
NNFO	38	40	18	23	15	48	43	29
NNPR	30	29	08	23	21	45	37	14
NNTAX	36	37	19	29	22	52	45	23
NNPS	45	57	30	32	07	46	47	37

r	*N*	*Percent*
.22 - (-.07)	37	27
.23 - .29 ($p < .05$)	20	15
.30 - .67 ($p < .01$)	79	58

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[a] Decimals have been omitted.

Table 4-9. Interbattery Factor Between CLD Tests and Project Talent Tests, Grade 9, Grade 12[a]

Test	*Number of Items in Test*	*Factor*	
		Grade 9	*Grade 12*
CTFO	18	46	47
CTPR	10	54	43
CTTAX	8	31	20
CTPS	5	48	46
TRFO	35	61	23
TRPR	20	53	64
TRTAX	8	48	39
TRPS	10	47	46
ETFO	18	48	53
ETPR	11	18	56
ETTAX	8	34	56
ETPS	6	41	75
NNCL	8	32	15
NNFO	25	53	47
NNPR	10	55	39
NNTAX	8	60	49
NNPS	5	27	56
Introductory High School Math	24	70	77
Math Information	23	81	79
Mechanical Reasoning	20	69	58
Visualization in 3D	16	47	71
Capitalization	33	68	55
English Usage	25	71	89
Biological Science Information	12	63	72
Physical Science Information	18	62	48
Eigenvalues		13.07	15.17
Percent of sum of eigenvalues		97	94

[a]Decimals have been omitted.

accounted for by the eight Project Talent tests and 17 CLD tests in Grade 9 and 94 percent in Grade 12.

The similarity of the loadings for Grade 9 and Grade 12 are striking. All eight Project Talent tests loaded quite high at both grades although Visualization in Three Dimensions, English Usage, and Biological Science Information were somewhat higher in Grade 12 than in Grade 9 while Mechanical Reasoning, Capitalization, and Physical Science Information loaded somewhat higher in Grade 9 than in Grade 12. The pattern of the loadings for the CLD tests were similar to Project Talent, although in general, the loadings are lower for the separate CLD tests. This is to be expected since each battery contributes 50 percent of the covariance for each factor identified, and there are 17 CLD tests and 8 Project Talent tests. Twelve of the 17 CLD tests had loadings of .30 or higher in both Grades 9 and 12. Six CLD tests loaded .10 or higher in Grade 9 than in Grade 12;

five loaded similarly higher in Grade 12 than in Grade 9. The *cutting tool* and *noun* tests tended to load higher in Grade 9, and the *equilateral triangle* tests loaded higher in Grade 12.

We have seen that the CLD tests and the educational achievements in Grade 6 and the CLD tests and the educational achievement and aptitude tests in Grades 9 and 12 loaded on the same factor. How may this factor structure be interpreted? It appears that the conceptualizing and problem-solving abilities tapped by the CLD measures also underlie students' achievement in mathematics, English, science, and other subjects that have a heavy emphasis on concepts, principles, structures of knowledge, and problem solving. It will be recalled that the CLD levels tests measure the operations necessary for attaining the respective levels of the concepts, while the other three tests measure the understanding of principles, the understanding of taxonomic relations, and problem-solving skills. Differential development of the abilities underlying these aspects of cognitive development appear to be powerfully related to how well students achieve.

Chapter 5

Logical Reasoning in Middle Childhood: A Study of the Piagetian Concrete Operations Stage

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GENERAL INTRODUCTION

Any person setting out to conduct normative research based upon Piagetian theory does so with genuine feelings of humility and trepidation. Humility appears appropriate in the face of the enormous volume of original ideas and associated research that have come from Geneva. The sheer amount of Piaget's published works alone (over 1,500 citations as of 1974) probably ranks him with such individuals as Wundt and Freud and presents major bibliographic and archival difficulties (see the *Catalog of the Jean Piaget Archives*, and the January 1970 issue of the *American Psychologist* (1970, *25*(1), 65-79) for a comprehensive listing of his published works up to that time). One then must add the work of Piaget's various collaborators (e.g., see the numerous volumes of the *Études D'Épistémologie Génétique* series) and large number of secondary and derived studies currently available in the literature (Flavell, 1970). Feelings of trepidation arise when the complexities and operational vagueness (from the typical Anglo-American logical positivist perspective) of the Piagetian system become apparent. Piaget's theory quite naturally is a product of the European tradition of philosophical and psychological inquiry with emphases upon logical analysis and clinical assessment of children in quasi-natural settings. As such it represents an orientation somewhat different from the Western experimental and psychometric traditions.

Nonetheless, Piagetian theory and the inspired research is itself an organic entity that must be viewed as changing and modifiable over historical time (see Piaget, 1970a, p. 703, fn.). Thus, additional empirical research that seeks to evaluate the developmental congruence between the stage independent and stage dependent assumptions of Piagetian theory and the changing behavioral patterns of growing children is appropriate and is essential to a better understanding of genetic epistemology. It is in this spirit that the present investigation was undertaken. We sought to evaluate the performances of a group of children aged 6 to 15 years on a representative array of logical reasoning tasks in a modified cross-sequential sampling and assessment design (Wohlwill, 1973). As the age range implies, the major focus was the concrete operations period of middle childhood.

STAGE INDEPENDENT ASPECTS

It has been suggested that these general aspects of the Piagetian "world view" (Reese & Overton, 1970) may be distinguished by two questions; the synchronic and the diachronic (Flavell, 1963, pp. 262-264; Feldman & Toulmin, 1976). The synchronic aspects of the Piagetian system concern the complex of events native to a particular time period or stage and largely ignore the historical antecedents (genetic precursors in Piaget's case) and transition factors or processes that govern cognitive growth across the life span. In contrast, diachronic aspects refer to the stage independent assumptions of the general theory and emphasize the change processes that govern the transitions across the major and minor developmental subperiods. In this latter category, Piaget identifies four major determinants of cognitive growth. These are (1) organic growth factors that are biologically constrained but *not* reducible to heredity or genetic preprogramming alone, (2) functional exercise and experience with the physical world, (3) social transmission, and (4) the equilibration process (Piaget & Inhelder, 1969, pp. 154-159). Maturation, as Piaget makes clear

> is undoubtedly never independent of a certain functional exercise where experience plays a role (and further) . . . the maturation of the nervous system simply opens up a series of possibilities . . . without giving rise to an immediate actualization of these possibilities so long as the conditions of material experience or social interaction do not bring about this actualization (and finally), this actualization presupposes certain conditions of physical experience such as the manipulation of objects, etc., which is also essential for logic, and certain social conditions such as the regulated exchange of information, mutual control, etc. (Piaget, 1967, pp. 119-120).

Of course the most important and unique determining process for Piaget (indeed it may be seen as assuming distinctly superordinate status) is equilibration. As we have stated previously,

> Equilibration refers to the self-regulatory process which governs the functional relations within the individual's action systems and provides the conceptual setting for the organism-environment interactions. Attempts to designate operational definitions for the equilibration processes and related equilibrium states have typically evidenced circular reasoning and have been restricted to differential response probabilities in rather specific task formats, such as conservation measurement (Flavell, 1963, pp. 244-249; Furth, 1969, pp. 261). In certain instances the term is used to describe the *internal* organizing factors which underlie sensorimotor action schemes and higher level representational schemas and their coordination (Langer, 1970). In others the equilibration "mechanics" are said to mediate the macro-level exchanges between the active organism and its surrounding milieu. In the latter sense, a state of equilibrium is perhaps most readily viewed as subsuming the assimilation and accommodation processes. A preponderance of one process over the other denotes the unstable conditions typical of the initial preparatory phases of a particular stage or developmental period. A complementary, balanced relationship between the assimilatory and accommodatory aspects is the hallmark of established stage-specific functioning and serves as the defining criterion for adaptation (Klausmeier and Hooper, 1974, p. 8).

The complementary "constructs" of organization and adaptation are operationally present throughout the life span and thus provide the essential continuity in Piaget's system (the label functional invariants may be applied here). The former term (organization) leads one to a consideration of structure (and eventually stage descriptions) while the latter refers to functional aspects particularly the assimilation and accommodation dyad. Assimilation (which may be considered as temporally preceding accommodation, although it can only be abstractly viewed as a separate entity) is the incorporative aspect of an operative action or scheme, that is, "A taking in of environmental data, not in a causal mechanistic sense, but as a function of an internal structure that by its own nature seeks activity (Furth, 1969, p. 260)." Thus, incoming stimulation (aliment) is modified vis-à-vis the individual's current structural status. Correlatively, accommodation is the outgoing aspect of an operative process and signifies the modifications that occur in the individual's cognitive organization as a natural consequent of environmental intrusions.

From the Piagetian perspective, all cognitive acquisitions including object, self, and self-object relationship concepts are the outcome of an active constructive process. Thus, Piaget may be termed a "constructive rationalist" (Beilin, 1971a, p. 92). His constructivism is neither idealist nor realist in basic orientation since knowledge depends *both* upon the available mental structures and the products of physical and social experience. Such conceptual categories (taken in the Kantian sense of this term) as space, time, relationships, classes, and quantitative invariants are not present at the onset but develop over time in a series of interesting evolutions. These interesting evolutions are marked by definite systematic structural changes that involve the stage dependent aspects of Piagetian theory.

STAGE DEPENDENT ASPECTS

Stages and developmental periods are perhaps the best known and most controversial component of the Piagetian system. Recent conceptual analyses of the Genevan version of developmental stages include essays by Beilin (1971b), Brainerd (1978c), Feldman and Toulmin (1976), Flavell (1971), Hooper (1973), Pinard and Laurendeau (1969), and Wohlwill (1973, chapter IX, pp. 190-239). These accounts have highlighted the theoretical emphases and associated empirical issues associated with stages as descriptive, metaphorical, and (potentially) explanatory constructs. Piaget identifies four major criteria for cognitive stages, hierarchization, consolidation, integration and structuring, and equilibration (Piaget, 1956, 1960, 1970a).

> Hierarchization refers to the well-known postulate of stage invariance or transitivity of progression throughout the developmental sequence. Later stages or levels can never precede the earlier acquisitions; i.e., in the sequence of stages, A-B-C-D, all individuals demonstrate C capabilities after Stage B and prior to Stage D. Integration (per se) is a closely related concept which specifies that subsequent stages or levels "transform" earlier stage behavior into new structural entities. Prior stages are not simply replaced (in an additive or substitutive fashion), but rather become subordinated and subsumed within the higher level functioning. This implies that later stage-related behavioral manifestations cannot be exhaustively (i.e., the complete antecedent-consequent version of efficient causation) predicted or causally linked from information specific to earlier modes of responding although the converse relationship may hold (Hooper, 1973, pp. 303-304).

This last property or attribute of stage progressions concerns the "emergence" assumptions of Piaget's theory that is shared by all organismic viewpoints (Nagel, 1957; Reese & Overton, 1970).

Consolidation refers to the fact that each stage or period has a preparatory and stabilization phase associated with it (indeed, each intermediate stage in the Piagetian system is viewed as the culmination of all those that have come before and the starting point for all those to follow). Early stage-specific behaviors are transitory and "reminiscent" of the earlier modes of functioning. Typically old means (structures) are applied to the new ends (functional demands), Werner (1957). Gradually the child's new response repertory becomes generalized and more stable, which in turn sets the stage for the next developmental transition via the equilibration dynamic.

The structural and integration assumption is unquestionably the most salient aspect of a meaningful stage designation for Piaget. As one of us has commented previously:

> All the organizational simplicity and potential explanatory parsimony follows from the *structures d'ensemble* postulate. For Piagetian theory the behavioral outcomes of this assumption have been described under various labels, e.g., interrelationships between responses to different tasks involving the same operations (Wohlwill, 1963), operational convergence (Beilin, 1965), functional correspondence (Hooper, 1973), significant interitem correlations (Braine, 1959; Shantz, 1967), complete functional interdependence and organic connections (Pinard and Laurendeau, 1969), concurrences (Flavell, 1970, 1971), and interpatterning among sets of developing responses (Wohlwill, 1973). All of these designations highlight the intraindividual and interindividual uniformity across response domains which is to be expected of persons "located" in a given stage of development. Moreover, contrary to Brainerd's (1978c) conclusions, the *integration* assumption implies more than a merely additive notion of each successive stage including the earlier stage's dominant response prototypes, as subcases within the individual's behavioral repertory. Rather, the later (chronologically defined) and more complex (logically defined) stages or levels of development effectively transform and reorganize the earlier levels of responding into new functional entities. Thus lower order responses become subordinated in terms of their organizational properties as higher stages are achieved and assume different functional roles in contrast to those of the earlier periods. This is the essence of the qualitative changes associated with stage transitions in organismic theory (Pinard and Laurendeau, 1969; Werner, 1957; Werner and Kaplan, 1963) and are exemplified in the changing relationships among sensory-motor, perceptual, and logico-symbolic functioning across the human life-span. Increased differentiation (the additivity and specificity assumption) is always accompanied by a concomitant increase in hierarchic integration (Werner, 1957). What this means in behavioral terms, of course, is that lower level response modes, e.g., the perceptual, assume qualitatively different functional salience after higher level potentials, e.g., the symbolic, are attained. This in no manner denies the possibility that earlier dominant response categories may continue to evidence progressive development during later stages and may

> indeed become temporarily characteristic during periods of stress or functional disruption (Flavell, 1971, 1977; Langer, 1970).
>
> This necessarily leads to major problems in accurately "diagnosing" an individual as belonging in any developmental stage (except of course the initial stage in any developmental progression). In particular it makes somewhat problematic the developmental synchrony or "abruptness" assumptions usually associated with Piagetian stage transitions. However, some recent writers have questioned the logical necessity of synchronous stage related behavioral acquisitions within the Piagetian system (Feldman and Toulmin, 1976; Flavell, 1970, 1971, 1977) and have proposed alternative models of development which retain the stage construct albeit in forms rather more complex and diversified than the conventional Genevan version (e.g., Wohlwill, 1973, pages 190-239) (Hooper, 1979, pp. 3-4).

These theoretical considerations pertain to all the general periods of cognitive growth, the sensorimotor (infancy), preoperational (early childhood), concrete operations (middle childhood), and formal operations (adolescence and adulthood), but they are singularly germane to the concrete operations case. This period has received the attention of the majority of neo-Piagetian researchers.

THE STRUCTURE OF CONCRETE OPERATIONS PERIOD THOUGHT

There is a considerable body of original and derived research that is concerned with concrete operations period functioning. The original Genevan sources include Inhelder and Piaget (1958, 1964), Piaget (1942, 1970a, 1970c, 1971, 1972), Piaget and Inhelder (1941, 1956), Piaget, Inhelder, and Szeminska (1960), and Piaget and Szeminska (1952). The logical concept content domains include classes, relations, number, conservation of quantitative invariants, logical inference, and probability, in addition to the related infralogical concepts dealing with space, geometry, time, velocity and movement, and distance. Representative reviews of this research and the secondary replication studies are found in Brainerd (1978b), Flavell (1970, 1977), and Sigel and Hooper (1968).

The major characteristics of concrete operational thinking may be described in psychological qualities *or* in formal properties. This follows from the fundamental Piagetian assumption that a formal logical model is *isomorphic* to psychological functioning. Thus, such defining properties of logico-mathematical structures (Piaget, 1972) as composition, associativity, general and special identities, and reversibility find psychological counterparts in behaviors that exemplify an understanding of transformations rather than static configurations, decentration rather than centration, inferred reality rather than perceived appearances, and reversible operations rather than irreversibility as evidenced in the negation of classes and reciprocity of relations (Flavell, 1963, 1977). As we have stated earlier:

> In the strict sense, operativity becomes possible only during the middle childhood years. It is during this time that the child's cognitive activities become organized into coherent and integrated systems. The term "operation" is used to refer to these systems both as something the child *has* at his disposal and as something the child *does* (Furth, 1969). Concrete operations are internalized representations of coordinated actions which are themselves organized into larger coordinated systems called groupings or *groupements.* Viewed as something the child does, operations include the logico-mathematical activities of adding, subtracting, dividing, setting terms in correspondences (as in the cases of classes and relations), and the infralogical activities with a specific spatiotemporal context which deal with quantity, measurement, space, and time concepts (Flavell, 1963, p. 166). Viewed as something the child has, operations must be considered as parts of a higher order structural organization (Klausmeier and Hooper, 1974, p. 9).

The term Piaget employs to describe the superordinate structures of the concrete operations period is groupement. *These structures* are proposed as interrelating and subsuming *all* logical reasoning during the middle childhood period. While nine separate groupements have been described by Piaget, the eight primary cases include four dealing with logical classes and four counterpart cases concerned with logical relations. Brainerd (1972b) has succinctly described the major dimensions of the groupement structures as follows:

> Piaget contends that a given structure S is engendered by either a cognitive operation and its *inverse*—if S is one of the four "class" groupements—or by a cognitive operation and its *reciprocal*—if S is one of the four "relational" *groupements.* In theory, the notion of "operation" can here be taken to mean almost any internalizable property of overt action. In practice, however, the notion may be restricted to the set-theoretic operations of *union* and *intersection* and the arithmetic operations of *addition* and *multiplication.* Thus each *class groupement* owes its existence to the fact that the cognizer is capable of simultaneously uniting and subtracting classes, while each *relational groupement* owes its existence to the fact that the cognizer is instantaneously aware of the reciprocal implications of the equivalence and difference relations produced by arithmetic addition and multiplication. To illustrate, the capacity simultaneously to add two subordinate classes to yield a superordinate class and to subtract one

> subordinate class from the superordinate class to yield the remaining subordinate class engenders a certain class *groupement* structure. Also to illustrate, the knowledge that A < B and B < C simultaneously implies both A < C and C > A engenders a certain relational structure. Although one may quibble with these claims about the origins of *groupement* structures on theoretical grounds, there is no denying their obvious empirical implication: If one wishes to conduct research on these structures, then one most certainly will have to focus on the unions and intersections of hierarchical classes and their inverses—on the one hand—and on the addition-multiplication of equivalence-difference relations and their reciprocals—on the other [Brainerd, 1972b, p. 5].

In view of the fact that children's logical groupement-related performances are theoretically predicted to emerge and demonstrate final mastery in parallel fashion, developmental synchrony is anticipated for the classificatory and relational concept domains. This prediction follows (Q.E.D.) from the orthodox Piagetian conception of stage structure and integration (described above) and underlies the key assumption of stage as a ***structure d'ensemble.*** Relative consistent performances (low intraindividual variability) should be found for concept tasks (equated for general item difficulty and content distinctions) operationally designed to assess behaviors derived from common structural components. Thus equivalent tasks assessing, for example, class, relations, and natural number concepts should evidence similar relative item difficulties (cross-sectional data), congruent developmental acquisition sequences (longitudinal data), and highly predictive time lag correlational patterns (factor analyses and panel-type relationship analyses with multiple assessment points).

A final issue concerns the possible role of such "performance" factors (as theoretically distinguished from structural or competence determinants) as stimulus salience, instructional set, response and attentional demands, and short-term memory requirements on logical concept acquisition. One of these potential influence variables, short-term memory span, was selected for study. As has been summarized previously:

> It was hypothesized that, in view of the surface task demands, individual differences in auditory memory span would predict a significant proportion of the variance of the class inclusion task, while visual memory would be more strongly related to combinatorial reasoning. If this were the case, it was argued that these performance aspects of the tasks should be taken into account in drawing inferences to underlying competence. In particular, relationships due to shared operational structures could be more accurately reflected by examining partial correlations, with the

> influence of individual differences in memory and the differing memory demands of the tasks statistically controlled.
>
> If it proved to be the case that shared variance among logical operations tasks is due, to a measurable extent, to shared perceptual demands of these tasks, one information-processing approach would lead the investigator to discount that portion of the relationship as "performance variance," and to estimate the degree of structural similarity of logical operations behaviors only from that portion of the relationship not so "explained." However, this approach depends on the view that memory is a passive capacity, the development of which *causes* the emergence of logical structures (McLaughlin, 1963). More recent, dynamic views of memory development suggest that emerging logical structures may themselves contribute to the development of memoric skills (e.g., Pascual-Leone, 1970; Norman and Rumelhart, 1975). Under this approach, it would clearly not be appropriate to partial out memory as a common cause of a spurious component of the relationships under study, since memory growth, to some extent, would be a *result* of this development (Swinton, Sipple, Hooper, and Hougum, 1977, pp. 2-3).

We now may examine the general results patterns for the Piagetian longitudinal study.

SUMMARY OF PIAGETIAN STUDY RESULTS

Preliminary Considerations

It will be recalled that the primary focus of the Piagetian investigation was the concrete operations period of middle childhood. Accordingly, a representative array of logical concept tasks and short-term memory measures were administered to children at the kindergarten, third-, and sixth-grade levels (see Table 2-5 of Chapter 2). The target concept domains included relations (seriation and transitive inference of length and weight), classification ("some" and "all" understanding, dichotomous sorting, class inclusion, and combinatorial reasoning), natural number (number conservation, cardinality, arithmetic addition), and conservation (identity and equivalence conservation of length and weight). The kindergarten-Grade 1-2-3 group of children and the Grade 3-4-5-6 group received *all* the tasks each year including the 64-item groupement battery and the three memory measures. The Grade 6-7-8-9 group received only the groupement battery, class inclusion, combinatorial reasoning, and the short-term memory tests.

No attempt was made to specify and assign children to conventional Piagetian stage categories (see, however, previous reports by

Gonchar, 1975, which used a comprehensive stage designation strategy for the number concept domain; and Dihoff, 1975; and Hooper, Toniolo, & Sipple, 1978, which employed pass/fail criteria). We elected to treat the data in a more conventional manner (by American standards) and have employed parametric variance analyses, trend comparisons, and so on. While it has been argued (Overton & Reese, 1973, p. 84) that models such as Piaget's that assume "organized complexity" rather than linear causality are not appropriately evaluated with statistical inference approaches that assume additivity and linear summation of interaction effects, it may be countered that the conventional ANOVA and related parametric correlational approaches are indeed suitable *initial* strategies for the present normative analyses designed to assess the original Genevan contentions (Hooper, 1976). The summary reported in this chapter is in no way intended to be a comprehensive account (the complete analyses and discussion will be presented in a later monograph (i.e., Hooper, Sipple, & Swinton, in preparation). Interim reports that have been previously published include a preliminary planning paper (Hooper & Klausmeier, 1973), a report of a pilot investigation using similar tasks (Hooper, Sipple, Goldman, & Swinton, 1979), multidimensional scaling analyses (Dihoff, 1975, 1976b), effects of language factors on classification performances (Dihoff, 1976a), investigations of identity conservation and equivalence conservation and related transitive inference (Brainerd & Hooper, 1975, 1978; Hooper & Toniolo, 1976; Hooper, Toniolo, & Sipple, 1978; Toniolo & Hooper, 1975), natural number concepts (Gonchar, 1975), theoretical papers discussing life span analyses of Piagetian tasks (Hooper, 1976) and the present test batteries (Hooper, Brainerd, & Sipple, 1975), a report of a related training study with kindergarten and first-grade children (Hooper, Wanska, Peterson, & DeFrain, 1979), an initial analysis of memory variables and logical reasoning (Swinton, Sipple, Hooper, & Hougum, 1977), and a summary of the first two years of data collection for the present longitudinal sample (Hooper, Swinton, & Sipple, 1976).

Continued Participation (Selective Survival) Effects. Selective participation effects have been cited as a potential confound in longitudinal analyses (e.g., Baltes, 1968; Wohlwill, 1973) and have been found to bias the resulting normative generalizations significantly (e.g., LaBouvie, Bartsch, Nesselroade, & Baltes, 1974, Riegel & Riegel, 1972; Riegel, Riegel, & Meyer, 1967). In the present Piagetian investigation, the mean percentage of "dropout" subjects per year was 8.5 percent, indicating moderate sample attrition from the longitudinal group across the three-year duration. A total of 46 subjects did not participate in all four years' assessments: 19 subjects from the initial kindergarten sample, 13 subjects from the third grade, and 14 from the sixth grade. Comparisons of these groups' first-year performances with those of the four-year longitudinal group indicated nonparticipant superiority of sixth-grade subjects only on groupement VIII (means of 7.71 vs. 6.83, $t = 5.27$, $df = 58$, $p < .03$). For the younger subject subsamples, two comparisons involving the same task proved to be significant. Continuing participants' some-all understanding task scores were higher than those of the "dropout" group for the kindergarten group (means of 3.63 vs. 3.21, $t = 4.14$, $df = 58$, $p < .05$) and for the combined kindergarten and third-grade sample (means of 3.74 vs. 3.47, $t = 4.12$, $df = 118$, $p < .05$). Comparisons approaching significance were as follows: sixth-grade means of 7.50 ("dropout" sample) versus 6.74 (longitudinal sample) on groupement VII, $t = 3.74$, $df = 58$, $p < .06$; third-grade means of 19.77 ("dropout" sample) versus 16.96 (longitudinal sample) on the combined class groupements, $t = 3.68$, $df = 58$, $p < .06$; kindergarten means of 3.16 ("dropout" sample) versus 1.90 (longitudinal sample) on the equivalence conservation of weight task, $t = 3.90$, $df = 58$, $p < .06$; kindergarten, third-, and sixth-grade combined means of 6.70 ("dropout" sample) versus 6.05 (Longitudinal sample) on groupement VII, $t = 3.63$, $df = 178$, $p < .06$. It is clear from these results that few distinct performance biases are evident in the longitudinal sample and that the remaining sample may be considered representative of the original age-level cohorts.

Repeated Measurement Effects. Specific provision was made to evaluate the possible effect of repeated testing on the primary longitudinal sample children by assessing cohort-matched groups of subjects in the second and third year of the investigation (see Table 2-2 of Chapter 2). A series of analyses of variance was conducted to evaluate these potential biases.

Analysis of variance (sex, grade level, and longitudinal group vs. testing control group) was employed (the means and standard deviations of the longitudinal and testing control group samples are reported in Hooper, Swinton, & Sipple, 1976, pp. 35-40). For the subset of measures that were administered to all subjects in the second year the evaluations indicated significant grade level main effects for 14 or 15 indices (see analysis summaries presented in Tables A5-1 to A5-5). Assessment group differences (repeated measurement effects) approached significance for the class inclusion task ($F = 3.59$, $df = 1, 216$, $.10 < p > .05$) and for the combined class groupements (I–IV) score ($F = 3.89$, $df = 1,216$, $p < .05$). In both cases the core longitudinal group scores were consistently higher at all age and grade levels. Sex differences and higher order interactions were generally nonsignificant for all the variables.

In the case of the first- and fourth-grade subjects who received the full task battery no cases of significant testing main effects were observed (see summaries presented in Tables A5–3 to A5–5). However, two instances of significant treatment group grade level interactions resulted, by that is, conservation of length (identity task format) $F = 6.48$, $df = 1, 142$, $p < .05$, and conservation of length (equivalence task format) $F = 5.63$, $df = 1, 142$, $p < .05$. Inspection of the means revealed the following: longitudinal group superiority (means of 4.46 vs. 3.63; $t = 2.88$, $df = 70$, $p < .01$) at the first-grade level and control group superiority (means of 5.83 vs. 5.30; $F = 2.12$, $df = 76$, $p < .05$) at the fourth-grade level for the length conservation identity task. Similarly, in the equivalence format length conservation task case, the first-grade longitudinal sample subjects were superior to their control group counterparts (means of 4.04 vs. 2.50; $t = 4.83$, $df = 70$, $p < .001$) while the converse relationship was true for the fourth-grade comparisons (means of 4.96 vs. 5.33; $t = 1.25$, $df = 76$, N.S.).

For the overall logical concept task array, it may be concluded that there is minimal evidence for pervasive repeated measurement effects in the subsequent year's assessments.

In similar fashion, analyses of variance were conducted during the third assessment year of the present Piagetian study (Tables A5–6 to A5–10 report the summary values for these analyses). The main considerations (over and above the anticipated grade level distinctions that are significant in 23 of the 27 cases; the exceptions include groupement VI, some-all understanding, length transitivity, and equivalence conservation of weight) indicate 10 cases of assessment condition main effects. Five of these instances produced F values with an associated probability $< .01$ (groupements IV and VIII, class inclusion, cardinality, and equivalence conservation of length). All of these difference contrasts favored the longitudinal sample subjects, that is, combined grade and sex subsample means of 6.57 versus 5.70 (groupement IV), 7.22 versus 6.22 (groupement VIII), 4.34 versus 3.77 (class inclusion), 4.07 versus 3.57 (cardinality), and 5.37 versus 4.29 (equivalence conservation of length). In addition, the class inclusion relative longitudinal and testing control group differences were most notable at the younger age levels (mean differences of 1.11, .40, and .27 for the second, fifth, and eighth grades, respectively; testing condition × grade level interaction $F = 3.24$, *df* 2,202, $p < .05$), and these distinctions were more evident for males at the second- and fifth-grade levels and for females at the second- and eighth-grade levels (i.e., testing condition × grade level × sex interaction $F = 6.71$, *df* 2, 202, $p < .01$).

Primary Results

Analyses of Variance. Tables A5-11 to A5-16 present the means and associated standard deviations for the various groupements scores, the traditional Piagetian measures, and the three short-term memory tests for each age and grade level across the four assessment years. The mixed model: factorial ANOVA summaries (between subject factors of sex and grade level, and within subject factor of assessment year) are shown in Tables 5-1 to 5-3. Table 5-1 presents the various groupements contrast results and reveals significant grade level main effects for all the subscore measures and a notable absence of gender distinction main effects. No grade × sex interactions approached significance. Year of assessment main effects, as anticipated, were significant for all 14 of the score and variable combinations. While assessment year × sex interaction values were uniformly nonsignificant, a sizeable number of assessment year × grade level interactions were significant (11 of the 14 F values have an associated probability $< .05$). These grade × assessment year interactions reflect the varying difficulty patterns *within* the groupement battery as evidenced in differential growth for certain grades selectively for certain between assessment year contrasts. This is shown in greater detail in the trend analyses section that follows. The counterpart ANOVAS for the remaining cognitive measures administered to the entire subject sample over the four-year interval are presented in Table 5-2. Grade level main effects are significant for all five cases. There was a marginal superiority present for female class inclusion performances, and these differences were more notable for the initial assessment years (i.e., male and female combined grade mean differences of .51, .26, .11, and .03 for the first, second, third, and fourth assessment years, respectively).

Table 5-3 presents the ANOVA summaries for the remaining measures that were administered only to the younger cohorts. As expected for the present assessment design grade level main effects were present for all the indices. In similar fashion significant assessment year main effects were found for the majority of measures (the single exception was transitivity of weight). Sex distinctions were absent. Assessment year × grade level interactions were significant

Table 5-1. Analysis of Variance Summary for the Groupement Tasks with Two Between Subject Factors (Sex and Grade Group) and One Within Subject Factor (Assessment Year)

	Sources of variance[a]															
	Sex		*Grade*		*Sex × Grade*			*Year*		*Year × Sex*		*Year × Grade*		*Year × Sex × Grade*		
Variable	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error between*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error within*
Groupement I	7.44	1.70	110.58	25.33^{c}	7.31	1.67	4.36	48.49	32.99^{c}	1.30	.89	4.27	2.91^{c}	.99	.68	1.47
Groupement II	3.67	.73	109.04	21.78^{c}	5.64	1.13	5.01	41.06	26.84^{c}	.50	.33	4.33	2.83^{b}	1.19	.78	1.53
Groupement III	25.69	2.70	109.38	11.49^{c}	6.00	.63	9.52	42.86	13.11^{c}	.41	.13	1.31	.40	3.81	1.17	3.27
Groupement IV	.07	.01	188.97	23.17^{c}	.88	.11	8.16	112.61	45.04^{c}	3.25	1.30	8.05	3.22^{c}	2.50	1.00	2.50
Groupements I-IV composition	50.55	1.13	993.53	22.27^{c}	51.55	1.16	44.61	453.35	42.53^{c}	5.03	.47	39.55	3.71^{c}	1.84	.17	10.66
Groupements I-IV inversion	8.26	.93	199.03	22.40^{c}	6.27	.71	8.89	85.98	29.85^{c}	.85	.30	10.48	3.64^{c}	4.44	1.54	2.88
Groupements I-IV combined	99.69	1.34	1982.42	26.59^{c}	66.75	.90	74.56	914.85	62.92^{c}	9.12	.63	38.29	2.63^{b}	5.32	.37	14.54
Groupement V	1.79	.97	15.51	8.37^{c}	.19	.10	1.85	13.17	12.04^{c}	.15	.14	1.33	1.21	2.82	2.56^{b}	1.10
Groupement VI	2.77	1.23	26.14	11.58^{c}	1.09	.48	2.26	11.64	12.25^{c}	.59	.62	5.31	5.59^{c}	.28	.29	.95
Groupement VII	.02	.01	29.73	6.94^{c}	.93	.22	4.28	53.16	29.05^{c}	.22	.12	2.88	1.57	1.87	1.02	1.83
Groupement VIII	5.54	1.49	68.77	18.46^{c}	1.92	.51	3.73	36.28	23.71^{c}	.73	.48	3.39	2.22^{b}	1.68	1.10	1.53
Groupements V-VIII composition	16.98	1.34	195.81	21.22^{c}	6.51	.71	9.23	138.07	51.33^{c}	.38	.14	10.61	3.94^{c}	1.67	.62	2.69
Groupements V-VIII reciprocity	1.17	.17	76.63	11.31^{c}	.62	.09	6.77	67.66	36.77^{c}	.40	.22	6.94	3.77^{c}	3.77	2.05	1.84
Groupements V-VIII combined	27.08	.93	514.61	17.58^{c}	11.12	.38	29.27	395.21	59.07^{c}	1.57	.22	33.72	5.04^{c}	8.28	1.24	6.69

[a] Degrees of freedom: Sex = 1, Grade = 2, Sex × Grade = 2, Error between = 128, Year = 3, Year × Sex = 3, Year × Grade = 6, Year × Sex × Grade = 6, Error within = 384.

[b] $p < .05$.

[c] $p < .01$.

Table 5-2. Analysis of Variance for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span, Auditory Sequential Memory, and Visual Orientation Memory with Two Between Subject Factors (Sex and Grade Group) and One Within Subject Factor (Assessment Year)

	Sources of variance[a]															
	Sex		*Grade*		*Sex X Grade*			*Year*		*Year X Sex*		*Year X Grade*		*Year X Sex X Grade*		
Variable	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error between*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error within*
Class inclusion	6.91	3.33	43.98	21.18[b]	5.28	2.54	2.08	22.92	29.96[b]	1.55	2.01	3.41	4.43[b]	1.27	1.66	.77
Combinatorial reasoning	2.30	.03	4040.17	54.33[b]	61.42	.83	74.37	1471.84	43.74[b]	75.13	2.23	235.79	7.01[b]	15.84	.47	33.65
Auditory memory span	37.39	.15	4865.19	19.64[b]	493.46	1.99	247.73	2127.30	57.08[b]	37.02	.99	165.69	4.45[b]	55.04	1.48	37.27
Auditory sequential memory	2.53	.01	7669.30	17.80[b]	464.67	1.08	430.78	3071.90	44.88[b]	20.78	.30	419.13	6.12[b]	61.87	.90	68.44
Visual orientation memory	4.28	.29	1053.95	70.85[b]	45.28	3.04	14.88	524.01	121.58[b]	5.81	1.35	76.93	17.86[b]	6.90	1.60	4.31

[a] Degrees of freedom: Sex = 1, Grade = 2, Sex X Grade = 2, Error between = 128, Year = 3, Year X Sex = 3, Year X Grade = 6, Year X Sex X Grade = 6, Error within = 384.

[b] $p < .01$.

Table 5-3a. Analysis of Variance Summary for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks with Two Between Subject Factors (Sex and Grade Group) and One Within Subject Factor (Assessment Year)

	Sources of variance[a]															
	Sex		Grade		Sex × Grade			Year		Year × Sex		Year × Grade		Year × Sex × Grade		
Variable	MS	F value	MS	F value	MS	F value	Error between	MS	F value	MS	F value	MS	F value	MS	F value	Error within
Some-all	.96	3.73	2.28	8.89[b]	.10	.41	.26	.67	4.04[b]	.05	.29	.08	.48	.01	.07	.17
Dichotomies	.29	.29	41.11	41.12[b]	.43	.43	1.00	22.98	62.11[b]	.30	.81	3.37	9.11[b]	.14	.38	.37
Cardinality	4.07	1.91	62.76	29.52[b]	3.43	1.61	2.13	30.56	43.04[b]	1.13	1.60	16.80	23.66[b]	.44	.62	.71
Arithmetic addition	20.42	2.02	847.93	83.76[b]	4.65	.46	10.12	432.40	93.80[b]	6.48	1.41	456.80	99.09[b]	10.17	2.21	4.61

[a] Degrees of freedom: Sex = 1, Grade = 1, Sex × Grade = 1, Error between = 84, Year = 3, Year × Sex = 3, Year × Grade = 3, Year × Sex × Grade = 3, Error within = 252.

[b] $p < .01$.

Table 5-3b. Analysis of Variance Summary for the Seriation and Transitivity Tasks with Two Between Subject Factors (Sex and Grade Group) and One Within Subject Factor (Assessment Year)

	Sources of variance[a]															
	Sex		*Grade*		*Sex × Grade*			*Year*		*Year × Sex*		*Year × Grade*		*Year × Sex × Grade*		
Variable	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error between*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error within*
Seriation	.48	.35	76.51	55.94[c]	4.10	3.00	1.37	31.11	59.83[c]	1.25	2.40	11.27	21.67[c]	.50	.97	.52
Transitivity of length	.02	.01	39.29	16.43[c]	1.85	.77	2.39	5.21	2.68[b]	1.21	.62	3.76	1.93	.06	.03	1.95
Transitivity of weight	2.70	2.63	7.39	7.18[c]	.12	.12	1.03	.51	.50	1.25	1.21	.25	.25	.28	.27	1.03

[a] Degrees of freedom: Sex = 1, Grade = 1, Sex × Grade = 1, Error between = 84, Year = 3, Year × Sex = 3, Year × Grade = 3, Year × Sex × Grade = 3, Error within = 252.

[b] $p < .05$.

[c] $p < .01$.

Table 5-3c. Analysis of Variance Summary for the Conservation Tasks with Two Between Subject Factors (Sex and Grade Group) and One Within Subject Factor (Assessment Year)

	Sources of variance[a]															
	Sex		*Grade*		*Sex × Grade*			*Year*		*Year × Sex*		*Year × Grade*		*Year × Sex × Grade*		
Variable	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error between*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*Error within*
Conservation of number	.66	.20	139.57	43.10[c]	.18	.05	3.24	37.53	25.88[c]	.81	.56	24.29	16.75[c]	1.25	.86	1.45
Identity conservation of length	1.64	.40	74.05	18.21[c]	.66	.16	4.07	34.43	24.25[c]	.77	.54	5.35	3.77[b]	6.70	4.72[c]	1.42
Equivalence conservation of length	11.38	1.61	93.65	13.26[c]	6.96	.99	7.06	89.26	33.94[c]	.60	.23	16.36	6.22[c]	9.50	3.61[b]	2.63
Identity conversation of weight	11.14	2.15	79.67	15.39[c]	4.77	.92	5.18	22.75	14.49[c]	2.06	1.31	3.26	2.08	2.89	1.84	1.57
Equivalence conservation of weight	6.70	.82	193.96	23.79[c]	22.78	2.79	8.15	63.56	20.57[c]	3.35	1.08	13.18	4.27[c]	8.49	2.75[b]	3.09

[a] Degrees of freedom: Sex = 1, Grade = 1, Sex × Grade = 1, Error between = 84, Year = 3, Year × Sex = 3, Year × Grade = 3, Year × Sex × Grade = 3, Error within = 252.

[b] $p < .05$.

[c] $p < .01$.

for the dichotomous sorting, cardinality, addition, seriation, number, and length (identity condition only) and weight conservation tasks. These latter interactions reflect the joint effects of initial task difficulty patterns and differential growth rates over the present grade and assessment year data points.

Trend Analyses. A series of trend analyses via an ANOVA approach were conducted in which the linear, quadratic, and cubic components were orthogonal. Evaluation of higher order interactions with the grade level and sex factors were also provided. These results are summarized in Tables A5-17 to A5-21. As noted in the previous data summary section, grade level main effects were significant for all tasks, gender distinctions were notably absent (there is a suggestion of female superiority for the some-all understanding and class inclusion tasks); and no significant grade × sex interactions were observed.

The linear component was significant for all measures except weight transitivity. Significant grade level × linear component interactions ($p < .05$) were observed for 15 cases in the larger sample ($N = 134$), that is, groupements I, II, IV, VI, VIII, all six combined groupement cases, class inclusion, combinatorial reasoning, digit memory, and visual memory tasks. For the subsample of younger children ($N = 88$) similar grade × linear interactions were found for all the conservation tasks, dichotomous sorting, cardinality, arithmetic addition, and the seriation tasks. These expected distinctions in relative developmental rates across the present three target cohorts are exemplified in Figures 5-1 to 5-4 with the mean percentage of correct items for groupement IV (counivocal multiplication of classes), class inclusion, cardinality, and seriation, respectively. No significant sex × linear component interactions were observed.

Significant quadratic components were observed for the following tasks: groupements V, VII, and VIII, total class negation, total relations composition, total relations reciprocity, combined relations, combinatorial reasoning, word memory, number conservation, equivalence conservation of length, dichotomous sorting, cardinality, arithmetic addition, and seriation. Interaction values for the grade level × quadratic component were significant for six cases, that is, combinatorial reasoning, word memory, visual memory, number conservation, cardinality, and arithmetic addition. These significant departures from monotonically increasing functions across the present annual assessment points is graphically represented in Figure 5-3 (cardinality), Figure 5-4 (seriation), and Figures 5-5 to 5-10 for the groupement VII (biunivocal multiplication of relations),

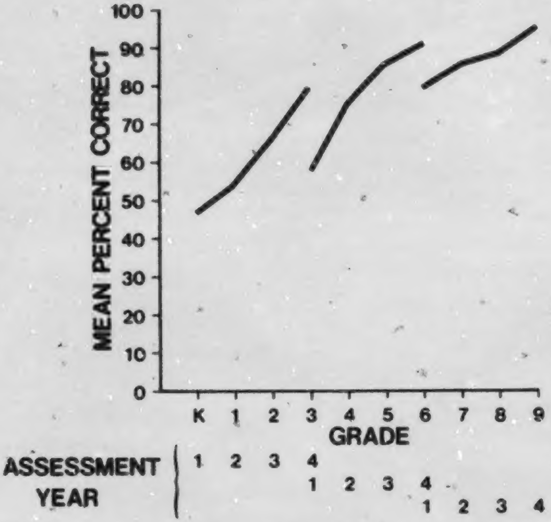


Figure 5-1. Mean Percent Items Correct on Groupement IV for Subjects Assessed in All Four Years

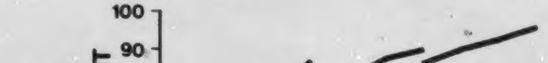


Figure 5-2. Mean Percent Items Correct on Class Inclusion for Subject Assessed in all Four Years

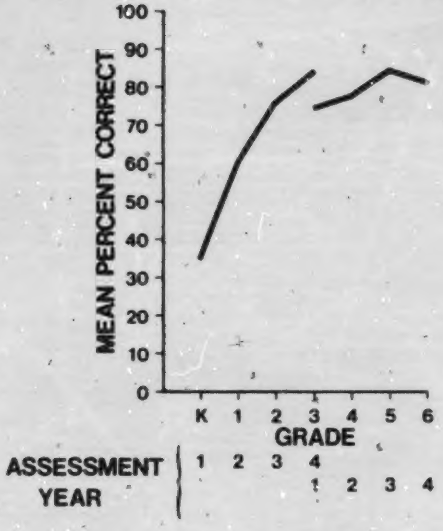


Figure 5-3. Mean Percent Items Correct on Cardinality for Subjects Assessed in All Four Years

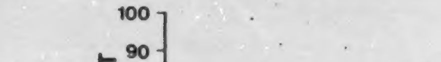


Figure 5-4. Mean Percent Items Correct on Seriation for Subjects Assessed in All Four Years

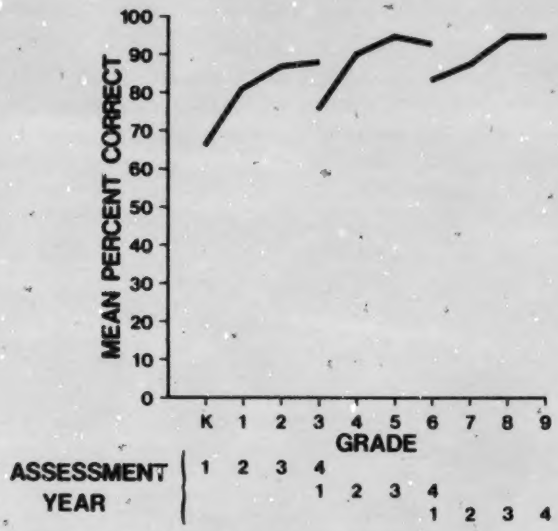


Figure 5-5. Mean Percent Items Correct on Groupement VII for Subjects Assessed in All Four Years

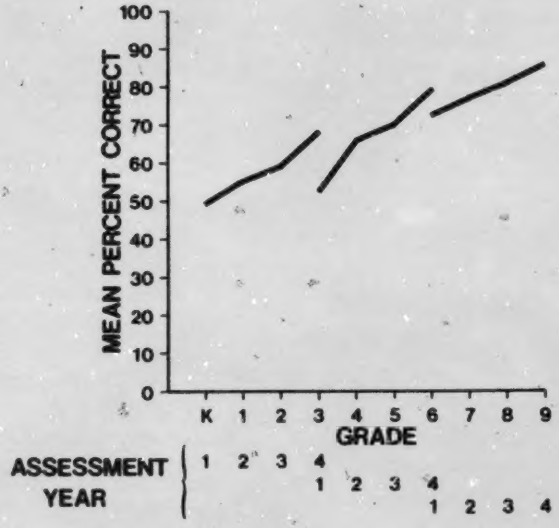


Figure 5-6. Mean Percent Items Correct on Groupements I to IV Combined for Subjects Assessed in All Four Years

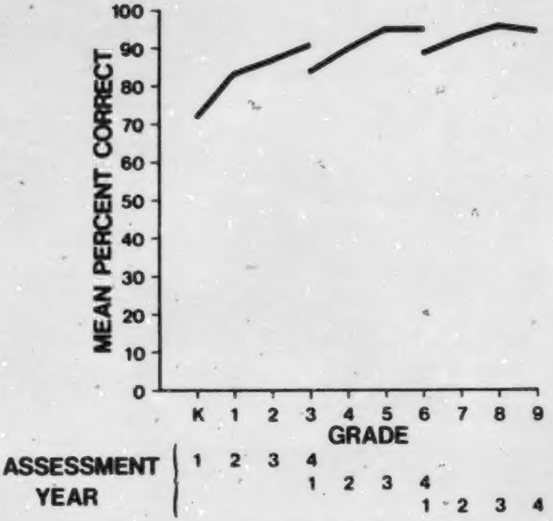


Figure 5-7. Mean Percent Items Correct on Groupements V to VIII Combined for Subjects Assessed in All Four Years

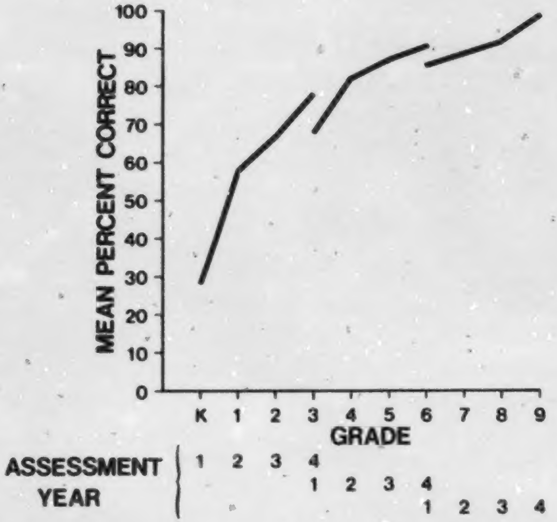


Figure 5-8. Mean Percent Items Correct on Combinatorial Reasoning for Subjects Assessed in All Four Years

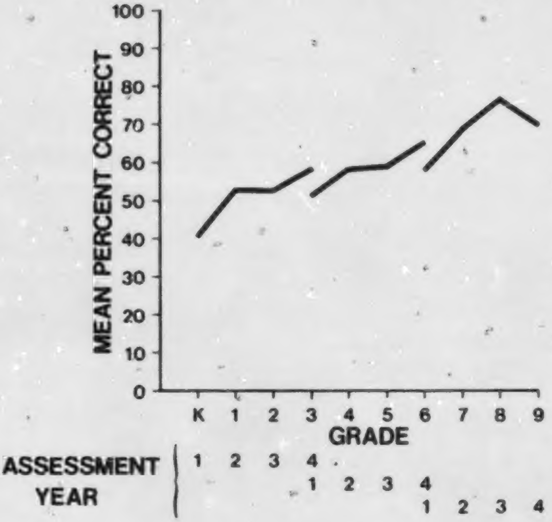


Figure 5-9. Mean Percent Items Correct on Auditory Memory (Word) Span for Subjects Assessed in All Four Years

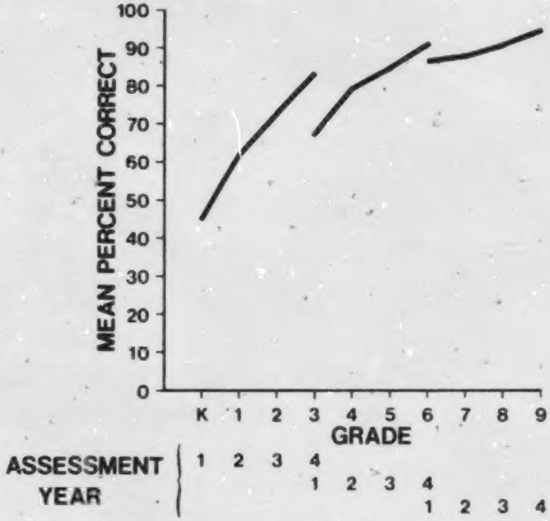


Figure 5-10. Mean Percent Items Correct on Visual Orientation Memory for Subjects Assessed in All Four Years

combined classes, combined relations, combinatorial reasoning, word memory, and visual orientation memory tasks, respectively. Two cases for the younger subjects are shown in the patterns for number conservation and equivalence conservation of length (Figures 5-11 and 5-12).

Finally, an instance of a significant cubic component was observed in groupement II: secondary addition of classes (see Table A5-17 and Figure 5-13). In addition, the cubic × grade level interactions were significant for the word and digit memory tasks, $p < .05$ (see Table A5-18).

In summary, as Tables A5-17 through A5-21 clearly indicate, the major trend component for the present concrete operations task array and subject age range is linear. As expected there are also numerous cases of trend component interaction with the age level cohorts. These are shown graphically in the representative figures selected. It is also obvious that the particular developmental growth patterns vary depending upon the tasks or task "clusters" at issue. Thus the patterns shown for the larger sample responses to the class groupements (I to IV) are different from those shown for the combined relations counterparts (V to VIII), that is, compare Figures 5-6 and 5-7. General classificatory understanding shows relatively concordant score increases across the present 5- and 13-year-old age range, while relations scores start much higher (72 percent vs. 49 percent of items passed at the kindergarten level) and show distinct "ceiling" effects by the fifth grade level (see also the pattern for class inclusion performance in Figure 5-2). In constrast, the younger subsample trend patterns for cardinality (Figure 5-3) and seriation (Figure 5-4) are certainly very similar and are in general accord with the concurrence predictions of Piaget and Szeminska (1952); see also Gonchar (1975). In general one would expect much more pronounced growth rates for tasks such as conservation of quantitative invariants in the kindergarten to third-grade cohort as compared to the third- to sixth-grade case. This is shown in the cases for number conservation (see Figure 5-11) and equivalence conservation of length (see Figure 5-12). There is a 36 percent increase in average number conservation scores for the younger cohort compared to a 5 percent increase for the older group. Length conservation acquisition appears somewhat later, and this is shown in the increase values of 54 percent and 22 percent for the two cohorts, respectively (see Figure 5-12). (See note 2.)

A Representative Case of Developmental Prediction: Memory Variables and Two Logical Reasoning Tasks. It is obvious that

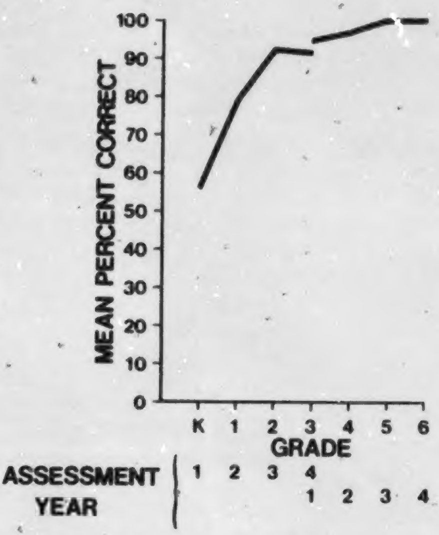


Figure 5-11. Mean Percent Items Correct on Conservation of Number for Subjects Assessed in All Four Years

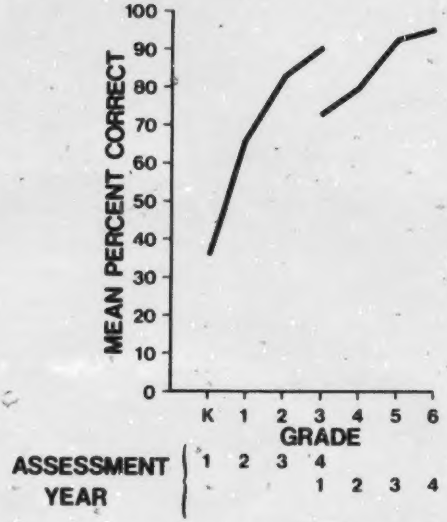


Figure 5-12. Mean Percent Items Correct on Equivalence Conservation of Length for Subjects Assessed in All Four Years

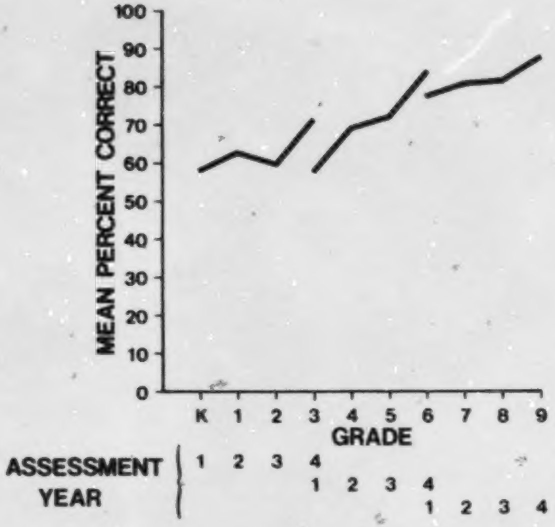


Figure 5-13. Mean Percent Items Correct on Groupement II for Subjects Assessed in All Four Years

inferences concerning the synchrony or asynchrony of developmental emergence of Piagetian logical operations structures are ultimately based upon subjects' performances on tasks designed to embody such cognitive structures. As we have emphasized previously, orthodox Piagetian theory predicts a tight interrelationship among structures and tasks characteristic of a given developmental period or stage, for example, concrete operations, and a sequential or predictive relationship between tasks derived from adjacent developmental periods, that is, concrete and formal operations (Hooper, Swinton, & Sipple, 1976). This follows from the basic tenet that "Inherent in Piaget's general system is the view that the logical structure of mathematical and physical systems is paralleled by the order of psychological (logical thought) development (Beilin, 1971a, p. 116)."

Performance on any series of logical reasoning tasks, however, is apt to reflect sources of variability other than those based upon the presumed underlying cognitive structure, such as stimulus salience, instructional set, response and attentional demands, and short-term memory requirements (see Flavell, 1977, pp. 220-235, for a recent discussion of these diagnosis and related causal inference problems). The present analysis examines the relationships among two measures of auditory memory span, visual orientation memory, and a representative index of the concrete operations period (class inclusion) and of the formal operations period (combinatorial reasoning). Two cross-lagged correlation models are applied to the data to investigate the degree to which causation might be viewed as reciprocal, rather than as flowing strictly from perceptual to logical operation development (Labouvie, 1974). It may be seen that although based on differing assumptions, the causal analyses yield similar results in these data.

Over the three-year period, the most stable instruments were the auditory memory tasks (r = .71, word and digit = .66, followed by visual memory (r = .55) and combinatorial reasoning and class inclusion (r = .41 and .35, respectively). the best single predictor of Piagetian performance three years later was initial status on the visual memory task, yielding slightly, but not significantly greater, predictive power than did initial status on the Piagetian performances themselves. Final year combinatorial reasoning performance shared more variance with earlier memory tasks than did class inclusion, but both tasks initially appeared more related to this visual memory task than to auditory span. It should be noted that the present correlation analyses report data for the combined grades (N = 134).

The intercorrelations among the various measures for the time intervals are shown in Table 5-4. While all variables show significant correlations with age, combinatorial reasoning and visual memory in year one are highly age-related, while digit memory in year two and year four class inclusion exhibit weak dependence on age ($r <$ .20). It should be kept in mind that the age span of the sample, initially 5 to 12 years, and ultimately 8 to 15, inflates correlations with age. For example, while the correlation of year four combinatorial reasoning with age is .43, the nearly 5 to 1 ratio of standard deviations results in a raw regression coefficient of combinatorial reasoning on age of only .08; that is, it requires about a one-year difference in age to result in a one-point predicted increment on the year four combinatorial reasoning task.

To allow for the possibility that relationships among tasks might be artifacts of mutual age dependence, age was allowed to enter the regression equations used to predict final status on the two reasoning tasks from previous years' levels on reasoning and memory tasks. As Table 5-5 reveals, the inclusion of the age variable inflates the regression weights for the other variables in predicting class inclusion, but acting as a suppressor variable deflates the impact of other predictors of combinatorial reasoning. In none of the predictive regression equations, however, does the addition of age cause the contribution

Table 5-4. Correlations Among Two Piagetian Tests and Three Memory Tests for the Four Annual Assessments: Three Combined Longitudinal Blocks[a,b]

		Year one						Year two					Year three					Year four			
		Age	*CR*	*CI*	*WM*	*DM*	*VM*	*CR*	*CI*	*WM*	*DM*	*VM*	*CR*	*CI*	*WM*	*DM*	*VM*	*CR*	*CI*	*WM*	*DM*
Year one	CR	.63																			
	CI	.47	.49																		
	WM	.41	.40	.36																	
	DM	.39	.36	.29	.70																
	VM	.76	.64	.47	.39	.37															
Year two	CR	.43	.41	.35	.27	.21	.39														
	CI	.40	.38	.44	.24	.28	.39	.30													
	WM	.37	.23	.31	.55	.52	.34	.22	.30												
	DM	.20	.14	.19	.42	.50	.17	.33	.34	.64											
	VM	.57	.47	.38	.27	.31	.65	.43	.39	.30	.32										
Year three	CR	.39	.43	.31	.28	.17	.42	.39	.34	.02	.13	.24									
	CI	.29	.29	.32	.29	.16	.27	.32	.44	.24	.22	.40	.22								
	WM	.51	.34	.23	.67	.65	.43	.33	.36	.62	.56	.39	.30	.34							
	DM	.42	.32	.28	.58	.63	.39	.37	.31	.57	.65	.39	.24	.26	.74						
	VM	.46	.33	.28	.28	.22	.58	.32	.42	.25	.27	.59	.38	.36	.37	.37					
Year four	CR	.43	.41	.22	.33	.28	.46	.42	.43	.27	.28	.35	.47	.34	.41	.38	.37				
	CI	.19	.25	.35	.20	.22	.36	.14	.41	.20	.21	.34	.20	.38	.26	.33	.29	.45			
	WM	.30	.35	.38	.71	.71	.35	.22	.20	.63	.59	.35	.20	.22	.68	.65	.24	.30	.31		
	DM	.50	.47	.33	.64	.66	.51	.39	.32	.56	.50	.46	.32	.22	.70	.74	.44	.39	.33	.69	
	VM	.44	.29	.32	.30	.31	.55	.28	.41	.28	.18	.48	.41	.32	.34	.24	.61	.37	.29	.25	.35

[a] Significance value at .05 level = .174.

[b] Significance value at .01 level = .228.

Table 5-5. Prediction Equations: Fourth Year Piagetian Tasks from Year 1 to 4 Variables, Without and With Age as a Predictor

	Standardized Regression Weights						Multiple	
							R	*F*
Year 4CI			+.237CI_1[c]			+.243VM_1[f]	.412	$13.39_{(2,131)}$
From Year 1	-.271 A[a]		+.274CI_1			+.433VM_1	.446	$10.78_{(3,130)}$
Year 4CR				+.179WM_1[d]		+.388VM_1	.486	$20.22_{(2,131)}$
From Year 1	.161 A			+.157WM_1		+.274VM_1	.496	$14.15_{(3,130)}$
Year 4CI			.330CI_2			+.214VM_2	.456	$17.20_{(2,131)}$
From Year 2	-.101 A		+.334CI_2			+.260VM_2	.463	$11.83_{(3,130)}$
Year 4CR		.320CR_2[b]	+.328CI_2				.523	$24.66_{(2,131)}$
From Year 2	.224 A	+.243CR_2	+.262CI_2				.557	$19.48_{(3,130)}$
Year 4CI			.319CI_3		+.246DM_3[e]		.451	$16.72_{(2,131)}$
From Year 3	-.011 A		+.320CI_3		+.250DM_3		.451	$11.07_{(3,130)}$
Year 4CR		.374CR_3	+.189CI_3		+.243DM_3		.578	$21.70_{(3,130)}$
From Year 3	.184 A	+.322CR_3	+.162CI_3		+.185DM_3		.598	$17.97_{(4,129)}$
Year 4CI		.389CR_4		+.195WM_4			.484	$20.05_{(2,131)}$
From Year 4	-.098 A	+.402CR_4		+.226WM_4			.486	$13.42_{(3,130)}$
Year 4CR			.316CI_4		+.211DM_4	+.207VM_4	.548	$18.62_{(3,130)}$
From Year 4	.363 A		+.379CI_4				.572	$31.79_{(2,131)}$

[a] A: Age.

$rACR_4 = .434; rACI_4 = .188$.

[b] CR: Combinatorial Reasoning.

[c] CI: Class Inclusion.

[d] WM: Word Memory Span.

[e] DM: Digit Memory Span.

[f] VM: Visual Orientation Memory

of the other variables to fall below significance. On the other hand, for predicting final combinatorial reasoning from year one status, the addition of auditory and visual memory, the only two significant task variables, raises the multiple *r* only .06 from the correlation (.43) obtained from age alone, suggesting that much of the variance in the system can be explained by age dependence.

Examining the standardized regression weights of the variables from each year entering significantly into the prediction of final level on reasoning tasks, and the resulting multiple correlations, it is apparent that combinatorial reasoning is uniformly more predictable from previous years' tasks than is class inclusion. However, class inclusion appears somewhat more stable in that, in all three prediction equations, earlier levels of performance on this task are significant predictors of final status, while knowledge of initial level on combinatorial reasoning does not add significantly to the prediction of final combinatorial reasoning status attainable from year one word memory span and visual memory. Furthermore, in years two and three, class inclusion enters as a significant predictor of combinatorial reasoning, but combinatorial reasoning never predicts class inclusion. There is no support for the hypothesis that the more visual combinatorial reasoning task would be better predicted by previous levels of visual memory, while class inclusion would be better predicted by the verbal digit or word memory span tasks. Only in the concurrent year-four analyses, in which the variables are of course not included as "predictors" of themselves, is there some indication that combinatorial reasoning is more related to visual memory, although each reasoning task is concurrently linked to one of the auditory memory variables. Over the three years of predictions, for both reasoning tasks predictors shift from visual memory to auditory memory as supplements of the earlier levels of the reasoning tasks themselves.

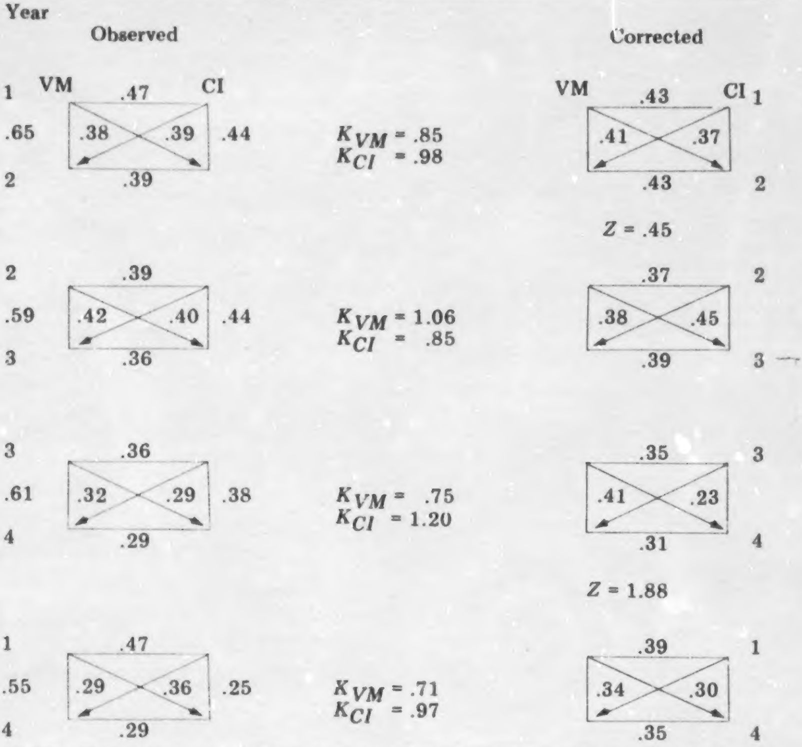
The concurrent year-four regressions, suggesting a word memory span component for class inclusion and digit span and visual memory requirements for combinatorial reasoning, do serve to underscore the fact that neither task is a "pure" measure of a logical operation and remind us that is is impossible to operationalize the measurement of an hypothesized structure without introducing performance variance. Thus, one might agree that the relative difficulty of the two logical operations might better be determined by first predicting scores on each from a knowledge of subjects' previous and concurrent levels on perceptual tasks with which the reasoning tasks share method variance, but which are clearly not measures of the cognitive structures of interest.

The residuals from these predictions might then be thought of as measures of attainment on the logical tasks with "irrelevant" variance stripped away. This use of memory variables as covariates is reasonable if in fact they represent a capacity or substrate for the development of logical operations. If, on the other hand, the perceptual skills are to some extent resultants of developing logical operations skills, removing their apparent contribution will, if reliability of measurement is sufficient, overcorrect, taking out variance properly attributable to the logical tasks. In particular, while the year-four combinatorial reasoning with class inclusion correlation is .45, the partial correlation with digit memory and visual memory taken into account is .33.

Attribution of cause from correlational evidence is an area where angels traditionally fear to tread. Two approaches, both described by Kenny (1973, 1975), were applied to these data in an attempt to tease out direction of developmental influence. The first, presented in the 1973 volume, *Structural Equation Models in the Social Sciences*, and elaborated in a 1975 article, corrects the synchronous and lagged correlations of two variables for presumed changes in reliability over time. If a variable is more reliable at time 2, it will be easier to predict, making it look more like an effect than it should. Changes in reliability are inferred from the pattern of correlations of the variables with other variables measured at the two points in time. The assumption is made that variables are moving in time as a rigid system, that is, that variables are neither approaching nor receding from one another in the observation space. Thus, the corrected synchronous correlations should be approximately equal if the assumptions hold. In this case, four other variables were available to obtain independent estimates of K_x, the ratio of the reliabilities of variable x at time t_2 to t_1. Tables 5-6, 5-7, and 5-8 show the observed and corrected correlations and corresponding values of K for the logical tasks and visual memory for all adjacent years and for other predictors of year-four logical tasks in the years in which they played significant roles in the regression equations.

Observing the adjacent year cross-lagged correlations between visual memory and class inclusion (Table 5-6), we see that the "quasistationarity" assumption of equal corrected synchronous correlations is met quite satisfactorily for the first two years, but that the corrected cross-lagged correlations are not sufficiently discrepant to lead to any inference about the direction of causality. From years three to four, the communality method leads to a marked estimated drop in the reliability of the class inclusion measure, making the corrected cross-lagged correlations more discrepant than the observed values.

Table 5-6. Cross-Lagged Correlations: Visual Memory (*VM*) and Class Inclusion (*CI*)


Year
Observed
Corrected
1 VM .47 CI
.65 .38 .39 .44
2 .39
K_{VM} = .85
K_{CI} = .98
VM .43 CI 1
.41 .37
.43 2
Z = .45
2 .39
.59 .42 .40 .44
3 .36
K_{VM} = 1.06
K_{CI} = .85
.37 2
.38 .45
.39 3
3 .36
.61 .32 .29 .38
4 .29
K_{VM} = .75
K_{CI} = 1.20
.35 3
.41 .23
.31 4
Z = 1.88
1 .47
.55 .29 .36 .25
4 .29
K_{VM} = .71
K_{CI} = .97
.39 1
.34 .30
.35 4

The difference, suggesting a tendency for changes in class inclusion to be causally prior to changes in memory, barely fails to attain significance at the .05 level ($Z = 1.88$), but in both the analyses for years three to four and years one to four, corrected synchronous correlations may be too different (.35 vs. .31 and .39 vs. .35) to make the hypothesis of quasistationarity appealing.

Turning to the cross-lagged relationships between visual memory and combinatorial reasoning (Table 5-7), the communality method leads to reliability corrections that attenuate the observed differences in cross-lagged correlations from years one to two and from years two to three, but a sharp increase in the year four estimated reliability of combinatorial reasoning amplifies the excess of $r_{CR_3VM_4}$ over $r_{VM_3CR_4}$ raising the difference to highly significant level ($Z =$

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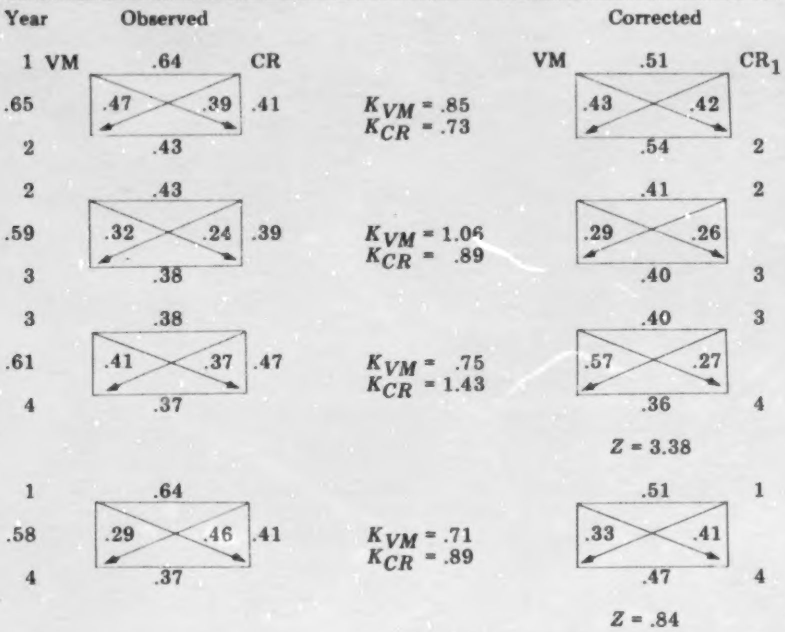
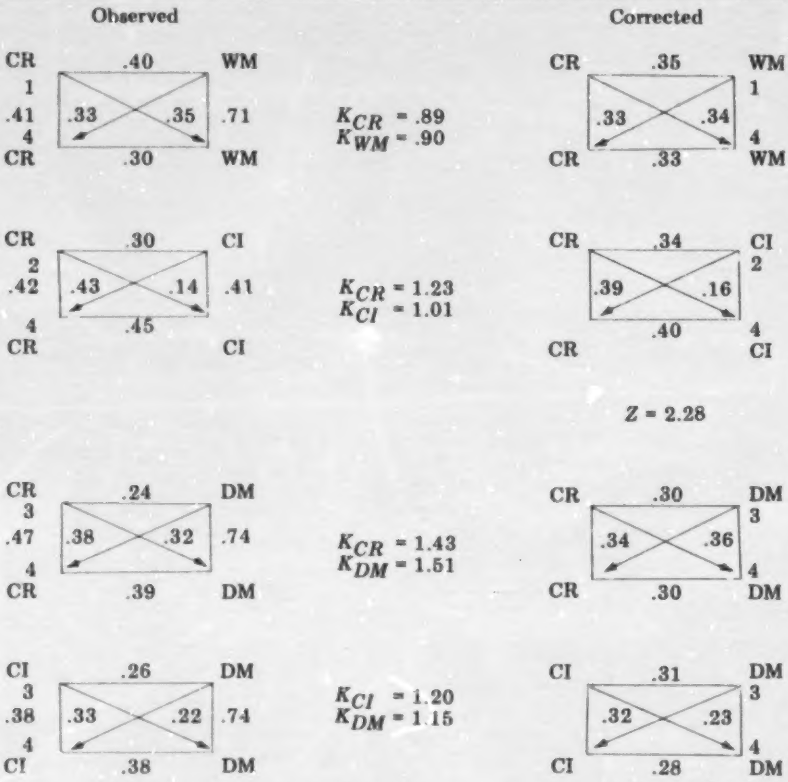
Table 5-7. Cross-Lagged Correlations: Visual Memory (*VM*) and Combinatorial Reasoning (*CR*)


Year
Observed
Corrected
1 VM
.64
CR
VM
.51
CR1
.65
.47
.39
.41
KVM = .85
KCR = .73
.43
.42
2
.43
.54
2
2
.43
.41
2
.59
.32
.24
.39
KVM = 1.06
KCR = .89
.29
.26
3
.38
.40
3
3
.38
.40
3
.61
.41
.37
.47
KVM = .75
KCR = 1.43
.57
.27
4
.37
.36
4
Z = 3.38
1
.64
.51
1
.58
.29
.46
.41
KVM = .71
KCR = .89
.33
.41
4
.37
.47
4
Z = .84

3.38), and suggesting that growth in combinatorial reasoning leads to growth in visual memory. Since such a developmental influence could hardly alter a fundamental physiological capacity, if we accept this finding, then the most reasonable causal link would seem to be not growth in memory space but a mediational process of change in means and efficiency of memoric encoding due to the emergence of new logical structures. However, the years one to four results, while not significant, are apparently in the opposite direction from those of each of the intervening adjacent years, suggesting that the hypothesis that change is occurring as a motion of a rigid system, with increases and decreases in correlations reflecting only changed reliabilities rather than relative directions of vectors in the system, may not be tenable.

Table 5-8 presents the observed and reliability-corrected cross-lagged correlations among year four class inclusion and combinatorial performances and those remaining previous years' variables that entered significantly into the regression equations. The only signifi-

Table 5-8. Cross-Lagged Correlations: Word Memory, Digit Memory, and the Piagetian Variables


Observed
Corrected
CR 1 .40 WM 1
.41 .33 .35 .71
CR 4 .30 WM 4
K_{CR} = .89
K_{WM} = .90
CR 1 .35 WM 1
.33 .34
CR 4 .33 WM 4
CR 2 .30 CI 2
.42 .43 .14 .41
CR 4 .45 CI 4
K_{CR} = 1.23
K_{CI} = 1.01
CR 2 .34 CI 2
.39 .16
CR 4 .40 CI 4
Z = 2.28
CR 3 .24 DM 3
.47 .38 .32 .74
CR 4 .39 DM 4
K_{CR} = 1.43
K_{DM} = 1.51
CR 3 .30 DM 3
.34 .36
CR 4 .30 DM 4
CI 3 .26 DM 3
.38 .33 .22 .74
CI 4 .38 DM 4
K_{CI} = 1.20
K_{DM} = 1.15
CI 3 .31 DM 3
.32 .23
CI 4 .28 DM 4

Partial Correlations Year 4
rCICR · VM = .38 rCICR · WM = .37
rCICR · VMDM = .33

cant difference among cross lags, $Z = 2.28$, occurs for year two class inclusion as a predictor of year four combinatorial reasoning. Again, however, the corrected synchronous correlations, .34 and .40, are sufficiently disparate to call into question the assumption on which the reliability corrections are based, although again, not significantly different with this sample size.

An alternative test offered in Kenny (1973) is based on the availability of additional occasions of measurement rather than of additional variables measured at two occasions. In this approach, reliabilities are not directly manipulated, but loadings on a postulated synchronous common factor and on a lagged factor are assumed to stay in a constant proportion. That is, observation vectors may converge or diverge over time, but they are assumed to do so at a steady rate. Kenny offers three wave causal contrasts in two forms: one form involving synchronous correlations and one only lagged correlations. It is easy to generalize these equations for the solution of four-wave data, and this has been done in Table 5-9. The first two columns represent the values of the two components of the test based on synchronous correlations. If the value of the left column is significantly more positive than that of the middle column, cause is presumed to flow from x to y. The discrepancy is greatest for CR to VM and in the other direction, from CI to CR, supporting the findings obtained through reliability correction.

However, the fact that small negative values appear for at least one component of each contrast suggests that the synchronous correlations rx_2y_2 and rx_3y_3 are slightly inflated by shared method variance. To the extent that the negative values are not only because of a sampling error, the assumptions of this test can also be questioned. The last column of Table 5-9, however, offers a test that does not depend on synchronous correlations, being affected by only the factors hypothesized to influence time-lagged correlations. This test in fact does not depend on synchronous data at all, and it again leads to the conclusion that combinatorial reasoning exerts a causal influence on the development of visual memory and that class inclusion is developmentally prior to combinatorial reasoning.

Partial correlation, removing covariance predictable from initial

Table 5-9. Four-Wave Causal Equations

X	Y	$rx_1y_2rx_2y_3rx_3y_4$ $-rx_2y_2rx_3y_3rx_1y_4$	$rx_2y_1rx_3y_2rx_4y_3$ $-rx_2y_2rx_3y_3rx_4y_1$	$\frac{rx_1y_2rx_2y_3rx_3y_4}{rx_1y_4} - \frac{rx_2y_1rx_3y_2rx_4y_3}{rx_4y_1}$
CR	VM	.014	-.041	.137
CR	WM	-.008	-.020	.036
CR	DM	-.020	-.012	.001
CR	CI	-.001	.026	-.118
CI	VM	.006	-.006	.038
CI	WM	-.015	-.004	-.015
CI	DM	-.017	.001	-.052

status on memory variables, considerably reduced the relationship between the two Piagetian tasks in the final year ($r_{CICR} = .45$, $r_{CICR \cdot VMDM} = .33$), suggesting that short-term memory factors account for a significant portion of the covariance of the two logical operations tasks. Examination of the cross-lagged panel correlations, however, suggests that using the memory variables as "passive" covariates may represent an oversimplification. In each of the three adjacent year comparisons, previous level of combinatorial reasoning predicts visual memory at a slightly higher level than visual memory predicts combinatorial reasoning ($r = .47$ vs .39; .32 vs .24; .41 vs .37). This suggests that visual memory task performance is not merely a background or capacity variable, but may itself be influenced by emerging operational structures. Class inclusion and digit span reveal an inconsistent pattern of direction of influence.

These results make it clear that individual differences in short-term memory development contribute to the relationships of these performances of concrete operations. However, the direction of apparent causality between combinatorial reasoning and visual memory in all years, and with auditory memory from years two to three, suggests that changes in memory performance may also be to some degree a *result* of the development of cognitive operations. To the extent that the assumptions of these tests are reasonable, the use of memory variables as covariates will be counterindicated, and the development of reciprocal causal models will be necessary to describe these relationships. Recent discussions of the complex interrelationship among memoric processes, metamemory, and cognitive developmental variables support this conclusion (Flavell, 1977; Hagen, Jongeward, & Kail, 1975; Kail & Hagen, 1976, and Piaget & Inhelder, 1973).

That class inclusion influences the emergence of combinatorial reasoning is not surprising. Class inclusion, as conceptualized and measured by Piaget, is one of the more difficult attainments of the concrete operations period, but combinatorial reasoning is necessary to formal operations. The finding of a causal link from combinatorial reasoning to visual memory may be less in accord with intuition or conceptual task analyses. Since McLaughlin (1963), information-processing theorists have thought of short-term memory as a capacity or working space limiting the complexity of cognitive operations. However, more recent approaches to memory development (e.g., see Flavell, 1977, pp. 183–218) have emphasized encoding, structuring and retrieval processes, and strategies as the entities in which development works its changes, rather than seeing development as the physiological growth of computing space.

DISCUSSION

It is probably worth repeating at this point that the present investigation was concerned with two major contentions within the Piagetian organismic system. These were the fundamental stage properties of integration and structuring and the derivative assumption of developmental synchrony for behavioral patterns putatively native to a particular stage or time period. The specific focus was directed toward the concrete operations period of middle childhood.

Initially, it was seen that continued participation (selective survival) influences were generally absent in the present study. In similar fashion an analysis of repeated measurement biases revealed few significant or consistent effects that would seriously question the longitudinal developmental trends (see, however, note 2). As would be expected for the present age and grade range, assessment interval, and concrete operations task arrays, notable developmental trends were evident. Thus, the main effects of age grouping and assessment year were highly significant for almost all of the task comparisons. Significant gender distinctions and sex and condition interactions were notably absent. Moreover, the trend analyses indicated that linear growth patterns were predominantly present in the great majority of the present task cases. Significant quadratic components (14 cases) and cubic components (1 case) highlighted the fact that the slope of the developmental gradients depended upon the age cohort in question. As would be expected growth was most marked for the younger cohorts.

In general, these result patterns are in accord with the expectations derived from both organismic theory à la Piaget and the original Genevan norms based on cross-sectional data (much of which did not permit direct comparisons across tasks). Yet the answer to the critical issue of *developmental synchrony versus developmental asynchrony* remains equivocal. Numerous cases of developmental task sequences (therefore denoting within-stage asynchrony) are evident. The more notable instances include the following cases. The emergence of classificatory skills in general appears to follow their relational counterparts (although some notable cases of synchrony are also present—see below). This is true for initial levels of mastery and later functional maturity. Thus the four class groupements cases were much more difficult than the four relations cases, and this was particularly true of the class composition operations. Within the classificatory task domain the "some and all" understanding task was distinctly easier (and therefore an earlier appearing developmental acquisition) than class inclusion, dichotomous sorting, or cardinality. The later three tasks appear to be of generally equivalent difficulty. This relative difficulty pattern agrees with Kofsky's (1966) earlier cross-sectional findings. Performance on the class inclusion task at all age and grade levels seems to exceed mastery of the initial four groupement tasks. Within the relations task domain transitive inference (as presently assessed) was much less difficult than seriation or the latter groupement task cases (V–VIII). Functional mastery of the seriation tasks appears to *follow* mastery of the four relations groupement cases.

Mastery of identity conservation preceded equivalence conservation (Brainerd & Hooper, 1975, 1978; Hooper, Toniolo, & Sipple, 1978), and each of these followed mastery of the counterpart transitive inference tasks. Within the eight groupement cases, as cited above, the class tasks are much more difficult than the relations tasks. Concerning the complimentary groupement operations skills, class composition mastery is much later in appearance when compared to class inversion skills. While these results may be traced to differential verbal demands or semantic understanding requirements of the composition versus the inversion test items (Dihoff, 1976a), they stand in marked contrast to the equivalent difficulty patterns for the relations composition and reciprocity operations items over the present age range.

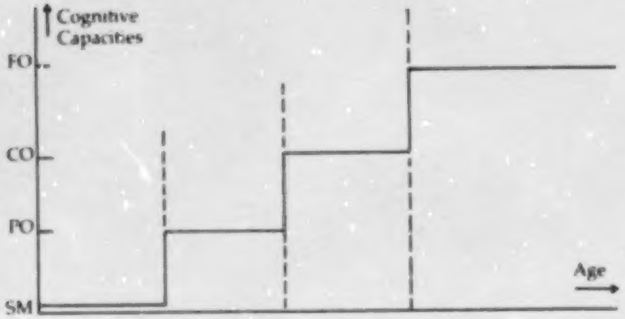
In contrast to these cases of developmental sequences within the period of concrete operations, a number of synchronous concurrences were also observed. Cardinality skills appear to develop in parallel with seriation skills (transitive inference excepted). Class inclusion understanding, cardinality, and dichotomous sorting (the latter involving a consistent and exhaustive classification on each of three attributes) appear to be of equivalent difficulty. Conservation of number, equivalence conservation of length, and equivalence conservation of weight indicate generally similar growth trends. Finally, and somewhat surprisingly, combinatorial reasoning (presumably a measure of early formal operations period concept abilities) and class inclusion reveal very similar developmental patterns.

Insofar as the three short-term memory span tasks are concerned, the means for forward word span exceed those for forward digit span at all age levels and assessment years. Visual orientation memory in contrast to these two auditory cases reveals much more accelerated developmental trends (steeper gradients) and higher mean percentage scores at all grade levels. As covered above in the final portion of the results section, short-term memory functions appear to have a mutually reciprocal developmental relationship with the present array of logical reasoning tasks.

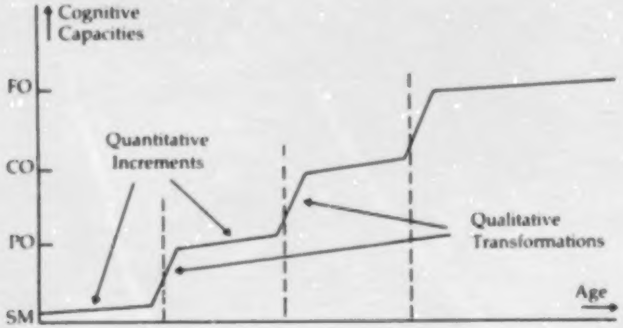
Concerning the actual validity of developmental stages the following generalizations are in order. One, there is no question that the younger children in the present study (in terms of age and grade level or initial assessment years) approached the logical reasoning tasks in a distinctly different way from that shown by their older counterparts or, indeed, by themselves in the later years of the study. Anyone (including the test administrators employed in the present project) who has questioned a nonconserving (i.e., preoperational stage) child compared to a conserving child knows immediately what these data convey. Of course, if one observes the mean score trends averaged across cohort groups and assessment years one is apt to conclude that developmental progressions are very gradual, indeed, even linear in character. In this regard, one definitive conclusion from the present data is patently obvious, that, is, quantitative and qualitative developmental patterns are not mutually exclusive. Rather, quantitative increments in one domain, for example, short-term memory, facilitate qualitative "leaps" in another area of cognitive functioning. Certainly the converse also holds true—that is, a mastery of certain logical skills, for example, transitive inference, appears to "transfer" to a counterpart cognitive domain, for example, conservation (Hooper, Toniolo, & Sipple, 1978).

What we are left with is a decidedly complex but not uninterpretable picture of cognitive growth. At the initial phase of any developmental period interindividual and intraindividual variability is most notable. Strong relations among task domains are at this time not to be expected. Gradually cognitive abilities emerge and facilitate each other. This would coincide with Wohlwill's cases of *reciprocal interaction* or *disequilibration and stabilization* (see Wohlwill, 1973, pp. 204-236). Eventually the complimentary task domains reveal a decidedly concordant mastery pattern.

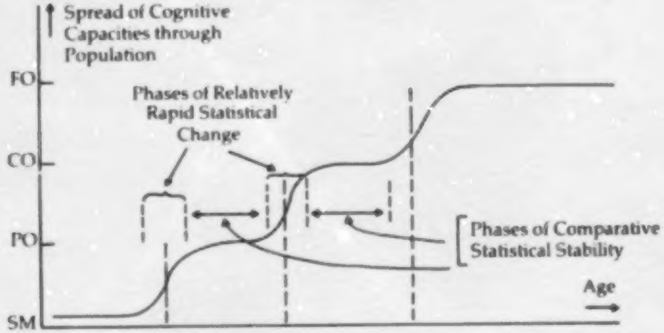
The picture of cognitive growth generated by the present study may be depicted by the third case shown in Figure 5-14. Rather than a simple step function of cognitive growth (Figure 5-14a) or a modified step function as suggested by Flavell (1971) (Figure 5-14b), the best approximation would appear to be a sigmoid sequence of stage transitions (Figure 5-14c) (these figures were originally presented in Feldman & Toulmin, 1976, p. 450). If one "expands" this figure of an idealized growth function to include an *array* of cognitive measures, one is immediately impressed with the inherent complexity of the generalized case. Variability coexists with across-task consistency. Still, this fundamentally is the essence of the human condition. Some abilities grow in a "lock step" fashion with other abilities while other abilities appear to be the genetic precur-


Cognitive Capacities
FO
CO
PO
SM
Age

A. Simple Step Function Model of Stage Transition


Cognitive Capacities
FO
CO
PO
SM
Quantitative Increments
Qualitative Transformations
Age

B. Modified Step Function (after Flavell, 1971)


Spread of Cognitive Capacities through Population
FO
CO
PO
SM
Phases of Relatively Rapid Statistical Change
Phases of Comparative Statistical Stability
Age

C. Sigmoid Sequence of Stage Transitions

Figure 5-14. C.F. Feldman and S. Toulmin. Logic and the Theory of Mind. In W. J. Arnold (ed.), *Nebraska symposium on motivation 1975: Conceptual foundations of psychology*. Lincoln: University of Nebraska Press, 1976, 450.

sors of later appearing higher order cognitive acquisitions. In a very real sense *both* cognitive consistency and variability are present in this investigation's data.

At a more technical level, the major question of cognitive décalages remains to be resolved. Are the cases of sequences briefly outlined above best considered to be instances of horizontal or vertical décalages? If they are judged to be the former, then few problems are raised for general Piagetian theory. If we wish to contend that a genuine vertical décalage was at issue (e.g., between the general relational vs. the general classificatory concept cases), then it remains to be demonstrated on what logical or psychological grounds this *across stage sequence tests* (Pinard & Laurendeau, 1969). Recently a new form of task interrelationship, the oblique décalage, has been suggested. This has been described as "indicated in differential achievement between conservation, for example, and classification and ordering tasks, which are informed by common concrete operational structures but different component structures" (Beilin, 1976, p. 78, footnote 2). It would seem that, despite their ad hoc basis, oblique déclages are designed to encompass both within and across stage phenomena and thus may be appropriate to the present case. Certainly within stage developmental concurrences have been observed relatively rarely (e.g., see Braine, 1959; Gonchar, 1975; Toussaint, 1974; and a recent review by Flavell, 1977). Indeed, Flavell concludes that *non*concurrence among theoretically related cognitive items is the rule and concurrence relationships the special exception (Flavell, 1977, p. 248).

The possibility exists, of course, that the concrete operations period is perhaps best viewed as a two-part sequence with certain earlier appearing abilities or skills (e.g., identity conservation, some-all understanding, and transitive inference) preceding certain later cognitive skills such as those dealing with class inclusion relationships and cardinal number (Pinard and Laurendeau, 1969). But this suggestion brings us full circle to the original issues of horizontal versus vertical décalage; that is, if there is really a two-stage developmental sequence, then what are the logical and functional prerequisites present in one task domain relative to the other? For natural number concepts Piaget's contention that classes and relations are logically coordinate and therefore should be psychologically parallel achievements is certainly intuitively appealing (Gonchar, 1975). Yet the majority of recent research results (including the present investigation) appears to favor the primordial status of relational concepts over the classificatory cases and thus favor Peano's views over either Piaget or Whitehead and Russell's theories (Brainerd, 1978b, 1979c). There do not seem to be any logical or task requirement analysis bases for this earlier emergence of relational as contrasted with classificatory concept acquisitions.

Nevertheless, the general concept of a cognitive stage for the middle childhood behavioral domain may be alive and well in places other than Geneva! Insofar as the concrete operations focus is concerned we agree with the following recent comments:

> Such evidence changes the meaning of "stage" in an interesting way. For a major stage like concrete operations, we might have expected a very brief period of change and transition, during which concrete operations emerge and mature, followed by several years of relative stasis and quiescence, during which the child is more or less stably and unchangeably concrete-operational in his thinking. If, instead, the child actually continues to perfect, generalize, and solidify his grasp of weight conservation throughout most of middle childhood and perhaps also well into adolescence, the stage of concrete operations is all change and transition, with little or no stasis and stability. Thus the stage itself, and not the transition to it, becomes the period of continuous growth and change. Because of this continuous growth and change, one cannot predict the child's responses to concrete-operational tasks merely from the knowledge that he is in the concrete-operational stage, as one could have if being in that stage meant continuing to have essentially the same mental structure for a period of years. This loss of predictability reduces the scientific value of the stage concept, but I do not think it makes it valueless. Suppose that all concrete-operational skills developed concurrently in an interdependent, mutually facilitative fashion. The fact that all these synchronous, closely interacting developments took a long rather than a short time to be completed would not mean that the term "stage" could not be applied meaningfully and usefully to this developmental pattern. We simply would have a more dynamic concept of stage, one that refers to an extended process of concurrent, interdependent developmental changes. Wohlwill thinks the concept of stage is theoretically useful, and the sort of stage he has in mind is of this dynamic sort (Wohlwill, 1973, Chap. 9). (Flavell, 1977, pp. 246–247)

This commentary reinforces the evaluation offered a number of years ago, that is, "Any monolithic attempt to describe or encompass the totality of cognitive growth from birth to maturity is certain to ignore the immense complexity and inherent spontaneity of man as an intellectual organism or to greatly overestimate our current level of sophistication in the behavioral sciences" (Sigel & Hooper, 1968, p. 5).

The final evaluation of the present normative investigation leads one to a "compromise" conclusion. While developmental synchrony was more evident in the initial and final assessment years for the younger age cohorts (ranging from 6 to 11 years of age), the transition from preoperational forms of logical reasoning to a full mastery of the logical groupements specified by Piaget as underlying the concrete operations period appears not to be a punctate, discontinuous episode in the child's life. Rather, the acquisition of concrete operations appears to involve an extended developmental process marked by unevenness and "reciprocal interweaving" among the supposedly unitary logical concept domains. However, instead of prematurely rejecting stages at this point, we prefer to consider the stage construct as an heuristic device for guiding our future research efforts and associated theoretical speculations. Thus the primary question concerns what modifications are necessary in the Piagetian concrete operations stage "designation" to permit it to play a continued role as a potential explanatory construct but in more complete accord with developmental reality.

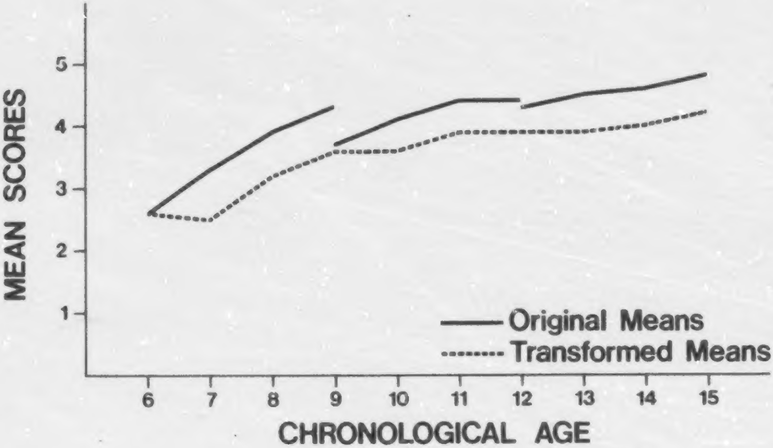


Figure 5-15. Class Inclusion Task Mean Scores Adjusted for Retest Effects

NOTES

1. We wish to extend our gratitude for their cooperation to Lyman Olson, Administrative Assistant; Virginia Carlson, Ralph Crowe, Arthur Engel, Robert Harris, Merlin Kenitzer, W.W. Kirchoff, George Kolak, Ralph Napper, Merlin Nundahl, and Peter Peterson, elementary school principals; Laverne Larsen and

John Smiley, junior high principals; and the staff and the students of Royce, Waterman, Merrill, Cunningham, Converse, McLenegan, Todd, Gaston, Burdge, and McNeel Elementary Schools and Roosevelt and Lincoln Junior High Schools of the Beloit, Wisconsin, Public School System. We wish to acknowledge the diligent efforts of Susan Anderson, Nikki DeFrain, Roberta Dihoff, Arthur Gonchar, Patricia Cone-Gorham, Rosie McMahan, Priscilla Roth, Richard Stern, and Tom Toniolo in administering the tasks. The authors also wish to thank Dr. Charles Brainerd of the University of Western Ontario for consulting on theoretical matters throughout the development of the study. We are grateful to Craig Hougum and Ed Haertel for providing computer programming services and to Janet Lindow and Suzy Gregg for typing the manuscript.

2. It is clear that the present trend analyses may be biased to unknown degrees by repeated testing effects and cohort distinctions (or, most likely a combination of these interactive influences). In order to "adjust" the individual child's performance patterns to account for these potentially biasing influences a special correction procedure was employed. This method and the associated results, which are reported in detail in Hooper, Sipple, and Swinton (in preparation), adjusts each individual's score to remove both retest bias and cohort differences. The resulting scores represent performances that would have been obtained if an actual full-range longitudinal study had been undertaken and no retest biases were present. The corrected trend analysis are reported in Hooper, Sipple, and Swinton (in preparation), and a representative case for class inclusion reasoning scores is shown in Figure 5-15.

Chapter 6

Relationships Among CLD Measures and Measures Based on Piagetian Theory

Herbert J. Klausmeier

Twenty-eight measures of cognitive development used in the comparative study to be reported in this chapter were based on an information-processing theory of cognitive learning and development (CLD). Two sets of measures of cognitive development based on Piagetian structural theory were used. (See Feldman & Toulmin, 1976; Flavell, 1977; and Brainerd, 1978b, for a discussion of the major constructs of Piagetian structural theory.) One set consisted of 13 familiar Piagetian tasks and the other included 16 groupement tests constructed by Brainerd (1972b). Both sets are described in detail in Chapter 2 and are mentioned later in this chapter.

None of the measures of development based on CLD and Piagetian theory are identical and the explanatory constructs are different. CLD theory employs the constructs of *levels of conceptual development* and of the *mental operations* necessary to attain the successively higher levels whereas Piagetian theory uses *stages* and *psychological structures.* Despite these differences, attaining concepts and their uses as critical elements of a total pattern of cognitive development should be able to be described and accounted for by both theories. Therefore, comparing and relating the dimensions of development based on both theories should contribute substantially to extending our knowledge of the course and nature of cognitive development. (The terms, dimension of development, developmental item, and item of development refer to what was measured by each test used in this comparative study.)

In describing the course of cognitive development, many developmentalists, including Wohlwill (1973), indicate that it is desirable to identify the year of life in which any dimension, or item, of development first becomes functional in normally developing individuals and also the year that it reaches either its full functional level or the maximum level at which normally developing individuals attain it. Determining the rate at which the item develops and also the form of the developmental curve is also desired. Normative information of this kind is needed in diagnosing and then predicting any maturing individual's pattern of development.

In this comparative study, Wohlwill's ideas were followed, except that the time of the initial functioning of some of the dimensions could not be identified directly from the observed data because the children when first tested in either kindergarten or Grade 1 were already functioning above a beginning level. The maximum level for a few dimensions also could not be ascertained because some students had not achieved full functional maturity when in Grade 9.

Various procedures for comparing the dimensions of development based on the two theories were employed in the present study. The means for each of the 28 CLD tests, 13 familiar Piagetian tests, and 16 groupement tests developed by Brainerd were compared. (Not all these tests were administered to the students of every grade.) In a second comparison of the test means, the rank order of the emergence and of the mastery, or full functioning, of all the items of development were identified. From the rank orders one can infer whether items predicted by a theory to emerge at about the same time did or did not emerge in accordance with the prediction. A similar inference can be drawn about full functioning of the items. (Full functioning of a developmental item was inferred when the test mean was 85 percent correct or higher, the criterion set for mastery of the test.)

Correlations were obtained among all the tests administered each year and also at the one-year and two-year measurement intervals. These rs were examined primarily to deal with the predicted relationships among tests of the same domain—CLD, Piagetian, and groupement—and between tests of the different domains. Of particular interest were those dealing with the groupement structures and the familiar Piagetian tests of the concrete operations stage.

Relationships also were identified by conducting interbattery factor analyses. Because the number of students who received the three test batteries was small in relation to the total number of tests administered, each student's scores from certain tests of each battery were combined and factor analyses were performed on the correlation matrices based on these combined scores. This permitted inferences to be drawn about the attributes of development common to CLD and Piagetian theory.

TECHNICAL CHARACTERISTICS OF TESTS

As indicated in Chapter 2, four tests within each of four batteries were constructed to measure the mental operations that are assumed by CLD theory to be necessary for children and youth of school age to attain the same concept at four successively higher levels; and another three tests of each of the four batteries were constructed to measure understanding principles, understanding taxonomic relations, and solving problems involving the particular concept (Klausmeier, Bernard, Katzenmeyer, & Sipple, 1973; Klausmeier, Ingison, Sipple, & Katzenmeyer, 1973a, b; Klausmeier, Marliave, Katzenmeyer, & Sipple, 1974).

Twelve Piagetian tasks were identified and adapted to measure performances directly related to Piaget's stage of concrete operations (Hooper, Brainerd, & Sipple, 1975). These tasks had been used by various experimenters in earlier research dealing with Piagetian formulations. A test of the Piagetian stage of formal operations, combinatorial reasoning, was also constructed and administered.

Brainerd (1972b) constructed a 64-item test organized into 16 subtests to measure Piaget's groupement structures. He used the tests in research to determine the extent to which Piaget's (1942, 1972) explanatory construct of psychological structures coincides with the observed facts of cognitive functioning in middle childhood. Brainerd chose Piaget's groupement structure because it is the psychological structure on which the concrete operations stage rests (Piaget, 1972). Both Flavell (1971) and Brainerd (1972b) indicated that in 1972 Piaget and others had not indicated means by which the groupement structure could be verified empirically. Therefore, elaboration of the Brainerd test battery and Piagetian theory is warranted.

Piaget postulated eight groupement structures, one subset of four involving "class" groupements and another subset of four "relations" groupements. Concerning the origin of the groupement structures, Piaget presumed that a given groupement structure is engendered by either a composition operation or its inverse operation. According to Brainerd,

> . . . each *class* groupement owes its existence to the fact that the cognizer is capable of simultaneously uniting and subtracting classes, while each

> *relational* groupement owes its existence to the fact that the cognizer is instantaneously aware of the reciprocal implications of the equivalence and difference relations produced by arithmetic addition and multiplication. To illustrate, the capacity simultaneously to add two subordinate classes to yield a superordinate class and to subtract one subordinate class from the superordinate class to yield the remaining subordinate class engenders a certain class groupement structure. Also to illustrate, the knowledge that $A < B$ and $B < C$ simultaneously implies both $A < C$ and $C > A$ engenders a certain relational structure. Although one may quibble with these claims about the origins of groupement structures on theoretical grounds, there is no denying their obvious empirical implication: If one wishes to conduct research on these structures, then one most certainly will have to focus on the *unions and intersections of hierarchical classes and their inverses*—on the one hand—and on *the addition-multiplication of equivalence-difference relations and their reciprocals*—on the other. (Brainerd, 1972b, p. 5).

Brainerd (1972b) derived three crucial predictions based on Piaget's groupement theory. First, developmentally all eight groupements will emerge and develop synchronously, not successively; second, developmentally each composition operation and its inverse will be acquired in synchrony, a composition operation cannot follow after its inverse operation; third, the correlation between the composition operation and its inverse operation within each of the eight groupements will be significantly larger than the correlation of a composition or inverse relation of one groupement with the composition or inverse relation of another groupement.

Brainerd constructed a 64-item test to measure the groupement structures, eight items for each structure. Within each class structure, four items were constructed for the composition operation and four for its inverse; similarly four items were constructed for each operation of each relations groupement structure. The tests were administered to approximately 100 children of five age levels, ranging from 4.5 to 5.5 years through 8.5 to 9.5 years. Contrary to the first two predictions, Brainerd found that the structures did not develop synchronously; some emerged earlier and also reached full functional maturity earlier than others. Perhaps more important, the relational groupements were found to emerge and become fully functional earlier than the class groupements. The inverse relations of subtraction and division were invariably acquired earlier in life than the paired composition operations of addition and multiplication. For the third critical prediction, the within structure rs were not found to be significantly higher than the between structure rs.

With this brief introduction to the tests of the three domains, we may examine the characteristics of the tests and also other information presented in Table A6-1 in the appendix.

The numbers of students who took part or all of the tests of the CLD, Piagetian, and groupement batteries were 37 for the Grade 1-2-3 group, 45 for Grades 4-5-6, and 44 for Grades 7-8-9. The reliability coefficients shown in Table A6-1 of the appendix range from .00 to 1.00. In all cases where the reliability coefficient is .00 the test mean is 95 to 100 percent correct. The single coefficient of 1.00 was associated with a mean percent correct of 95. It may be appropriate to consider further the relationship of the low reliability coefficient to the difficulty of the tests.

Since one purpose of the CLD study was to chart the course of cognitive development during the school years, Grades 1-12, tests were administered in the next grade after that in which students of the same age and other characteristics in earlier pilot studies functioned above a chance level. Thus, as shown in Table A6-1, seven CLD tests were not administered in Grade 1 and five not in Grade 2. When some of these tests were administered for the first time, their reliability coefficients were relatively low, since a considerable number of the students were still functioning at a chance level. However, determining an early developing individual's beginning manifestation of a cognitive dimension requires that the test that measures it must be administered at an age or grade in which slow developing students still perform at a chance level.

It was also necessary to administer the tests in one or more grades after the mean was 85 percent correct in order to evaluate when each student, including the slowest developers, first achieved full functioning of the developmental item measured by the test. Eighty-five percent correct was the criterion established for determining mastery of any test. This percent correct on a test corresponds to full functioning of the developmental item measured by the test. Administering tests when the mean percentage correct is very high results in relatively low Hoyt reliability coefficients and also occasional abnormally high ones.

Most of the reliability coefficients were between .40 and .99 (Table A6-1). A higher proportion were above .60 for the Piagetian measures than for the groupement measures, and for the groupement measures than the CLD measures. The lower coefficients for the CLD measures are attributable to the less homogeneous content of the CLD tests.

Tests with reliability coefficients below .20 based on any year of testing any longitudinal group of students were used in computing test means; however, they were not used in computing correlations.

This level was set in order to retain a maximum number of tests while still eliminating those with near chance level reliability coefficients.

COMPARISONS BASED ON 57 SEPARATE TEST SCORES

Comparisons Based on Means for Separate Tests

The means for each test at each grade were examined to arrive at a best estimate as to whether the pattern of the test means across the successive grades based on the two theories was more alike or different. The means for each set of tests were also rank ordered and analyzed to estimate likenesses and differences between the two theories and to determine whether the observed results were in accordance with predictions based on the theories.

Pattern of Test Means. Inspection of the test means for each of the three longitudinal blocks of students (Table A6-1) shows that in the large majority of cases the means increased for the successive intervals of measurement until 85 percent correct or higher was achieved. The incidence of increases and occasional decreases appeared to be about the same for the CLD, Piagetian, and groupement means. Based on these observations, one would infer more similarities than differences in growth of the developmental items included in the two theoretical approaches. (Statistical analyses of the means for the Piagetian and groupement tests were reported in Chapter 5 and for the CLD tests in Klausmeier & Allen, 1978, and are not repeated here.)

The increases were expected but the decreases were not; therefore the decreases merit further consideration. The means for the students when in Grade 2 on four tests of *cutting tool* were lower than in Grade 1. Means on two groupement tests were also lower in Grade 2 than in Grade 1. In some cases a mean score for Grade 5 or Grade 6 was lower than for the prior grade. The mean for the problem-solving test of *equilateral triangle* decreased from Grade 7 to Grade 8 by 1 percent and for understanding principles of *noun* from Grade 8 to Grade 9 by 3 percent. Ten of 16 of the groupement tests had means in Grade 7 equivalent to 90 percent correct or higher. Small fluctuations, including lower means at Grade 8 or Grade 9 occurred on these tests.

Most of the preceding decreases occurred when the means were already high in the year before the decrease occurred. These and the other decreases probably resulted from error of measurement rather than being a true decrease. However, the possibility of a true decrease on one or more tests cannot be ruled out.

The means for 18 of the 28 CLD tests, 5 of 13 Piagetian, and 10 of 16 groupement tests were lower in Grade 4, the first year that the Grade 4-5-6 students were tested with all four CLD batteries, than in Grade 3, the third year that the Grade 1-2-3 students had been tested. Similarly, the means for 16 of 20 CLD tests and 10 of 16 groupement tests were lower in Grade 7 than in Grade 6. The differences between the means statistically significant at the .05 level were as follows: 5 CLD and 1 groupement between Grade 3 and Grade 4 and 9 CLD between Grade 6 and Grade 7. Any lowering of the means for the older students, whether statistically significant at the .05 level or not, is not predicted by either theory, and a developmental curve based on the lower mean signifies not only cessation in growth of the item but a regression.

How may the lower means of the older students be accounted for? Klausmeier and Allen (1978) found significant cumulative retesting effects. There were also significant cohort effects, that is, control students who were two years younger than the longitudinal students scored higher than the longitudinal students. The present students were part of the same longitudinal groups. The error of measurement as inferred from the test reliability estimates given in Table A6-1 was also substantial Thus, the higher means in Grades 3 and 6 and the lower means in Grades 4 and 7 are regarded as resulting from the combined effects of repeated testing, differences connected with being born two years earlier, and error of measurement.

Rank Order of Initial Functioning and Mature Functioning of Dimensions. The rank order of the *initial functioning of the developmental items* of the CLD, Piagetian, and groupement domains was derived as follows. At the grade at which a test of each battery was first administered, the percent correct on the test was noted. The percents for all the tests of the particular battery were then ranked from highest to lowest and the highest percent correct was given a rank of 1. Not all CLD tests were administered until Grade 3. Tests administered for the first time in a higher grade were assigned a lower rank order, indicating later initial functioning.

The rank order of *mature functioning of the developmental items* within each domain was derived in the following manner. The grade was identified at which a mean of 85 percent correct or higher on a test occurred for the first time. Within any grade two or more tests with percentages of 85 and higher were ranked from highest to lowest. Tests first reaching 85 percent correct in the successively higher grades were assigned lower ranks, but higher numbers, than those in the preceding grade. There was one necessary exception to the criterion of 85 percent correct. In Grade 9 the scores not yet reaching 85 percent correct were also ranked according to their values.

In Table 6-1 the rank order of the test means within each of the three domains is given as are also the rank order coefficients for the three domains. The rank order coefficients are .99 for the CLD and groupement domains and .95 for the Piagetian domain. From these very high correlation coefficients we infer that the rank order of the initial functioning of the developmental dimensions within each domain was nearly identical to the rank order of achieving full functional maturity of the same dimensions. For example, initially

Table 6-1. Rank Order of Initial Score and of Mastery Score Within Each of Three Domains

Rank Order—CLD			*Rank Order—Piagetian*			*Rank Order—Groupements*		
Test	*Initial*	*Mastery*	*Test*	*Initial*	*Mastery*	*Test*	*Initial*	*Mastery*
CT co	(1)	1	SA	(1)	1	APC	(16)	14
CT id	(2)	2	SER	(8)	13	SPC	(4)	4
CT cl	(7)	7	TL	(3)	5	ASC	(13.5)	15
CT fo	(10)	13	TW	(2)	2	SSC	(3)	3
CT pr	(15)	16.5	DS	(11)	8	BMC	(15)	16
CT tax	(11.5)	12	Car	(10)	12	BDC	(12)	13
CT ps	(14)	20	CN	(6)	3	CMC	(13.5)	12
TR co	(4)	3.5	ICL	(5)	4	CDC	(11)	10
TR id	(6)	6	ECL	(9)	6.5	ADR	(5.5)	5.5
TR cl	(9)	11	ICW	(4)	6.5	RDR	(2)	2
TR fo	(16)	18.5	ECW	(12)	10	AER	(5.5)	7
TR pr	(28)	23	CI	(7)	9	RER	(1)	1
TR tax	(19)	15	CR	(13)	11	BMR	(9)	9
TR ps	(20)	21		r = .95		BRR	(7)	5.5
ET co	(4)	3.5				CMR	(10)	11
ET id	(4)	3.5				CRR	(8)	8
ET cl	(11.5)	9.5					r = .99	
ET fo	(17)	18.5						
ET pr	(23)	22						
ET tax	(18)	16.5						
ET ps	(22)	24						
NN co	(8)	8						
NN id	(13)	9.5						
NN cl	(21)	14						
NN fo	(25.5)	27						
NN pr	(25.5)	28						
NN tax	(24)	25.5						
NN ps	(27)	25.5						
	r = .99							

attaining the concrete and identity levels of the concept, *cutting tool,* at a beginning functional level came earlier than any other developmental dimension of the CLD domain; and mature functioning, based on the mastery criterion, of the same two levels preceded mastery of any other dimension. This pattern of the rank order of the initial functioning and final mastery of dimensions was nearly the same for the CLD, Piagetian, and groupement domains.

My purpose in analyzing the rank order data was to compare the performances of the same students from the CLD and Piagetian perspectives in order to gain a better picture of cognitive development. We have just noted the great similarity. No particular relationship in the rank order of the initial functioning and mature functioning of the CLD dimensions and Piagetian dimensions is presumed by the theories; therefore no further comparison of CLD and Piagetian measures are made. However, the rank order data merit consideration related to other matters based on the two perspectives.

As shown in Table 6-1, the concrete, identity, classificatory, and formal levels of each concept emerged and were mastered in the invariant sequence predicted by CLD theory. No predictions were made regarding the order of mastery of understanding principles, understanding taxonomic relations, and problem solving except that all would follow after mastery of the formal level of the same concept. This prediction also was upheld by the rank order data. The reader may wish to refer back to Chapter 3 where these same results were reported for the four larger longitudinal groups of students from which the samples for this comparative study were drawn.

Kofsky (1966) identified sequential substages in the development of classification skills, based on Piagetian theory. According to Kofsky's interpretation of Piagetian theory, conservation should precede horizontal reclassification, and horizontal reclassification should precede hierarchical reclassification. The tests measuring these three operations employed in the present study and the rank order of percentages correct in kindergarten and the rank order of mastery that occurred in later grades were as follows: conservation as measured by the test, conservation of number, rank order of 6 and 3; horizontal reclassification as measured by the test, dichotomous sorting, 11 and 8; and hierarchical reclassification as measured by the test, class inclusion, 7 and 9. While the ranks for horizontal and hierarchical reclassification are in reverse order from the prediction for initial functioning 11 versus 7, the three dimensions were mastered in the predicted sequence as indicated by the ranks of 3, 8, and 9. Kofsky found similar support for most of the Piagetian substages of classification.

Piagetian theory indicates that quantitative identity develops at the same time as equivalence. The observed ranks of the initial means in kindergarten and of the means in later grades indicative of mastery were identity conservation of length—5 and 4 and equivalence conservation of length—9 and 6.5; identity conservation of weight—4 and 6.5 and equivalence conservation of weight—12 and 10. Furthermore, in each case the identity operation was mastered one year earlier than its paired equivalence operation. This finding contradicts Piagetian descriptions regarding the order of emergence of specific items of development.

All of the eight groupement operations involving relations emerged and were also mastered before six of the eight involving classes, which is contrary to Piagetian theory as interpreted by Brainerd. In addition, contrary to Piaget's view of the psychological structure underlying the concrete operations, the emergence and mastery of every paired set of composition and inverse operations within each of the class groupement structures was opposite to the prediction formulated by Brainerd as being consistent with Piaget's theory. The students performed better on every subtraction test than its paired addition test and also better on every division test than its paired multiplication test. The most revealing comparison of the ranks of the Piagetian and groupement tests is that full functioning of some of the groupement structures was achieved earlier than many of the Piagetian dimensions of concrete operations while others were not functioning fully even in Grade 9, whereas all 12 familiar Piagetian items were functioning at a mature level at some time either during the primary grades or the intermediate grades. If the Brainerd assumptions are correct and if the groupement tests are valid measures of the psychological structures of the concrete operations stage, the groupement structure is not useful in explaining the concrete operations stage. One might even say that there is no stage in the sense of a tightly related, synchronous structure.

Grades in Which CLD, Piagetian, and Groupement Tests Were Mastered. Identifying the grade in which each of the 57 tests was mastered provided another interesting comparison of the various dimensions of development. Table 6-2 indicates the grade in which the various tests were mastered for the first time or at which the highest percentage correct was achieved. Eight CLD tests, seven groupement tests, and two Piagetian tests were mastered in kindergarten or Grade 1. The eight CLD tests were the concrete level of the four concepts, the identity level of all four except *noun*, and the classificatory level of *cutting tool*. The seven groupement tests included five of relations and two of classes. The two Piagetian tests

Table 6-2. Grade at Which Tests of Three Domains Were Mastered

Test	*Mean Percent*	*Grade*	*Test*	*Mean Percent*	*Grade*
CT co	99	1	CT tax	86	4
CT id	98	1	CMR	95	5
TR co	97	1	ECW	89	5
TR id	96	1	CR	86	5
SSC	96	1	Car	85	5
SA	95	Kg	CT fo	89	6
ET co	92	Kg	Ser	86	6
CT cl	94	1	CMC	85	6
SPC	94	1	NN cl	86	8
TW	89	Kg	TR tax	90	9
ET id	88	Kg	CT pr	79	9
RER	86	Kg	ET tax	86	9
RDR	85	Kg	BDC	85	9
ADR	87	1	TR fo	85	9
BRR	87	1	ET fo	85	9
NN co	86	1	CT ps	75	9
AER	85	1	APC	78	9
CN	97	2	TR ps	77	9
ET cl	95	2	ET pr	76	9
NN id	95	2	ASC	76	9
TR cl	92	2	TR pr	75	9
CRR	90	2	ET ps	71	9
ICL	85	2	NN tax	68	9
BMR	85	2	NN ps	68	9
TL	92	3	NN fo	65	9
ECL	91	3	BMC	63	9
ICW	91	3	NN pr	56	9
DS	90	3			
CDC	89	3			
CI	87	3			

were transitivity of weight and some-all. In Grades 2 and 3, 13 tests were first mastered as follows: CLD—classificatory level of *equilateral triangle* and *tree* and the identity level of *noun;* Piagetian—conservation of number, identity conservation of length, transitivity of length, equivalence conservation of length, identity conservation of weight, dichotomous sorting, and class inclusion; groupement—counivocal reciprocity of relations, biunivocal multiplication of relations, and counivocal division of classes.

From the above we see that by the end of the primary school years the following tests had been mastered: the concrete, identity, and classificatory levels of all four concepts except *noun;* 9 of the 12 Piagetian tests of concrete operations, 7 of 8 groupement tests involving relations, and 3 of 8 tests involving classes. Thus, most of the operations involved in classifying, whether measured by the CLD tests or the Piagetian tests, had been mastered by Grade 3; however, the groupements involving classes had not.

In Grades 4 through 6 the formal level and understanding taxonomic relations of *cutting tool* were mastered; the Piagetian measures of equivalence conservation of weight, combinatorial reasoning, cardinality, and seriation were, and another groupement test of relations and another test of classes were. In Grade 8, only the classificatory level of *noun* was mastered. In Grade 9, 3 of the 4 CLD tests involving the formal level, 3 of 4 involving taxonomic relations, 4 of the 4 involving understanding principles, and 4 of 4 involving problem solving were first mastered or reached their highest level correct. Four groupement tests concerned with classes also were first mastered or reached their highest level; two of these involved addition, one multiplication and one division.

These findings suggest that the formal level of most concepts and understanding various relationships among the concepts are mastered during Grades 4 to 9, the same grades during which the groupement structures dealing with the composition of classes are.

We may now summarize the comparisons of cognitive development from the two theoretical perspectives based on the analysis of the test means. Growth toward full functional maturity of the items based on the two theories proceeds in much the same manner in that mastery of some items occurs in kindergarten and of others not even in Grade 9. As we shall see later, developmental curves based on the test means of any domain vary greatly, but the amount of variation is about the same for the CLD domain and the combined Piagetian and groupement domains.

The rank order correlations of the initial functioning of the developmental items and of the final mature functioning of the same items within each domain range from .95 to .99. From these high and nearly equal correlations, we infer that the order in which the developmental items of each domain emerge is the same as that in which they reach final mature functioning.

Other conclusions of interest pertain to the CLD, Piagetian, or groupement domains separately or to a combination of two of them. The prediction of an invariant sequence in the attainment of the concrete, identity, classificatory, and formal levels of the same concept was upheld, as was also the prediction that the mastery of the formal level precedes mastery of understanding principles, understanding taxonomic relations, and problem solving. (These principles are treated more fully in Chapter 3.) As predicted by both CLD and Piagetian theory, mature functioning of the identity operations precedes mature functioning of the equivalence operations.

Conservation, horizontal reclassification, and hierarchical reclassification emerged in the same order as tested and reported earlier by Kofsky (1966).

The observed order of the emergence and full functioning of the developmental items of the concrete operations stage, as measured by the use of the 12 familiar Piagetian tasks of concrete operations, and the order of the emergence and full functioning of the psychological structures of the concrete operations stage, as measured by the 16 groupement tests, are not supportive of the groupement structures as an explanation of the concrete operations stage. Many of the relations groupements were mastered earlier than the classes groupements, and many of the composition operations were mastered later than their inverse operations.

By the end of the primary school years, the ability to classify, except purely symbolic material such as *noun*, is functioning at a mature level whether evaluated by the CLD classificatory level tests or by the familiar Piagetian tests involving classification. However, the groupements involving classes do not reach a similar level of functional maturity for several years.

Mastery of the formal level of most concepts and of understanding principles and taxonomic relations among concepts occurs during Grades 4 through 9, the same grades during which the class groupements are mastered.

We should bear in mind that these conclusions are based on the means of tests of three groups of students. Individual student's scores are not directly reflected in the mean. In the next section of this chapter, correlational results are presented that reflect the individual's scores.

Correlations Based on Separate Test Scores

Correlations were obtained for the separate tests of the three domains for each annual measurement and also for the longitudinal measurements at the one- and two-year intervals. Longitudinal Blocks A—Grades 1-2-3; B—Grades 4-5-6; and C—Grades 7-8-9—were included. The resulting matrices, given in TR Table 6-1 (Sipple, Allen, & Klausmeier, 1979), were examined primarily to ascertain the relationships among the three domains rather than within each because the factor structure for the CLD domain separately is given in Chapter 4 and many relationships pertaining to the Piagetian domain and the groupement domain are dealth with in Chapter 5. Examination of the correlation matrices here was limited to identifying specific relations of interest, particularly those bearing on the preceding conclusions that were based on group means.

Preliminary inspection of the correlation matrices indicated that, contrary to expectations, the large majority of *r*s, Grades 1-6, were not statistically significant at the .05 level. Furthermore, some of the statistically significant rs were negative rather than positive. With this unexpected finding, and since factor analyses were not performed because the number of students in each longitudinal block in relation to the number of tests was very small, key questions were answered by logical analysis of the percentages of statistically significant rs.

1. Are the rs between the CLD tests and the Piagetian and groupement tests positive and statistically significant? Statistically significant, positive, and moderately high rs are generally found among students' achievements in reading and mathematics. Low, positive, but often not significant rs are found among students' achievements in reading, handwriting speed, and art production. Correlations of the first pattern were expected since all the CLD tests, the Piagetian tests, and the groupement tests measure performances in the cognitive domain rather than in a combination of the cognitive, psychometer, and affective.

Table 6-3 presents the percentages of rs statistically significant between domains based on the annual measurements and the 1- and 2-year intervals. Table A6-2 in the appendix gives the tests that yielded the statistically significant negative rs. The percentages of rs statistically significant and positive between the CLD tests and the Piagetian tests based on the six annual measurements for Grades 1-6 ranged from 4 to 15 percent and for the six 1-year and 2-year intervals from 9 to 14. One to 2 percent of the rs based on longitudinal measurements were significant and negative for each interval of measurement, except the Grade 5-6 interval. Examination of the correlation matrices, TR Table 6-1 (Sipple, Allen, & Klausmeier, 1979), indicates that in general different tests rather than the same tests contributed to the significant rs at different times of measurement. Based on this low percentage of significant positive rs we conclude that development of the items measured by the CLD tests and Piagetian tests was relatively unrelated.

The percentages of statistically significant positive rs between the CLD tests and the groupement tests based on the annual measurements ranged from 3 to 20 percent for Grades 1 through 6 and from 3 to 18 percent for the longitudinal intervals. Higher percentages of statistically significant positive rs and lower percentages of negative

Table 6-3. Percentages of rs Statistically Significant ($p < .05$) Between Domains Based on Annual Measurements and Longitudinal Measurements

Grades of Annual Measurement	*CLD and Piagetian*		*CLD and Groupement*		*Piagetian and Groupement*		*Grades of Longitudinal Measurement*	*CLD and Piagetian*		*CLD and Groupement*		*Piagetian and Groupement*	
	(+)	*(-)*	*(+)*	*(-)*	*(+)*	*(-)*		*(+)*	*(-)*	*(+)*	*(-)*	*(+)*	*(-)*
	(156 total rs)		*(143 total rs)*		*(132 total rs)*			*(312 total rs)*		*(286 total rs)*		*(264 total rs)*	
1	15	0	4	3	11	2	1-2	10	1	4	3	6	3
2	13	0	8	1	9	0	2-3	9	2	6	1	12	0
3	6	4	3	1	10	0	1-3	9	1	3	4	11	1
	(135 total rs)		(180 total rs)		(108 total rs)			(270 total rs)		(360 total rs)		(216 total rs)	
4	4	2	8	1	11	0	4-5	9	1	14	1	15	0
5	8	0	14	1	10	0	5-6	14	0	18	1	13	0
6	15	0	20	0	30	0	4-6	12	2	15	1	13	0
			(170 total rs)							(340 total rs)			
7	NC[a]		36	1	NC		7-8	NC		46	0	NC	
8	NC		64	0	NC		8-9	NC		55	0	NC	
9	NC		28	0	NC		7-9	NC		43	0	NC	

[a] Not computed since only one Piagetian test of concrete operations was administered.

rs were found for the Grade 4-5-6 longitudinal block than for the Grade 1-2-3 block. For Grades 7-8-9, the percentages based on the annual measurements ranged from 28 to 64 and on the longitudinal interval measurements from 43 to 55. Negative rs (1 percent) were found only at Grade 7. The rs based on the interval measurements were lower than for the annual measurements but followed the same trend from lower to higher for the three successive longitudinal groups of students.

Thus, the dimensions of development measured by the CLD and groupement tests changed from relatively unrelated during the primary school years to quite highly related during the junior high school years.

The relatively small number of tests yielding statistically significant negative rs are shown in Table A6-2. There were a few more negative rs between the CLD and groupement domains than between the Piagetian and groupement. No theoretical basis was identified to account for any of the negative rs. Similarly, they could not be accounted for in terms of lower reliability coefficients of the tests.

2. Are the rs between the Piagetian and the groupement tests statistically significant and positive? All should be since Brainerd (1972b) constructed the groupement tests to measure the structures that Piaget indicated underly and explain the concrete operations.

As shown in Table 6-3, the percentages of statistically significant and positive rs between the Piagetian and the groupement tests based on the annual measurements ranged from 9 to 11 at Grades 1 through 5 and sharply increased to 30 percent at Grade 6. For the longitudinal measurements at one- and two-year intervals, the percentages of significant positive rs ranged from 6 to 15. Negative rs were found for Grade 1 and for the Grade 1-2 and Grade 1-3 intervals.

From the lack of more positive rs between the Piagetian and groupement measures we may infer that development of the various cognitive dimensions of the two domains proceeds quite independently, not in a highly correlated synchronous manner as is implied by the Piagetian construct of stages.

3. Are there more significant positive rs between the Piagetian and groupement tests than there are between the CLD tests and either the Piagetian or the groupement tests? Even though a very low proportion of significant positive rs was found between the Piagetian and groupement measures, that proportion should

be higher than between the CLD and either of these two domains inasmuch as the groupement tests were constructed to measure the structures underlying concrete operations functioning. The CLD tests were not constructed to measure either the concrete operations or the groupement structures.

As shown in Table 6-3, there were slightly more significant positive rs between the Piagetian and groupement tests than between the CLD and Piagetian tests at Grades 3, 4, and 5, but slightly more between the CLD and Piagetian tests at Grades 1 and 2. The largest difference in the percentages of positive significant rs was in Grade 6 where 30 percent of the Piagetian and groupement tests and 15 percent of the Piagetian and CLD tests were significant and positive, respectively.

For the longitudinal 1- and 2-year interval measurements, only small differences in the percentages between the domains were found. From 6 to 15 percent of the rs between the Piagetian and the groupement measures were significant and positive whereas from 9 to 14 percent of the CLD and Piagetian were. The higher percentages involving the CLD and the Piagetian items occurred at Grades 1-2: 10 percent versus 6 percent; and at Grades 5-6: 14 percent versus 13 percent.

There were also more significant positive rs and fewer negative rs between the piagetian and the groupement tests than between the CLD and the groupement tests in Grades 1, 2, 3, 4, and 6 but not in Grade 5. Similar comparisons were not made at Grades 7 through 9 because only one Piagetian test of concrete operations was administered in these grades. More Piagetian and groupement longitudinal measures correlated significantly and positively at the Grade 1-2, 2-3, and 4-5 intervals, whereas the opposite tendency was found at the Grade 5-6 and 4-6 intervals where the CLD and groupement measures tended to have more positive and significant rs.

In summary of the preceding, the substantially greater proportion of significant positive rs expected between the Piagetian and groupement measures than between the CLD and either the Piagetian or the groupement measures was not found.

4. Are there more statistically significant positive rs between the Piagetian tests measuring classification and the CLD tests measuring the four levels of concept attainment than between the same Piagetian tests and the CLD tests that measure understanding principles, understanding taxonomic relations, and solving problems that involve the particular concepts? (These are also referred

to as "uses" of the concepts.) More are expected between the Piagetian classification tests and the CLD concept levels tests. The Piagetian tests involving classification are class inclusion and dichotomous sorting; the CLD tests involving classification are the concrete, identity, classificatory, and formal levels of each of the four concepts.

Table 6-4 gives the percentages of significant positive and negative rs for the annual data for Grades 1-6 and also for the 1- and 2-year longitudinal intervals. From 17 to 22 percent of the rs in Grades 1, 2, and 3 between the Piagetian classification tests of class inclusion and dichotomous sorting and the CLD tests of the levels of concept attainment were statistically significant while none of the rs between the Piagetian tests and the CLD uses tests were. However, at Grades 4, 5, and 6, the percentages significant between the Piagetian tests and CLD concept attainment tests were about equal to those between the Piagetian tests and the CLD uses tests.

The pattern of significant rs for the longitudinal interval measurements was similar to that for the separate annual measurements in that there were more significant rs between the CLD concept attainment tests and the Piagetian classification tests during the primary school years and more between the CLD uses tests and Piagetian classification tests during the intermediate school years. In general,

Table 6-4. Percentages of rs Statistically Significant ($p < .05$) Between CLD Levels Tests and Two Piagetian Classification Tests and Between CLD Uses Tests and Two Piagetian Classification Tests

Grades of Annual Measurement	*CLD Levels with CI and DS*		*CLD Uses with CI and DS*		*Grades of Longitudinal Measurement*	*CLD Levels with CI and DS*		*CLD Uses with CI and DS*	
	(+)	(-)	(+)	(-)		(+)	(-)	(+)	(-)
1	22	0	0	0	1-2	11	0	0	0
2	17	0	0	0	2-3	17	0	6	0
3	22	0	0	0	1-3	18	0	19	0
4	8	0	6	0	4-5	8	0	19	0
5	17	0	22	0	5-6	25	0	28	0
6	42	0	33	0	4-6	33	0	17	3

however, the proportion of significant rs was quite low for both sets of relationships. The percentage of significant positive rs between the Piagetian classification tests and the CLD concept levels tests was much lower than expected.

5. Are more of the rs between the class groupement tests and the Piagetian classification tests of class inclusion and dichotomous sorting significant and positive than are the rs between the class groupement tests and the Piagetian relations tests of transitivity of length, transitivity of weight, and seriation? This is expected inasmuch as the class groupement tests and the Piagetian tests of class inclusion and dichotomous sorting were constructed to measure highly related dimensions of cognitive functioning.

The percentages of statistically significant rs as shown in Table 6-5 were equal or greater for the Piagetian classification tests and the class groupement tests, based on both the annual data and the longitudinal intervals, than for the Piagetian relations tests and the class groupement tests. The differences were not great during the primary school years while at the intermediate years they were large for both the annual and longitudinal data. This finding particularly at Grades 4-6 supports Piagetian theory regarding the structure associated with the concrete operations involving classification.

Table 6-5. Percentages of rs Statistically Significant ($p < .05$) Between Two Piagetian Classification Tests and the Class Groupement Tests and Between the Piagetian Relations Tests and the Class Groupement Tests

Grades of Annual Measurement	*CI and DS with Class Groupements*		*(Ser), TL, and TW with Class Groupements*		*Grades of Longitudinal Measurement*	*CI and DS with Class Groupements*		*(Ser), TL, and TW with Class Groupements*	
	(+)	*(-)*	*(+)*	*(-)*		*(+)*	*(-)*	*(+)*	*(-)*
1	8	0	0	0	1-2	8	0	0	3
2	17	0	0	0	2-3	8	0	11	0
3	0	0	0	0	1-3	21	0	8	0
4	42	0	8	0	4-5	33	0	17	0
5	42	0	0	0	5-6	33	0	4	0
6	67	0	17	0	4-6	29	0	13	0

6. Are more of the rs between the relations groupement tests and the Piagetian relations tests of seriation, transitivity of length, and transitivity of weight significant and positive than are the rs between the relations groupement tests and the Piagetian tests of classification?

Data based on the annual and longitudinal measurements are shown in Table 6-6. At only Grade 1 and Grade 6 was there a greater percentage of statistically significant rs between the Piagetian relations tests and the relations groupement tests than between the relations groupement tests and the Piagetian classification tests. In Grades 2, 3, 4, and 5, the opposite was true. The differences in the percentages in the opposite direction were quite large, for example, 27-0 at Grade 1, 50-25 at Grade 6, 0-30 at Grade 3, and 0-33 at Grade 5.

Results based on the longitudinal data indicate that there was an equal or smaller percentage of significant rs between the Piagetian relations tests and the relations groupement tests than between the Piagetian classification tests and the relations groupement tests for every measurement interval except Grades 1-3. The differences were substantial for the intervals at Grade 4-5 and Grade 5-6.

Although there were some inconsistencies, the general pattern is

Table 6-6. Percentages of rs Statistically Significant ($p < .05$) Between Two Piagetian Classification Tests and the Relations Groupement Tests and Between the Piagetian Relations Tests and the Relations Groupement Tests

Grades of Annual Measurement	*CI and DS with Relations Groupements*		*(Ser), TL, and TW with Relations Groupements*		*Grades of Longitudinal Measurement*	*CI and DS with Relations Groupements*		*(Ser), TL, and TW with Relations Groupements*	
	(+)	*(-)*	*(+)*	*(-)*		*(+)*	*(-)*	*(+)*	*(-)*
1	0	0	27	0					
					1-2	10	0	10	0
2	10	0	7	0					
					2-3	15	0	10	0
3	30	0	0	0					
					1-3	15	0	17	0
4	17	0	8	0					
					4-5	25	0	13	0
5	33	0	0	0					
					5-6	29	0	8	0
6	25	0	50	0					
					4-6	17	0	17	0

of more significant rs in the direction opposite to the direction predicted. Neither the greater proportion of higher rs for the classification tests with the relations groupement tests nor the substantial changes in the percentages of significant rs from one grade to the next and from one longitudinal interval to the next are consistent with predictions based on Piagetian theory.

7. Are all the rs among the 12 Piagetian tests at one- and two-year intervals statistically significant and positive? This is expected since, according to Piagetian theory, the stage of concrete operations (as other Piagetian stages) is characterized by synchronous development of the various dimensions of the stage. All the dimensions should emerge at about the same time and all should increase at about the same rate and reach full functioning over a relatively short period of years, that is, roughly age 7 to 11.

The percentages of significant rs for the Piagetian tests based on the measurements at one- and two-year intervals are given in Table 6-7. The percentages of rs statistically significant and positive among the Piagetian concrete operations tests for each of the two longitudinal groups of students were quite stable but relatively low, ranging from 15 percent for the Grade 5-6 interval to 24 percent for the Grade 1-2 interval. There were no significant negative rs.

The preceding percentages of significant rs include those between the same tests administered at the 1- and 2-year intervals. The number of these significant rs and the percentage of the significant rs that these tests contributed to the total percent of significant rs at each measurement interval are also given in Table 6-7. The rs based on the same tests contributed from 3 to 9 percent to the total of 15 to 24 percent that were significant. Thus, the percentages significant between the Piagetian tests ranged from 6 for the Grade 5-6 interval to 21 for the Grade 1-2 interval. This low percentage of significant rs is interpreted as not supportive of synchronous development of the items of the concrete operations stage.

8. Are all the rs among the 16 groupement tests at the one- and two-year intervals statistically significant and positive? This is expected because these tests were constructed to measure the structures that, according to Piagetian theory, explain performances during the stage of concrete operations.

The percentages of significant rs for the groupement tests at the longitudinal measurement intervals are given in Table 6-7. The total

Table 6-7. Percentages of rs Statistically Significant ($p < .05$) Within Piagetian and Groupement Domains Based on Longitudinal Measurements

Grades of Longitudinal Measurement	*Within Piagetian*		*N Same Piagetian Test*	*% of Total*	*Within Groupement*		*N Same Groupement Test*	*% of Total*
	(+)	*(-)*			*(+)*	*(-)*		
			(*N* = 12)				(*N* = 11)	
A1-2	24	0	5	3	13	0	1	1
A2-3	22	0	5	3	12	0	4	3
A1-3	19	0	5	3	6	0	1	1
			(*N* = 9)				(*N* = 12)	
B4-5	20	0	5	8	19	0	5	3
B5-6	15	0	6	9	33	0	10	7
B4-6	20	0	4	6	24	0	7	5
							(*N* = 10)	
C7-8					62	0	8	8
C8-9					79	0	10	10
C7-9					54	0	7	7

percentages of rs that were statistically significant ranged from 6 to 13 for the Longitudinal Block A intervals, 19 to 33 for the Longitudinal Block B intervals, and 54 to 79 for the Longitudinal Block C intervals. There were no significant negative rs for any of these longitudinal measurements.

The number of rs for the same test that were significant and the percentages that they contributed to the total percentage of significant rs were low for Longitudinal Block A; but the number and corresponding percentages increased for the successive longitudinal blocks. Despite this, the percentage of significant rs *between* the groupement tests was relatively high for Longitudinal Blocks B and C, ranging from 16 to 26 percent for Block B and 47 to 69 percent for Block C.

We may summarize the correlational data for the developmental items of both the Piagetian and the groupement domains. There was more relatedness of the Piagetian items for the Grade 1-3 intervals than for the Grade 4-6 intervals; however, the highest percentage of significant rs for any interval was only 21 for the Grade 1-2 interval. The percentage of significant rs between the groupement items was at or below 16 though the Grade 4-5 interval. At the Grade 5-6 interval the percentage was 26 and for the remaining one-year intervals it was 54 and 69. Thus, the pattern of significant *r*s was very different for the Piagetian and the groupement development items.

9. Are more of the rs at one- and two-year intervals within each of the eight groupement structures statistically significant than are the

rs between the different structures? For example, will the r between a composition and its inverse, such as the addition of primary classes and the subtraction of primary classes, be higher than the r between the composition of two different groupements such as the addition of primary classes and the addition of secondary classes? According to Piagetian theory regarding the groupement structures, any composition and its inverse should become functional simultaneously and develop in synchrony.

Three tests of the class groupements and three tests of the relations groupements were identified to examine the relationships, both within the same structure and between structures. The first three included the two groupement III tests, that is, biunivocal multiplication of classes (BMC), biunivocal division of classes (BDC), and counivocal multiplication of classes (CMC). As shown in Table 6-8, an equal number of significant rs, seven, was found between the composition test (BMC) and its inverse (BDC) and between BMC and the composition tests of the different structure (CMC). For the one-

Table 6-8. Correlations Between Tests of the Same Groupement Structure and Between Tests of Different Groupement Structures

Grades of Longitudinal Measurement	*Within Structure*	*Between Structures*	*Within Structure*	*Between Structures*
	BMC with BDC	*BMC with CMC*	*BMR with BRR*	*BRR with CRR*
1-2	.35^a	-.03	.22	.38^a
2-1	.10	-.09	.21	-.02
2-3	-.06	.21	-.01	.33^a
3-2	-.07	.25	-.01	.02
1-3	.19	.15	.16	.11
3-1	.31	.18	.09	.32
4-5	.09	.18	.04	.35^a
5-4	.28	.32^a	-.04	-.11
5-6	.28	.28	-.05	.16
6-5	.32^a	.33^a	.32^a	.15
4-6	-.26	.19	.46^a	.32^a
6-4	-.03	.25	.29^a	.15
7-8	.39^a	.36^a	.32^a	.83^a
8-7	.37^a	.36^a	.43^a	.61^a
8-9	.32^a	.42^a	.27	.67^a
9-8	.36^a	.29^a	.73^a	.38^a
7-9	.28	.19	.47^a	.19
9-7	.42^a	.31^a	.24	.28

$^a p < .05$.

year intervals starting in Grade 5-6, and continuing through Grade 8-9, the rs were generally significant for both comparisons and also nearly identical in value.

The three relations groupement tests selected were biunivocal multiplication of relations, biunivocal reciprocity of relations (both groupement VII), and counivocal reciprocity of relations from groupement VIII. Only 7 of the 18 comparisons showed higher rs within each structure than between the structures. Seven statistically significant rs were found for the within structures and eight for the between structures. The rs are very high for the between relations groupements in Grades 7-8 and 8-9, ranging from .38 to .83.

We may infer from this pattern of significant rs that there is no closer relationship between the emergence and full functioning of the within structures than of the between structures. Furthermore, there is no consistent relationship between either the within or the between structures across consecutive 12-month intervals until Grade 5-6. The relationship is reasonably consistent starting in Grade 7-8 and continuing thereafter for the 12-month intervals, but it is also equally as consistent and high for the between as the within structures.

COMPARISONS BASED ON 12 COMBINED TEST SCORES

The number of students participating in this comparative study in relation to the number of tests administered to them was too low to perform a factor analysis using all the separate test scores. Therefore, each student's scores from certain tests of each of the three domains were combined to get four CLD scores, five Piagetian scores, and three groupement scores.

The basis for combining the students' separate test scores was as follows. As shown in Chapter 4, the seven separate tests in each of the four CLD batteries starting in the intermediate grades tended to load on a single factor for each concept: *cutting tool*, *tree*, *equilateral triangle*, and *noun*. Based upon this consideration, a combined score for each of the four CLD batteries was obtained for each student that included the seven separate test scores of each battery.

Based on logical and theoretical considerations, each student's separate scores from various Piagetian tests were combined to yield four scores as follows: conservation, classification, relations, and number. The conservation score included identity conservation of length, equivalence conservation of length, identity conservation of weight, and equivalence conservation of weight. The separate scores from dichotomous sorting and class inclusion were combined for a classification score. A relations score included the combined scores of the tests of seriation, transitivity of length, and transitivity of weight. Number included the combined scores from the tests of conservation of number and cardinality. The score for combinatorial reasoning, the only formal operations Piagetian test, was kept as a single score. Based on logical considerations and also on results of factor analyses (using principal components method with orthogonal rotation), each student's separate test scores of the groupement battery were combined to yield three combined scores as follows: one based on the four tests of composition of classes, a second based on the four tests of inversion of classes, and a third based on the four tests of composition of relations and the four tests of inversion of relations.

Technical Characteristics of Combined Tests

The Hoyt reliability coefficient, standard error of measurement, mean, and standard deviation for each of the 12 combined tests for Grades 1-9 are given in Table A6-3 in the appendix.

The range of the Hoyt reliability coefficients for the combined tests of the three domains for each group of students were as follows:

	CLD	*Piagetian*	*Groupement*
Grades 1-2-3	.52-.87	.72-.95	.72-.85
Grades 4-5-6	.72-.93	.05-.93	.68-.91
Grades 7-8-9	.76-.93		.64-.94

The low reliability coefficient for one of the Piagetian tests is related to the fact that that test had a very high mean score and low standard deviation in Grade 5. Only two Piagetian tests were administered to the Grade 7-8-9 students.

Comparisons Based on Means for Combined Tests

The first grade at which the mean was equivalent to 85 percent correct or higher for each of the 12 tests was identified. The name of the test, the grade, the percent correct at that grade, and the rank from 1 to 12 follow on page 198.

The first test to be mastered was the groupement test, composition and inversion of relations, in Grade 1. Inversion of classes was first mastered in Grade 2 while composition of classes was not yet mastered in Grade 9. The five Piagetian tests were mastered from Grade 2 through Grade 5 with number being first and the formal

Name of Test	*Grade*	*%*	*Rank*
Composition and Inversion of Relations (Groupement)	1	85	1
Number (Piagetian)	2	89	2
Inversion of Classes (Groupement)	2	85	3
Relations (Piagetian)	3	89	4
Conservation (Piagetian)	3	88	5
Classification (Piagetian)	3	88	6
Combinatorial Reasoning (Piagetian)	5	86	7
Cutting Tool (CLD)	6	88	8
Equilateral Triangle (CLD)	9	87	9
Tree (CLD)	9	86	10
Composition of Classes (Groupement)	9	77	11
Noun (CLD)	9	75	12

operations test, combinatorial reasoning, last, at Grade 5. *Cutting tool* was mastered in Grade 6, *equilateral triangle* and *tree* in Grade 9, and the highest score on *noun* was in Grade 9.

It may be noted in passing that contrary to Piagetian theory as interpreted by Brainerd, composition of classes did not precede inversion of classes. Instead, inversion of classes was mastered in Grade 2 while composition of classes had not yet been mastered in Grade 9. In addition, contrary to Piagetian theory mastery of classes did not precede mastery of relations. Composition and inversion of relations was mastered in Grade 1 whereas inversion of classes and composition of classes, as noted, were mastered for the first time in Grade 2 and Grade 9, respectively.

Correlations Based on Combined Scores

Table 6-9 gives the percent of statistically significant rs for the tests within each of the three domains of measurement and also between the tests of the different domains. TR Table 6-2 (Sipple, Allen & Klausmeier, 1979) gives the correlation matrices on which Table 6-9 is based. The pattern of significant rs *within domains* is much the same for the CLD and groupement tests. The percentage of rs significant for these two domains range from 19 to 31 for the Grade 1-2 and 2-3 intervals. From the Grade 4-5 interval through 8-9 interval, the percentage of significant rs involving tests within each the CLD and groupement domains ranges from 67 to 100. The percentage of significant rs within the Piagetian domain was highest between Grades 2 and 3 (48) and lowest between Grades 5 and 6 (20).

The percentage of significant rs between the tests of any two domains is similar for the Grade 1-2 and 2-3 intervals, ranging from 17 to 27. From the Grade 4-5 and 5-6 intervals, the percentage of

Table 6-9. Percentages of Statistically Significant rs Within and Between Three Domains at 12- and 24-Month Intervals, Combined Test Scores

Grade Block and Grades	*Within*			*Between*			
	CLD	*Piagetian*	*Groupements*	*CLD and Piagetian*	*CLD and Groupements*	*Piagetian and Groupements*	*CLD and Piagetian and Groupements*
A1-2	31	32	22	23	21	27	23
A2-3	19	48	22	18	17	23	19
B4-5	88	44	67	18	33	37	28
B5-6	88	20	78	28	38	20	28
C7-8	100	—	89	88	96	67	89
C8-9	100	—	100	50	92	67	79
Total	71	36	63	26	49	30	35
A1-3	13	40	33	20	8	20	17
B4-6	63	28	66	25	50	40	36
C7-9	100	—	100	100	92	67	94

significant rs is higher between the CLD and the groupement tests than between the CLD and Piagetian tests. The percentage of significant rs between the tests of the three domains for the Grade 4-5 and Grade 5-6 intervals ranges from 18 to 38.

The percentage of significant rs between the CLD and groupement tests was very high, 96 and 92 for the Grade 7-8 and 8-9 intervals. Two-thirds of the rs between combinatorial reasoning and the CLD and groupement tests were significant. (Only the combinatorial reasoning test score is included in this analysis for Grade 7-8-9.)

The total percentage of significant rs involving the tests within each domain for the 12-month measurement intervals was highest for the CLD domain (71), next highest for the groupement (63), and lowest for the Piagetian (36). The percentage of rs statistically significant between the various domains was highest between CLD and groupements (49), then between Piagetian and groupements (30), and lowest, CLD and Piagetian (26).

As would be expected, the percentages of rs statistically significant for the 24-month interval were generally lower than for the 12-month intervals, although there were some exceptions. The percentage of significant rs for the tests within the groupement domain was higher for the Grade 1-3 interval than for either the Grade 1-2 or Grade 2-3 interval. Similarly, the percentage for the Grade 4-6 interval between the Piagetian and the groupement tests was higher than for the Grade 4-5 and Grade 5-6 intervals. For the Grade 7-9 interval, all rs for the tests within the CLD and groupement domains and all the rs between the CLD and Piagetian tests were statistically significant.

Factors Identified by Interbattery Analysis

In Chapter 4 the results of interbattery analyses were reported for CLD measures alone, based on the four longitudinal groups, Grades 1-12. The same interbattery analysis procedure was used in deriving factors using the combined scores. The measures taken at each of the one-year intervals and the two-year interval on the same students of each of the longitudinal blocks were treated as separate batteries. Thus the CLD, Piagetian, and groupement tests administered in year 1 to a longitudinal group of students were treated as one battery and the same tests administered the second year were treated as another battery. If the covariance accounted for by the first unrotated factor exceeded 90 percent, no rotation was done. When the covariance accounted for was less than 90 percent, additional factors were identified by rotation and the factors that were identified accounted for 100 percent of the covariance. The factors identified for Longitudinal Blocks A and B are presented in Table 6-10.

Table 6-10. Factors Identified Across Times of Measurement for Longitudinal Block A—Grades 1-3 and B—Grades 4-6, Combined Tests[a]

Test	*Grades*	*Factor*			*Grades*	*Factor*				*Grades*	*Factor*			*Grades*	*Factor*		*Grades*	*Factor*		*Grades*	*Factor*	
		1	*2*	*3*		*1*	*2*	*3*	*4*		*1*	*2*	*3*		*1*	*2*		*1*	*2*		*1*	*2*
CT	1	03	49	-18	2	(16)	52	27	-34	1	(09)	(08)	34	4	-04	59	5	79	41	4	32	(11)
TR	1	03	(-04)	61	2	(16)	63	13	(06)	1	(-27)	29	37	4	27	70	5	72	(05)	4	77	(27)
ET	1	-03	62	(29)	2	(27)	25	(13)	(22)	1	13	(-15)	60	4	08	59	5	63	37	4	55	(09)
NN	1	(25)	58	(29)	2	(10)	(03)	48	-25	1	35	(24)	49	4	24	50	5	51	(-03)	4	58	(-25)
Conservation	1	44	35	(03)	2	82	(-12)	(-03)	(01)	1	70	(02)	26	4	(25)	37	5	40	(-04)	4	34	43
Classes	1	40	53	(-06)	2	53	30	(22)	15	1	(22)	47	26	4	59	31	5	27	69	4	55	37
Relations	1	52	(15)	(-01)	2	(06)	58	(-14)	(12)	1	41	49	(-15)	4	66	-36	5	(14)	03	4	(-12)	62
Number	1	32	(27)	53	2	42	03	36	(-05)	1	(18)	44	47	4	27	30	5	(-28)	60	4	41	04
Combinatorial Reasoning	1	53	(01)	-11	2	11	25	(29)	53	1	50	07	(-02)	4	35	(-09)	5	(16)	13	4	(-20)	28
Composition of Classes	1	42	-31	-41	2	19	(15)	14	57	1	17	07	-39	4	56	47	5	54	47	4	56	35
Inversion of Classes	1	57	(07)	(19)	2	(-.03)	(-06)	59	37	1	(-16)	61	(12)	4	66	21	5	43	29	4	31	56
Composition and Inversion of Relations	1	71	08	36	2	55	33	-35	(29)	1	64	43	(09)	4	54	34	5	(09)	47	4	38	43
CT	2	32	20	57	3	(-01)	-17	45	15	3	(-26)	(11)	-05	5	67	44	6	51	27	6	54	(21)
TR	2	57	(22)	31	3	(01)	05	73	23	3	(-05)	38	08	5	71	09	6	57	(15)	6	59	(07)
ET	2	34	37	(24)	3	(-20)	60	(07)	(08)	3	57	(-27)	08	5	75	12	6	42	23	6	58	(-15)
NN	2	(-17)	54	(05)	3	(-03)	(14)	-04	76	3	-29	(22)	75	5	54	12	6	72	(29)	6	78	(16)
Conservation	2	20	81	(21)	3	71	(-04)	(-13)	(24)	3	35	(05)	54	5	(18)	53	6	72	(-18)	6	53	18
Classes	2	43	39	(12)	3	40	16	(10)	51	3	(28)	29	61	5	12	69	6	63	27	6	49	30
Relations	2	37	(-03)	(02)	3	(24)	38	(02)	(-07)	3	55	31	(-16)	5	13	49	6	(06)	56	6	(06)	61
Number	2	-16	(15)	57	3	11	45	-16	(15)	3	(29)	-07	18	5	-34	33	6	(18)	35	6	13	32
Combinatorial Reasoning	2	54	(-09)	41	3	67	43	06	11	3	50	53	(27)	5	34	(12)	6	(07)	43	6	(13)	45
Composition of Classes	2	45	14	-38	3	61	(26)	34	-10	3	41	49	18	5	25	69	6	60	26	6	59	39
Inversion of Classes	2	40	(-26)	(-12)	3	(14)	(30)	45	-02	3	(03)	61	(-04)	5	05	63	6	19	38	6	21	29
Composition and Inversion of Relations	2	68	30	-11	3	10	50	01	(28)	3	41	16	(-11)	5	01	28	6	(13)	72	6	09	64
Factor Covariance		8.00	5.67	3.62		5.92	5.91	3.86	3.87		6.24	5.75	4.52		7.50	7.37		7.14	6.96		7.32	6.94
Percent of Total Covariance		46	33	21		30	30	20	20		38	35	27		50	50		51	49		51	49

[a] Decimals have been omitted.

Note: Parentheses are used to indicate a test loading of less than .30 for both times of measurement.

Three factors were identified for the Grade 1-2 interval. The first factor is a Piagetian-groupement factor. Three of the five Piagetian tests—classes, relations, and combinatorial reasoning—and all three groupement tests loaded .30 or higher on this factor in both Grades 1 and 2. Three CLD tests—*cutting tool*, *tree*, and *equilateral triangle*—also loaded .30 or higher on this factor in Grade 2 but not in Grade 1. Factor 2 is a CLD-Piagetian factor with loadings from the CLD tests of *equilateral triangle* and *noun* and the Piagetian tests of conservation and classes. The loadings of the tests of conservation and classes shifted markedly between Grades 1 and 2. The CLD *cutting tool* test loaded above .30 in Grade 1 but not in Grade 2. The third factor has one test from each of the three domains with loadings of .30 or higher in both Grades 1 and 2: *tree*, composition of classes, and number.

For Grades 2-3, four factors were identified, not any one of which approximates any one of the three factors found for Grades 1-2. On all four factors only five tests had loadings of .30 or higher in both Grades 1 and 2: conservation and classes on factor 1, composition and inversion of relations and relations on factor 2, and inversion of classes on factor 3. These factors and the loadings of .30 or higher are not relatable to either CLD or Piagetian theory. Furthermore, the factor structure for the Grade 2-3 interval is very different from that for Grade 1-2, and also unrelatable to it.

Three factors were identified for the Grade 1-3 interval. Four tests with loadings of .30 or higher in both Grades 1 and 3 on the first factor were: composition and inversion of relations, conservation, relations, and combinatorial reasoning. Two tests, inversion of classes and relations, had loadings of .30 or higher for both Grades 1 and 3 on factor 2. The CLD test, *noun*, loaded .30 or higher on the third factor in both Grades 1 and 3. On this factor the other three CLD tests loaded .30 or higher only in Grade 1, whereas the Piagetian tests of conservation and classes loaded .30 or higher in Grade 3. These three factors identified for the Grade 1-3 interval show no similarity with those identified for either the Grade 1-2 or the Grade 2-3 intervals.

Two factors were identified for the Grade 4-5 interval. The first factor had consistently high loadings on the three groupement tests and three of the five Piagetian tests in Grade 4 but not in Grade 5 and on the four CLD measures only in Grade 5. Only the test of combinatorial reasoning had loadings of .30 or higher in both years on this factor. The second factor had high loadings on all four CLD tests in Grade 4 but not in Grade 5 and less high loadings on six of the Piagetian and groupement measures, not including combinatorial reasoning, in both Grades 4 and 5.

The first factor identified for the Grade 5-6 interval included the four CLD tests, composition of classes, and conservation, a total of six tests, with loadings of .30 or higher for both grades. On the second factor, two tests, number and composition and inversion of relations, loaded .30 or higher in each Grade 5 and 6.

Two factors were identified for the Grade 4-6 interval. The four CLD tests, the two Piagetian tests of conservation and classes, and the groupement test of composition of classes had loadings of .30 or higher on the first factor at each Grade 4 and Grade 6. Two Piagetian tests—classes and relations—and two groupement tests—composition of classes and composition and inversion of relations—had loadings of .30 or higher on the second factor in both Grades 4 and 6.

Three summary observations may be made regarding the results of the preceding factor analyses. First, the set of factors identified for the successive Grade 1-2, 2-3, 4-5, and 5-6 intervals differed greatly and inconsistently. Second, there was a greater tendency for the Piagetian and groupement tests than for the CLD tests to load on the same factor for the Grade 1-2, 2-3, and 4-5 intervals. Not until the Grade 5-6 and 4-6 intervals did all four CLD tests load consistently high on the same factor at both grades. Third, from the Grade 1-2 to the 5-6 interval there was a marked shift in the factors that were identified. At the Grade 1-2 interval one factor that accounted for 46 percent of the total covariance was identified. It was a Piagetian-groupement factor; three CLD tests had loadings of .30 or higher on it but only in Grade 2. At the Grade 5-6 interval, one CLD factor was identified on which all four CLD tests loaded .30 or higher in each Grade 5 and Grade 6; one Piagetian and one groupement test also had loadings of .30 or higher on it in each Grade 5 and 6. This factor accounted for 51 percent of the total factor covariance. This shift in the factor pattern might have been predicted inasmuch as the Piagetian and groupement tests were constructed to measure performances associated with the Piagetian stage of concrete operations corresponding to ages 7-11, whereas each combined CLD test measured performances associated not only with the attainment of the four consecutive levels of the same concept but also with understanding principles, understanding taxonomic relations, and problem solving. It will be recalled that mastery of the four combined CLD tests occurred in Grade 6 or in later grades, whereas mastery of six of the eight Piagetian and groupement tests had already occurred in Grades 1, 2, or 3.

The factors identified for Longitudinal Block 7-8-9 are presented in Table 6-11. One factor was identified at each interval, Grade 7-8, 8-9, and 7-9. The four CLD tests, the three groupement tests, and

Table 6-11. Factors Identified Across Times of Measurement for Longitudinal Block C—Grades 7-9, Combined Tests

Test	*Grades*	*Factor 1*	*Grades*	*Factor 1*	*Grades*	*Factor 1*
CT	7	75	8	65	7	65
TR	7	71	8	75	7	65
ET	7	78	8	78	7	73
NN	7	84	8	82	7	70
Composition of Classes	7	80	8	78	7	70
Inversion of Classes	7	57	8	57	7	58
Composition and Inversion of Relations	7	67	8	62	7	60
Combinatorial Reasoning	7	58	8	40	7	57
CT	8	64	9	71	9	56
TR	8	87	9	67	9	66
ET	8	78	9	69	9	58
NN	8	85	9	72	9	68
Composition of Classes	8	80	9	78	9	80
Inversion of Classes	8	60	9	77	9	68
Composition and Inversion of Relations	8	58	9	63	9	69
Combinatorial Reasoning	8	55	9	47	9	52
Eigenvalue		17.01		14.07		11.47
Percent of sum of eigenvalues (i.e., percent of covariance)		96		96		95

combinatorial reasoning had loadings of .40 or higher on the factor identified for each of the three measurement intervals. With this consistency in the factors for the three intervals, comparisons may be made of the values of the test loadings. The loadings on only two tests of the Grade 7-8 factor changed .10 or more from Grade 7 to Grade 8: *cutting tool*, .75 to .64, and *tree*, .71 to .87. For the Grade 8-9 factor, shifts of this magnitude occurred on two test loadings from Grade 8 to 9: *noun*, .82 to .72, and inversion of classes, .57 to .77. Three test loadings were involved for the Grade 7-9 factor: *equilateral triangle*, .73 to .58, composition of classes, .70 to .80, and inversion of classes, .58 to .68.

A few conclusions may be drawn regarding the factor structure across the years from Grades 1-2 through Grades 8-9. First, even when many separate tests are combined, the factor structure changes greatly and unrelatably from Grades 1-2 to Grades 2-3; however, from Grades 4-5 to Grades 5-6, some stability of the structure occurs. High stability in the test loadings occurs in Grade 7-8, Grades 8-9, and Grades 7-9; and essentially the same factor is identified for the three intervals.

The majority of the tests of the Piagetian stage of concrete operations and the groupement tests loaded .30 or higher on the same factor at both grades for the Grade 1-2 and 4-5 intervals but not for the Grade 2-3 and 5-6 intervals. The CLD measures loaded on one factor at both grades for the Grade 5-6 interval but not earlier. At no one-year interval up to and including Grade 5-6 did the Piagetian tests or the groupement tests load on the same factor as did the four CLD tests in Grades 5-6.

The four CLD tests, combinatorial reasoning, the Piagetian test related to the stage of formal operations, and the three groupement tests consistently loaded on the one factor identified for the Grade 7-8, 8-9, and 7-9 intervals. The combinatorial reasoning test did not have higher loadings on any of these factors than it did on any of the factors identified during the elementary school years. It might have been expected to since it purportedly is a measure of formal operations stage.

The preceding results corroborate the findings presented earlier in this chapter based on separate tests. One conclusion is apparent whether based on the CLD, Piagetian, or groupement measures separately or in combination. Cognitive development through age 9 is characterized by instability in the performances of the same students from one year to the next and by lack of integration of the mental operations underlying the developmental items. Based on the CLD and groupement test batteries but not the Piagetian, we also conclude that during the junior high school years there is much stability and integration and that both stability and integration emerge during the intermediate school years, corresponding to the ages of 10-12. We observed earlier in Chapter 4 that differentiation related to the various subject matters taught in school also occurs during the intermediate school years.

Finally, the results based on the several analyses of the Piagetian measures of the concrete operations stage do not support the concrete operations stage as descriptive of children's actual cognitive development during ages 7 to 11. Furthermore, the results based on both the Piagetian measures and the groupement measures do not support the stage of concrete operational thought as an empirically verifiable theoretical construct.

Chapter 7

The CLD Perspective in Retrospect

Herbert J. Klausmeier

Principles of cognitive development from age 6 to 18 are presented in this chapter as a summary and synthesis of the comparative study reported in this book and also of the more comprehensive longitudinal study that extended from Grade 1 through Grade 12, the results of which were summarized in Chapters 3 and 4. Some of the principles would have been essentially the same whether based on the CLD, familiar Piagetian, or groupement items used in the comparative study. However, the CLD and Piagetian constructs that are employed to account for the developmental progressions are quite different.

In Piagetian theory, the three key explanatory constructs are stage, structure, and equilibration. Stage is also used by Piaget as a descriptor of real-life developmental progression; and teachers and parents, as well as scholars, have been encouraged to believe that maturing individuals develop through the same four stages as described in the voluminous textbook and popular Piagetian literature. The corollary constructs in CLD information-processing theory are level of concept attainment rather than stage, mental operations and conceptual cores rather than structures, and executive control rather than equilibration. Level of concept attainment is used to indicate the progression for attaining the same concepts to increasingly mature levels. The CLD tests used in the present studies were constructed to yield accurate measurements of the progression of each level. The empirical evidence is supportive of the CLD constructs; therefore they will be employed in accounting for the developmental progressions. Further evaluation of the Piagetian constructs based on

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the empirical results also will be indicated. Brainerd and Murray present their views regarding various aspects of Piagetian theory in the next two chapters.

Before proceeding to the principles, I should like to indicate some of the antecedents of the present comparative study. Earlier, two intriguing but contrasting descriptions of developmental progression were prominent, namely, Olson's organismic age and Gesell's growth gradients. Research and theory regarding organismic age were summarized by Olson (1949). Growth gradients were presented by Gesell and Ilg (1949) for the years birth to age 10 and by Gesell, Ilg, and Ames (1956) for the years 10 to 16.

Olson's main proposition was that since individuals develop as total organisms, all areas of growth are closely related. The construct formulated to represent this kind of progression was organismic age. Olson derived the organismic age of the individual child when of any particular chronological age by computing the average of the following seven ages in months—height, weight, grip, dental, carpal, mental, and reading. Organismic age obtained in this manner presumably was a better indicator of the child's growth than were the seven areas considered independently. Furthermore, organismic age purportedly could be used in reliably predicting achievement in language arts, mathematics, science, and other fields.

Gesell, very different from Olson, identified the *growth gradients* for many different items of development related to ten separate maturational areas. Six of the 10 areas were motor characteristics, emotional expression, self and sex, interpersonal relations, school life, and ethical sense. The area of school life alone included many gradients in language arts, such as writing and reading; mathematics and science; and personal-social participation.

The growth gradients that Gesell and his colleagues identified and described in great detail were generally accepted by scholars, practitioners, and parents as reliable descriptions of how the boys and girls of middle and upper socioeconomic status at that time were developing. The available evidence was much less compelling for organismic age, although the idea of dealing with the individual as a totally developing organism had as much intuitive appeal then as it does today.

Two graduate students and I carried out an empirical test of organismic age (Klausmeier, 1958; Klausmeier, Beeman, & Lehman, 1958). Two groups of boys and girls were identified, one when in Grade 3 and another when in Grade 5. These groups were measured at one-year intervals when in Grades 3 and 4 and in Grades 5 and 6, respectively. The seven measures of organismic age mentioned earlier and measures of achievement in language arts and arithmetic were obtained for each child. (Getting carpal age required x-raying of the wrist and evaluation of the x-ray by a roentologist, and getting dental age required examination and evaluation by a dentist.)

In the first year of the study the high- and the low-achievers in reading, arithmetic, and language arts were identified from their test scores. The means for the five physical measures of the high achieving children were found not to be significantly different from those of the low achieving children; however, the high-achievers had significantly higher means for mental age and occupational rating of parent and a significantly lower incidence of behavioral problems (Klausmeier, 1958). For the one-year measurement intervals, the correlations between organismic age, the mean of the seven ages including M.A. and reading age, and arithmetic achievement and language-arts achievement were much lower (.04 to .58) than were the correlations between the regression equations based on the seven organismic measures and the achievements in arithmetic and language arts (.63 to .77). Based on the regression equations that were derived from the one-year measurement intervals, the five physical measures were found to add practically nothing to mental age and reading age in predicting achievement in either arithmetic or language arts. As a matter of fact, for the Grade 3-4 boys and girls separately and for the Grade 5-6 boys and girls separately, the five physical measures toegether added only from .02 to .06 to the multiple correlations that also included mental age and reading age. The four multiple correlations were high, ranging from .77 to .89. Clearly, development in the mental and physical areas was progressing very differently and combining the five physical measures with the two intellectual measures into a single organismic age yielded a totally inaccurate assessment of the child's development in both areas.

In retrospect, the concept of organismic age, including the means of obtaining it for the individual child, may appear to be overly simplistic. The fact is, however, that orthodox stage theory as reflected by the theoretical curves given in Figure 5.14a, b, and c, with its accompanying proposition of congruent, synchronous development of all of the items of each stage, implies that a reliable measure of any one item of a developmental stage is a good predictor of all the other items of the stage. According to orthodox theory, neither summing and averaging measurements nor using more sophisticated correlational techniques is needed to understand growth of the different items and structures of a developmental stage.

Despite the total incompatability of the constructs of a single organismic age and multiple growth gradients, both Olson and Gesell, as most other psychologists at that time, gave heavy emphasis to maturational factors as determinants of cognitive growth. Bruner (1964) and Bruner, Olver, and Greenfield (1966) spearheaded the departure from this hereditarian tradition. Bruner, though an interactionist rather than an environmentalist, consistently pointed to the significant role of cultural factors, including language and schooling, as determinants of cognitive growth.

Bruner (1964) did much to popularize Piagetian theory in America. However, the considerable evidence that Bruner and his coworkers (Bruner, Olver, & Greenfield, 1966) presented concerning growth of various cognitive items from early childhood to age 19 does not support the orthodox stage construct as described by Piaget. The forms of their normative curves for various developmental items including equivalence, perceptual recognition, relational concepts, and conservation correspond to those presented earlier in Chapter 3 for the various CLD items. A concurrent but not synchronous progression of the items for many years is evident. The form of the curves varies greatly for the different items.

Bruner accounts for cognitive development in terms of the successive emergence and increasing integration of the three modes of acting on the environment and representing experiences internally. The three modes, based upon action, imagery, and language, are called enactive, iconic, and symbolic. Although these three modes descriptively correspond to the three major epochs of development indicated by the sensorimotor period, the period of preparation and organization of concrete operational thought, and formal operational thought, as explanatory constructs they bear little resemblance to the Piagetian logico-mathematical structures. Bruner's explanation of development is based on the processes that occur during the developmental years and the forces that impel development, including cultural as well as biological, whereas Piaget is more concerned with representing the knowledge of the maturing individual at four clearly demarcated and nonconcurrent stages in terms of mathematical and logical structures.

Bruner's emphasis on the symbolic mode, including the mediation of experience through language as early as six years of age, encouraged me to pursue two main ideas, namely, having the name of the concept and its defining attributes is prerequisite to attainment of the formal level, and the development of the meaning of the word expressing the concept progresses simultaneously with attaining the four successively higher levels of the concept. I have gathered only correlative information about word acquisition and concept attainment for students of age 6 to 18. Other investigators have established that very young children starting before age two associate concept names with concept examples and the names when heard influence their behavior (e.g., see Nelson, G.K., 1976; Nelson, K., 1974; and Nelson & Bonvillian, 1973). In research starting in the 1960s on strategies for learning concepts and continuing up to the present time, I have attributed great importance to the role of language in concept attainment.

A final note about the cognitive and information-processing aspects of CLD theory may be in order before proceeding to the principles. It is a descriptive theory that encompasses concepts, principles, structures of knowledge, and problem solving. I have characterized the theory as both cognitive and information processing. I use the word, cognitive, as it is used also by Ausubel (1963, 1968) and Bruner (1964, 1973) to indicate an interactional approach to learning and maturation and to environment and heredity, rather than a behavioristic or stimulus-response view. Information processing implies that the attainment of concepts at successively higher levels and the development of increasingly comprehensive conceptual cores can be accounted for in terms of acquiring, organizing, storing, and retrieving information. Acquiring and organizing are regarded as active construction processes, not as passive reception processes. Cognitive information processing clearly does not imply that the maturing human being is conceived of as complex machine or device, analogous to an electronic computer, that accepts and uses the programs provided by external agents for passively receiving, storing, processing, and retrieving information that is provided.

DEVELOPMENTAL PROGRESSIONS

The four successively higher levels of the same concept are attained in an invariant sequence. The four levels are attained many years earlier for concepts that have many three-dimensional examples widely dispersed in the environment than for concepts that have only symbolic examples.

The invariant progression is accounted for in terms of internal and external conditions of cognitive learning and development. Necessary, and possibly sufficient, internal conditions for attaining the first three levels are: (1) mastery of the prior level, (2) functioning of at least one new mental operation in combination with the continuing operations, (3) functioning of the operations of the prior level on information of increasing quantity and complexity experienced in a greater variety of contexts, (4) intending to attain the next higher level, and (5) persisting until it is attained. In addition to these internal conditions, being able to discriminate the defining attributes of the concept and to name the concept and its defining attributes are prerequisite and necessary for carrying out the new operations at the formal level.

The mental operations include attending, discriminating, generalizing, remembering (organizing, storing, and retrieving information), and also the higher level operations of hypothesizing, evaluating, and inferring. Intending and persisting, as well as the other operations, are governed by the executive control mechanism of the individual.

The necessary external conditions of learning include experience and practice with examples of the concept and examples of related concepts, or with symbolic information for concepts that do not have perceptible examples. The amount and kind of experience and practice that may be necessary for initial learning vary for the four different levels of attainment and also for different kinds of concepts.

From age 6 to 9, individuals' performances of developmental items are characterized by instability from one year to the next. The items develop at a more rapid rate during these years than later years, but the separate items also develop at different rates and with inconsistent amounts of gain from one year to the next. Relatively few correlations for the one- and two-year intervals for the same items are positive and statistically significant and even fewer of the inter-item correlations are.

We should not conclude from this lack of stability during age 6 to 9 that the concrete, identity, classificatory, and formal levels of the same concept are not being attained in an invariant sequence. Instead, the concrete level of the different concepts, as well as the other levels, become initially functional at different years of age and the amount of increase of any level that is functioning is not consistent from one year to the next during age 6 to 9.

From age 10 to 12, high stability in performance of the CLD items occurs. The rate of increase of each item decelerates until maximum functioning of the item is achieved, and there is considerable consistency in the amount of gain for each item from one year to the next.

During the years 13 to 15, performances have stabilized. The rate of increase for the items continues to decelerate until maximum functioning has been achieved; and the amount of gain is about the same for the various items from one year to the next.

Having dealt with stability we may now consider integration and differentiation as inferred from the correlations and the factor analyses performed on the CLD data gathered annually and at one- and two-year intervals on the four groups of students, Grades 1-12.

From age 6 to 9 the items develop as a relatively unrelated collection of items. The unrelatedness of the collection of items is more similar to Gesell's idea of separate growth gradients than to Olson's organismic age idea or to Piagetian synchronous stages. From age 10 to 12 the items are integrated into a smaller number of correlated groups of items corresponding to the factors obtained, namely, one for each concept. Differentiation according to the content of the integrated groups of items commences somewhat earlier than does the stabilization of the item performances across consecutive years. Some differentiation based on the concrete-abstract dimension of concept examples, rather than of subject field knowledge, occurs during the primary school years. Almost complete differentiation according to the subject fields of mathematics, English, and science occurs from age 10 to age 12, corresponding to Grades 4 to 6.

Presumably individuals during these years learn to relate and organize large amounts of perceptual, conceptual, and symbolic information into conceptual cores, each core consisting of concepts, principles, and relationships among them. These internal conceptual cores, corresponding to semantic fields, include the words and other symbols that express the concepts, the attributes of the concepts, the principles, and the relationships among them. These conceptual cores are the individual's representation of the structures of knowledge of the subject field. The structures are included in the curricula of the school as well as in other sources of information. The amount of information that the same individual acquires, organizes, stores, and retrieves varies according to these subject fields.

Full differentiation according to subject fields, and possibly other bases of organizing knowledge, occurs during the junior high school years, corresponding to age 13 to 15, and continues thereafter. We note in passing that the pattern of differentiation discussed here indicates the developmental progression of intraindividual differences that will be discussed later. Intraindividual differences according to the different subject fields are large during the junior high school years and continue into the senior high school years, corresponding to ages 15 to 18.

The preceding progressions are for the CLD items only, not for the familiar Piagetian items or the groupement items except that, first, as was shown in Chapter 6, there is also lack of stability and integration of the Piagetian and groupement items throughout Grades 1-2-3 and possibly into Grades 4-5-6; and second, there is increasing stability and integration of the groupement items starting at about

Grade 5 or 6. The grade cannot be stated with preciseness for the Piagetian and groupement items because factor analyses were not performed separately for these two domains. For the same reason no statement is warranted for these two domains concerning differentiation.

Two attributes in addition to those of stability, integration, and differentiation that may be used in describing the progression of developmental items are concurrent and continuous. To what extent is there concurrent development of the items of the four CLD levels and of the Piagetian and groupement items of the stage of concrete operations? To what extent is there continuity or discontinuity between the four CLD levels and between the two Piagetian stages? (Items that develop in synchrony must also develop concurrently; items that develop concurrently may or may not develop in synchrony.)

Figure 7-1, that appeared earlier as Figure 3-1, contains the smoothed normative curves for the concrete, identity, classificatory, and formal levels of *noun*. As shown in Figure 7-1, the children made much progress during the primary school years toward full attainment of both the concrete and identity levels of *noun*. Functioning near a chance level in Grade 1, they attained the classificatory level of *noun* at a very rapid rate from Grade 1 through Grade 3; a less rapid but continuous rate of increase was shown by the older groups of students until near full attainment was reached in Grade 12. Functioning at essentially a chance level in Grade 2, the children attained the formal level at a slower rate than the classificatory level. We see in Figure 7-1 that there is concurrent development of the concrete, identity, and classificatory levels from Grade 1 through Grade 3 and of the identity, classificatory, and formal levels from Grade 2 through Grade 4. We also observe continuity rather than discontinuity between the successive levels.

Figure 7-2a, b, c, that appeared earlier as Figure 3-4a, b, c contains the smoothed normative curves for understanding principles, understanding taxonomic relations, and problem solving. Inspection of these curves indicates that growth in the understanding of principles, the understanding of taxonomic relations, and in problem solving continues from the primary school years through Grade 12, and there is concurrent development of these items of development from Grade 3 through Grade 12.

Figure 7-3 gives the smoothed curves for two familiar Piagetian tasks of the concrete operations stage, classification and seriation, and for combinatorial reasoning, an item of the stage of formal operations. Figure 7-4 gives the curves for four of the eight groupe-

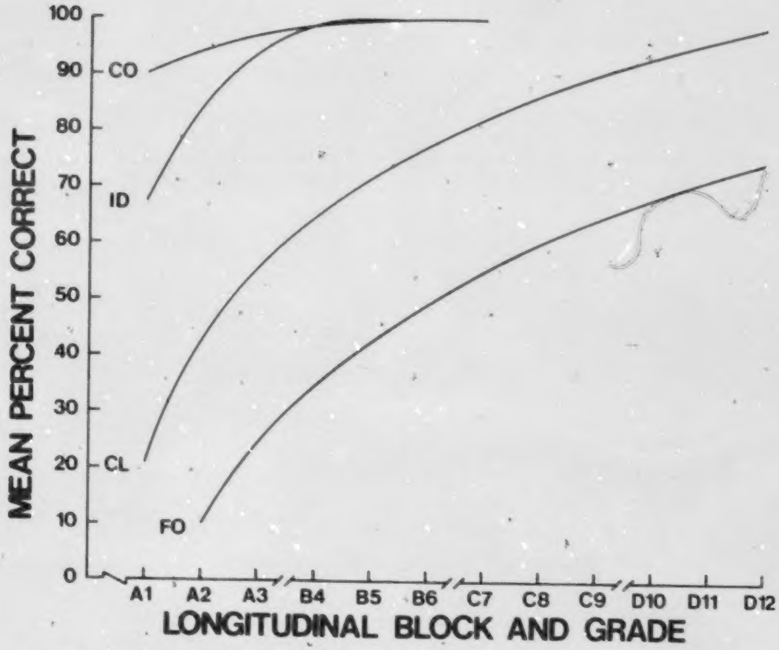

Figure 7-1. Developmental Curves for the Concrete, Identity, Classificatory, and Formal Levels of *Noun*

ment items dealing with classes, namely counivocal and biunivocal multiplication of classes, the composition operations, and counivocal and biunivocal division of classes, the inverse operations.

Inspection of the curves for the six items of the concrete operations stage—seriation and class inclusion and the four groupement items—indicates concurrent development of the six items of this stage, except seriation, from kindergarten through Grade 9. We hasten to point out that classification and seriation were the last of the 12 Piagetian items to reach full functional maturity, but we also note that they were already functioning well above a beginning level in kindergarten. The four classification groupement items were also among the latest to achieve maturity but they too were already functioning above the beginning level in kindergarten.

Inspection of the curve for combinatorial reasoning, the only item of the formal operations stage included in this comparative study, and the six Piagetian and groupement items for the concrete operations stage indicates concurrent development of this formal operations

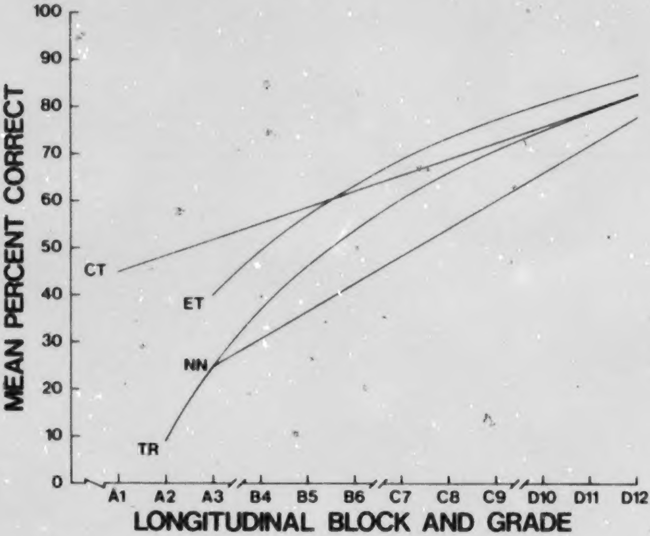

Figure 7-2a. Normative Development Related to Understanding Principles

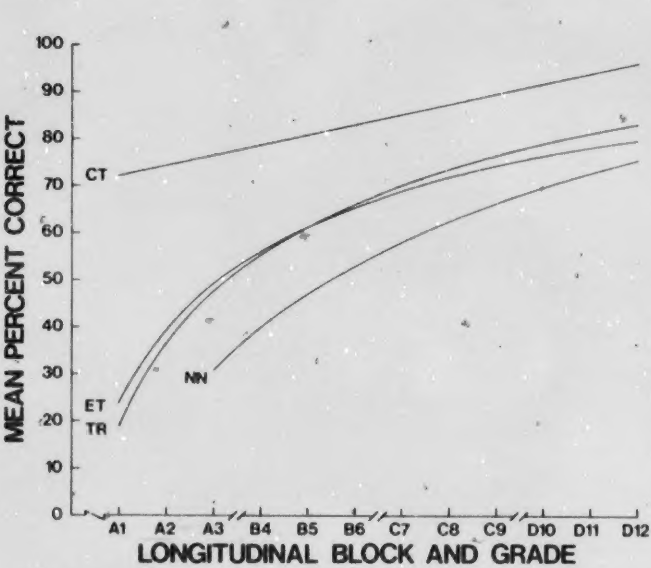

Figure 7-2b. Normative Development Related to Understanding Taxonomic Relations

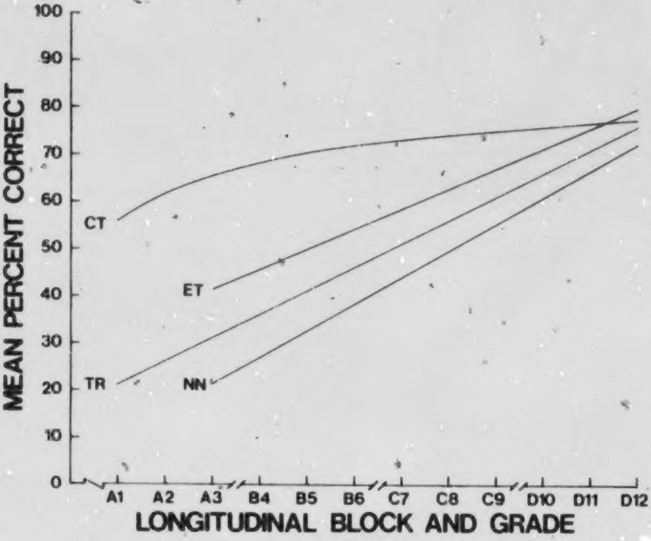

Figure 7-2c. Normative Development Related to Problem Solving

item with those of the concrete operations stage from kindergarten through Grade 9. Further, there is no suggestion of discontinuity between the concrete operations stage and the stage of formal operations. On the contrary, there is greater continuity than between any two of the successively higher levels of the concept *noun*.

Hypothetical curves for the Piagetian stages appeared earlier in Figure 5-14a, b, c. These curves were presented originally in Feldman and Toulmin (1976). The forms of the hypothetical curves bear practically no resemblance to the curves for the familiar Piagetian items given in Figure 7-3 or for the groupement items given in Figure 7-4.

How may the observed continuity from one level of concept attainment to the next and the large amount of nonsynchronous concurrent development of all the items, whether CLD- or Piagetian-based, be accounted for? CLD theory indicates that they can be accounted for primarily in terms of the individual's continuing to perform the operations of the prior level on information of increasing quantity and complexity experienced in a greater variety of contexts while also performing the new operations of the next level. The new operations of the next higher level emerge as a product of learning and maturation and become functional while the operations of the prior level continue to function on more complex informa-

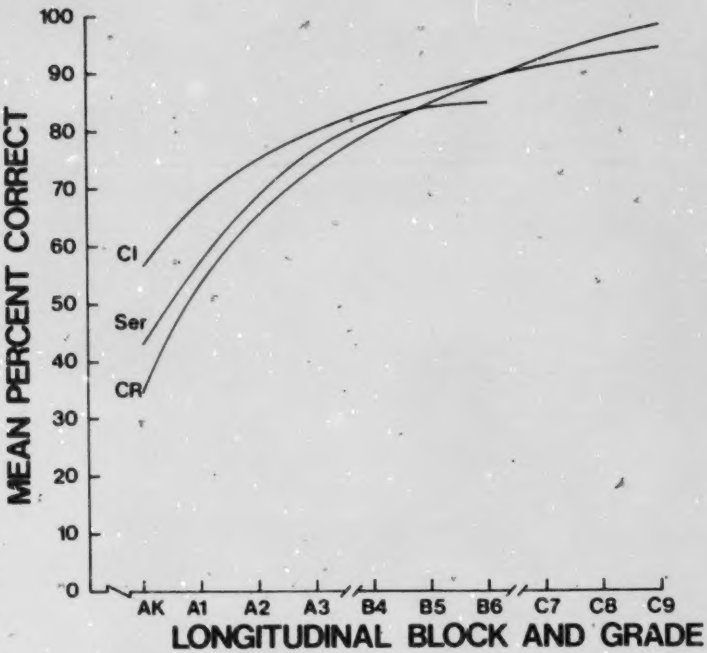

Figure 7-3. Normative development related to combinatorial reasoning (CR), Seriation (Ser), and Class Inclusion (CI).

tion. Integration of the continuing and new operations makes possible the progression of the prior levels toward full attainment and concurrently the initial functioning of the new level.

The preceding principles regarding concurrence and continuity of the CLD items are derived not only from the analysis of normative curves based on the mean scores but also on correlational and factor analyses of each individual student's test scores, as reported in Chapters 4 and 6. The correlations among the tests of each CLD concept battery were very low for the Grade 1-2, 2-3, and 1-3 measurement intervals. Not until the Grade 5-6 interval were there consistently high correlations, and these were much higher for the within-battery tests than for the tests of the different batteries. This suggests that development had probably become synchronous at Grade 6, not for the 28 items of all four concepts but for the 7 items of each concept. This pattern of the 7 developmental dimensions of each concept progressing in synchrony is not compatible with orthodox stage theory.

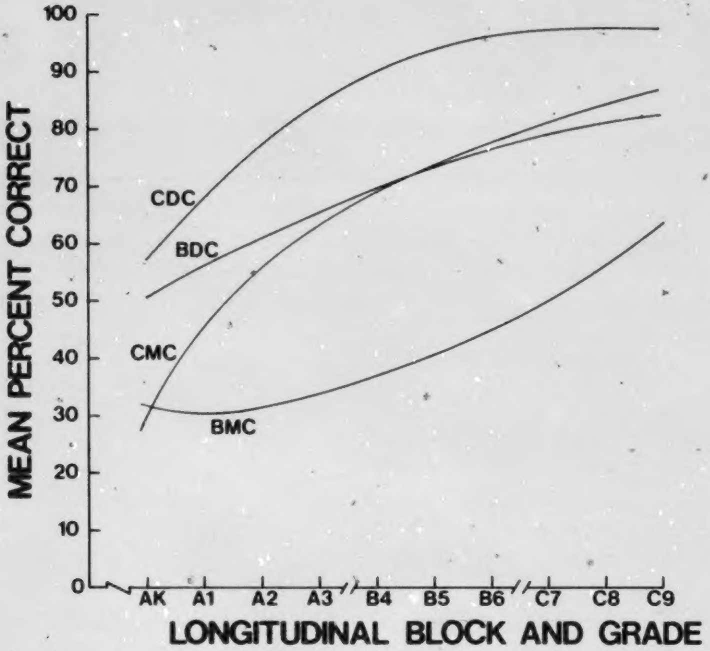

Figure 7-4. Normative development related to biunivocal multiplication of classes (BMC), counivocal multiplication of classes (CMC), biunivocal division of classes (BDC), and counivocal division of classes (CDC)

We saw in Chapter 6 that most of the correlations for the CLD, familiar Piagetian, and groupement tests for Grades 1-2, 2-3, 1-3, 4-5, and 4-6 were low and not statistically significant, and the factors derived were unrelatable from one successive measurement interval to the next. We also noted that the pattern of correlations and factor loadings for combinatorial reasoning, the only formal operation, were similar to those for the other Piagetian and groupement items. Based on all the information, it seems clear that the Piagetian items of the concrete operations stage develop in a nonsynchronous concurrent manner rather than in synchrony from age 7 through 11 and that there is much continuity rather than discontinuity between the stage of concrete operational thought and formal operational thought.

We saw in Chapter 5 that Hooper, Swinton, and Sipple indicated sufficient support for synchronous progression of the Piagetian items to conclude: "the general concept of a cognitive stage for the middle childhood domain may be alive and well in places other than Geneva! . . . instead of prematurely rejecting stages at this point we prefer to consider the stage construct as an heuristic device for guiding future research efforts and associated theoretical speculations." The discrepancy between their support and my disconfirmation of the Piagetian items progressing in synchrony is probably relatable to the different statistical procedures that we used. The conclusion of Hooper et al. regarding synchrony is based, not on correlational and factor analyses, which take into account individual's patterns of growth, but on analysis of variance performed on the test means. As was indicated in Chapter 5, a report of the Piagetian data that will include correlational analyses is being prepared (Hooper, Sipple, & Swinton, in preparation).

INTER- AND INTRAINDIVIDUAL DIFFERENCES

The CLD information gathered at three 12-month intervals on the four longitudinal groups of students was analyzed to identify interindividual differences in attaining the consecutively higher levels of the same concept and in understanding principles, understanding taxonomic relations, and solving problems. The same information was used in a systematic attempt to identify and describe intraindividual differences in these significant dimensions of cognitive growth.

Exceedingly large interindividual differences characterize students' development of these items. For example, 52% of the Grade 6 students had mastered the formal level of *equilateral triangle*, whereas 48% had not. The magnitude of the difference across grades is even more dramatic. For example, 3% of the Grade 3 children had mastered the formal level of *equilateral triangle*, whereas 21% of the Grade 12 students had not.

As might be expected, the developmental patterns were quite different for the most rapid and the slowest developers. The rate of growth of the rapid developers across the three annual measurements was more consistent than that of the slow developers; there were fewer plateaus and sharp dips in their developmental curves. The difference in the performance levels between the rapid and slow developers increased from the younger to the older ages and it was very large for Grades 10-11-12. It is possible that cognitive growth ceases or actually decreases for many slow developers from Grade 10 to Grade 12.

Intraindividual variability in attaining the concept levels and uses is discernible during Grade 1 to 3 on the basis of the concrete-abstract dimension of concept examples and according to subject matter fields during Grades 4 to 6. From Grade 6 through Grade 12 the intraindividual variability is very pronounced; separate factors were indentified in each of these grades for each of the four concepts: *cutting tool, tree, equilateral triangle,* and *noun.* A principle of highest significance may be inferred from the results of these and other factor analyses, namely, that intraindividual cognitive development from about age 11 onward is sharply and clearly differentiated according to content fields. The individual's cognitive functioning is not unitary or global; it is not characterized by equally high or low performance for all concepts. Rather, individual students vary within themselves in their attainment of different concepts, their understanding of the related principles and taxonomic relations, and their development of problem solving skills.

According to CLD theory, the wide range of *interindividual differences* among the students in their levels of concept attainment result in part from differences in their ability to perform the necessary mental operations on increasingly complex information and in motivational conditions related to intending and persisting. The *intraindividual variability* is accounted for by differences within the same individuals in their ability to perform the operations on different kinds of content and also possibly on motivational conditions.

In passing, we note that intraindividual variability is not compatible with the idea of congruent, synchronous progression of the items of a stage. However, orthodox Piagetian stage theorists ignore the widely observed facts regarding human intraindividual variability.

LANGUAGE

The relationship between the acquisition of words, word meanings, and concepts is of particular interest in the study of conceptual development. We may examine the relationship between word meanings and the four levels of concept attainment, the relationship between conceptual cores and semantic fields, and the facilitative effects of language on concept attainment.

Assume that an individual acquires the name of a concept at about the same time as attaining the concrete level and then attains the three successively higher levels of the same concept. The meaning of the word parallels the attainment of the four levels; the individual's meaning of the word that expresses the concept is the same as the individual's mental construct at each level. At the formal level of attainment the individual has the name not only of the concept but also of the defining attributes. The word meaning at the formal level is the societally accepted meaning of the word. Miller and Johnson-Laird (1976) refer to this word meaning as a lexical concept. In their theory, having the meaning of a word enables the individual to ascertain what is and what is not an entity that can or cannot be labeled by the word. Having the word meaning also enables the individual to indicate the function or purpose of the entity that the word labels. Finally, having the meaning can help the individual retrieve from secondary memory everything that the individual knows about the particular entity.

A group of related lexical concepts forms a semantic field. Just as having a lexical concept requires knowing the defining attributes of the particular concept, so also a semantic field requires having formed a conceptual core. The conceptual core includes the defining attributes of the generic concept and the critical attributes of the coordinate concepts comprising the taxonomy or other structure of knowledge.

The facilitative effects of language in concept attainment merit attention also. Language is widely used in directing the attention of the young child to an object, person, process, quality, or relation. This directing of attention facilitates attainment of the concept at the concrete and identity levels. Language is also used to point to an attribute of a generic class or to the critical attribute of a coordinate class. This facilitates attending and discriminating at both the classificatory and formal levels. The facilitative effects of being taught the names of the concepts and the attributes have been interpreted as due to mediating between the environmental input and the individual's responding, enhancing the distinctiveness of the concept examples, and enhancing the individual's sensitivity to the defining attributes of the concept. Although theorists do not agree about the internal mechanisms that might explain the facilitative effects, it is clear that instruction that gives the name of the concept is helpful in increasing intraclass similarity and decreasing interclass similarity. This in turn enables individuals to classify members of coordinate concepts that have the same generic attributes and are much alike in critical attributes.

Increasing competence in language is also associated with more sophisticated strategies for acquiring and remembering information and for using the strategies under appropriate circumstances. Focusing strategies generally have been found to be more effective than scanning strategies in acquiring concepts. Rehearsal, elaborative, and organizing strategies are useful in processing and storing information.

The increasing competence of the individual in learning and using the strategies contributes to the gradual and continuous development of concepts to successively higher levels, to more mature understanding of principles and taxonomies, and to more effective problem solving during the school years.

The preceding synthesis is limited to the descriptive aspects of cognitive development during the school years. Mental operations, executive control, transfer, motivation, and other constructs of CLD theory are dealt with in Chapter 1.

QUALITY OF EDUCATION AND THE RATE OF COGNITIVE DEVELOPMENT

After reviewing cross cultural studies, including those he conducted involving Wolof children of Senegal, Africa, Bruner (1973) concluded that is was always the schooling variable that made qualitative differences in directions of growth. He noted that Wolof children who had been to school were more different intellectually from unschooled children living in the same bush village than they were from city children in the same country or from Mexico City, Anchorage, Alaska, or Brookline, Massachusetts.

Our studies dealing with socioeconomic status, curriculum, and instruction also indicate the critical role of education in cognitive performances. We found a combination of low socioeconomic status and lack of instruction in geometry to be associated with immature conceptualizing skills and also with low achievement in geometry. Furthermore, children of low socioeconomic status who did not receive instruction remained at an immature level of conceptualizing, continuing to classify improperly through use of the nondefining variable attributes rather than classifying properly through use of the defining attributes and the supraordinate-subordinate names. It appeared that the student's curricular experiences were related not only to the level of concept attainment, but also to the very means of conceptualizing that make higher achievements possible. It is probable that some students will continue not to master concepts at the formal level unless the quality of their education is improved.

Instruction can be arranged to facilitate the learning of single concepts, multiple concepts, and structures of knowledge. When it is introduced at the proper time to supplement regular classroom instruction, large gains are made by the students during short time intervals. For example, in one experiment conducted with third-grade children in four 50-minute sessions, the percentage of experimental children passing the formal level of the concept, *tree*, increased from 3 percent on the pretest to 68 percent on the posttest (Klausmeier, Schilling, & Feldman, 1976).

In another experiment with four groups of Grade 4 children, 60 percent of experimental group 1 and 64 percent of experimental group 2 attained the formal level of *equilateral triangle* on a posttest, whereas 7 percent of control group 1 and 11 percent of control group 2 did (McMurray, Bernard, & Klausmeier, 1974).

Through three lessons to teach multiple concepts and the taxonomy of which they were a part, high school seniors were brought from functioning at a chance level to functioning at a high level (Bernard, 1975). A comparison of the achievements of the group receiving placebo lessons and of the groups getting other lessons designed to teach mastery of the concepts and understanding of taxonomic relations is most revealing. The mean percents correct for these groups on the three tests were: mastery of the concepts, 30 and 78; understanding taxonomic relations, 26 and 74; and placement of the concept names in an outline of the taxonomy, 33 and 83. The effectiveness of the instructional materials for teaching these multiple concepts, and also the single concepts of the earlier experiments, results from the precision with which the learning process is guided throughout the lessons.

This synthesis of research findings may be concluded by indicating that the purpose of the studies was to obtain a better understanding of cognitive development, both from an information-processing perspective and a Piagetian perspective. A description has been provided of the developmental progressions related to concepts, understanding principles, understanding taxonomic relations, and problem solving for each year of schooling, Grade 1 through Grade 12; and interindividual differences in the rate of development and intraindividual variability in areas of development have been delineated. A fairly comprehensive description of development from kindergarten through Grade 6 based on Piagetian measures has been presented. It appears that the course of development as inferred from the CLD measures can be accounted for by CLD constructs of cognitive learning and development but not by orthodox Piagetian stage constructs. On the other hand, the observed course of cognitive development as inferred from the Piagetian and groupement measures cannot be accounted for adequately by orthodox stage constructs.

❋ *Chapter 8*

Concept Learning and Developmental Stage

Charles J. Brainerd

When I was first asked to write this chapter, I must confess that I had something very different in mind from the chapter as now written. What I originally had in mind was a very wide-ranging essay that would deal with the contrasting philosophies of cognitive development embodied in CLD and Piagetian theory. This sort of chapter would have been enjoyable to write and, I think, would have been fun to read. Besides, since I have served as consultant to the project that spawned the data reported in this book since its inception, I have often thought about the global points of disagreement between CLD and Piagetian theory. Occasionally, in case I had forgotten, the author gently reminded me that I should be thinking about this matter.

After reading the preceding chapters, it soon became apparent that an essay dealing with philosophical matters would not be exactly in keeping with the empirical scope of this monograph. Instead, a chapter that focuses on a single empirical question and brings the available data to bear on that question seemed more appropriate. The particular question that I have selected is arguably the most important issue at stake in research on human concept development. What is the relationship between children's concept learning and the more general process of cognitive development? Or, more explicitly, does cognitive development impose very tight constraints on children's ability to learn concepts or is learning constrained in only a weak and general way? This question has been discussed—hotly debated would be a more accurate description—in the concept development literature for the better part of two decades. It is a qustion that seems to be lurking in the background of all of the preceding chapters, especially Chapter 1. Finally, it is a question to which CLD and Piagetian theory seem to offer very different answers. Consequently, some discussion of it appears essential to a comparative evaluation of the two theories.

Despite the centrality of the learning-development question, it is not explicitly analyzed in earlier chapters. No doubt this is a consequence of the fact that the data being reported, longitudinal studies rather than learning experiments, are not well equipped to answer the question. However, extensive data are available from other sources, in particular from learning experiments with Piaget's concrete operational concepts. It may be possible to obtain some closure on the learning-development issue by considering some of these data. That is what I propose to do in this chapter.

In the first section below, the respective positions of CLD and Piagetian theory on the relationship between concept learning and cognitive development are summarized. The remarks about Piagetian theory do not seem particularly controversial to me because they are based on statements appearing in any number of secondary sources (e.g., Beilin, 1971; Brainerd, 1977, 1978a, 1978b; Strauss, 1972) and in Genevan writings on learning (e.g., Inhelder, Sinclair, & Bovet, 1974; Piaget, 1970; Sinclair, 1973). The remarks about CLD, on the other hand, are more conjectural because they are based on my interpretation of some of Klausmeier's writings. In the second section below, I summarize findings from several experiments that are relevant to the issue of how closely cognitive development constrains children's concept learning. In the third section below, I summarize five new experiments, two with perspective-taking concepts and three with conservation concepts, that provide detailed quantitative information on the extent to which children's concept learning covaries with their prior knowledge of the concepts being learned.

THE THEORIES

Piagetian Theory

It is generally acknowledged that Piaget's approach to cognitive development imposes strong developmental contraints on children's concept learning. In fact, not too many years ago most investigators believed that the theory implies that stage-related concepts such as conservation, classification, seriation, horizontality, and so on, simply cannot be taught to children. In retrospect, this particular interpretation has turned out to be an oversimplification. Moreover, it has been explicitly repudiated by some of Piaget's co-workers. For example, Sinclair reports that although some early learning experiments conducted in Geneva produced negative results, "Piaget and his collaborators did not conclude that any kind of learning procedure would be useless. To the contrary, such a conclusion would go counter to the developmental theory that is resolutely interactionist and constructivist" (1973, p. 57). Similar disavowals may be found in Inhelder et al. (1974). Although it now seems that the prediction of zero learning effects does not follow from Piaget's "resolutely interactionist and constructivist" theory, it is perhaps of historical interest that these disavowals began to appear shortly after the publication of several successful learning experiments conducted by investigators outside Geneva.

Although the theory does not rule out learning effects, it clearly limits the amount of improvement in a concept that can be expected to accrue from training: "learning represents only a sector or a phase of development itself, arbitrarily provided by the environment . . . but remaining subject to the general constraints of the current developmental stage" (Piaget, 1970a, p. 713). And: "learning is subordinate to the subjects' levels of development. If they are close to the operational level, that is, if they are able to understand quantitative relations, the comparisons they make during the experiment are enough to lead them to compensation and conservation. But the farther they are from the possibility of operational quantification, the less they are likely to use the learning sequence to arrive at a concept . . ." (Piaget, 1970a, p. 716).

Piaget appears to be telling us at least two things in comments such as these. First and foremost, he is saying that children's acquisition of concepts cannot be explained by invoking the usual principles of learning—discrimination, reinforcement, association, and the like. It is cognitive development, a spontaneous process that is ostensibly governed by its own unique principles (equilibration, adaption, organization, etc.) that controls concept acquisition. Learning, by comparison, is "no more than a sector of cognitive development which is facilitated by experience" (Piaget, 1970a, p. 714). Second, the extent to which children acquire some concept in a learning experiment invariably is limited by "the general constraints of the current developmental stage" (Piaget, 1970a, p. 713). Assuming that we have methods of measuring both the current cognitive stages of our subjects and the strength of learning effects, it follows that learning effects will "vary very significantly as a function of the initial cognitive levels of the children" (Piaget, 1970a, p. 715). The nearer the current stage is to the level at which the to-be-trained concept "spontaneously" develops, the more likely a child is to learn that concept as a result of training, and conversely.

It should be evident by now that the Piagetian view of learning is basically a readiness theory, where we define a readiness theory as any hypothesis that assumes that certain developmental prerequisites must be layed in before subjects can benefit from learning experiences. The developmental prerequisites in Piaget's theory, of course, are in the nature of stages and structures. Specifically, learning is characterized not as an independent process but, rather, as a dependent process consisting of *learning how to apply previously acquired cognitive structures to novel situations.* (The current level of cognitive structuration, in turn, defines the current stage of cognitive development.) If the cognitive structures associated with the stage at which the to-be-trained concept normally emerges are present, then learning experiences can operate on these structures to produce concept acquisition. but the structures themselves cannot be trained in any strict sense. They develop very slowly through the interaction of maturational and environmental factors, a process called equilibration in Piagetian theory. If the relevant structures are not yet present (i.e., if the current stage is far below the one at which the target concept first appears), then training is futile: "teaching children concepts that they have not acquired in their spontaneous development . . . is useless" (Piaget, 1970b, p. 30).

Children's ability to learn a concept, then, is positively correlated with their stages of cognitive development. Since most experiments in which this prediction has been examined have been concerned with conservation concepts, let us consider how this general prediction can be translated into some specific statements about conservation learning. To begin with, Piaget (e.g., 1952; Piaget & Inhelder, 1941) usually describes developmental changes in given conservation concepts in terms of three global stages. During Stage I, which corresponds to the preoperational period of mental growth, children show no evidence of conservation. During Stage II, which corresponds to the transition phase between preoperations and concrete operations, children conserve in some situations but not in others. For example, Stage II children may conserve following a small stimulus deformation but not following a large one. Similarly, Stage II children may *predict* conservation prior to deformation but fail to conserve after the deformation has been executed (e.g., cf. Brainerd & Brainerd, 1972). During Stage III, which corresponds to the concrete operational period, children conserve in all or nearly all situations. In Piaget's original studies, this three-stage description is little more than a crude framework around which to organize the finer details of children's performance on conservation problems. However, it is possible to embody this description in a mathematical model and test its goodness of fit to conservation acquisition data (see Brainerd, 1979a, 1979b).

Suppose that we now conduct a conservation learning experiment. First, we administer conservation pretests to a sample of children. Second, we classify the children as Stage I, Stage II, or Stage III on the basis of their pretest performance. Since Stage III children perform perfectly, we discard these subjects, Stage I and Stage II children are retained for training. Third, we administer a training procedure of some sort to the remaining children. Finally, we administer a series of posttests to determine how much the children learned. From what has already been said, it is clear what Piagetian theory predicts about posttest performance—a given amount of training should produce more improvement in Stage II children than Stage I children.

Actually, the theory provides at least three grounds for this prediction. First and most obviously, Stage I subjects are farther "from the possibility of operational quantification" than Stage II subjects, that is, Stage I subjects are farther from the stage at which conservation spontaneously emerges than Stage II subjects. Second, the prerequisite cognitive structures for conservation are believed to be present during Stage II but not during Stage I. Since Stage II is a transitional period, preoperational and concrete operational structures coexist during this phase; hence, learning can operate on the latter to produce conservation acquisition. Only preoperational structures are believed to be present during Stage I. Third, Stage II is described as an unstable state, a "sensitive period" if you will, during which children are especially likely to benefit from learning experiences. The instability of Stage II is supposedly a consequence of the presence of two distinct levels of cognitive structuration.

CLD

It is useful to think of Piagetian theory as a top-down model. A rather complex collection of general prerequisites is built into the organism, where the building process involves largely unspecified interactions between maturational and environmental variables, and concept learning somehow "evolves" from these prerequisites. I find it equally useful, at least for purposes of comparison, to think of CLD as a bottom-up theory. While concept learning is a dependent process with respect to cognitive development in Piagetian theory, it is relatively unfettered in CLD. In fact, I think a reasonable case can be made for the conclusion that cognitive development depends on concept learning in CLD. The basic desideratum of CLD appears to be the fact that children learn concepts. Cognitive development, on the other hand, seems to be treated as an abstraction from the concept learning process. It is essentially a high-altitude description of what results from many individual concept learning events. (This is why I think it is useful to view CLD as a bottom-up theory.) I briefly consider these points in what remains of this section.

I said that a case can be made for the conclusion that cognitive development depends on learning, not the reverse, in CLD. There are several grounds for this statement. The most obvious one is that cognitive development is simply not formulated as an independent process in CLD. Whereas there is much discussion of "the laws of development" in Piagetian theory, there is little in CLD. Children's progression through the concrete, identity, classificatory, and formal levels of concept attainment is not described in terms of developmental principles such as organization, adaption, equilibration, and the like. Rather, it is described in terms of learning principles that operate within the context of unspecified maturational variables: "The cognitive structure of the individual at any point in time includes everything the individual has *learned.* The products, or outcomes, of learning may be categorized as perceptual information, verbal information and skills, concepts, principles, structures of knowledge including taxonomies, and problem-solving skills, including strategies of learning and of remembering. *These products are learned initially* . . . Concepts, principles, structures of knowledge including other hierarchies, and problem solving, the primary concerns of CLD theory, *are powerful outcomes of learning* in the cognitive domain" (pp. 1-7, my italics). And: "Development is regarded as a reciprocal interaction of learning and maturation that occurs across relatively long time spans, *not as preceding learning*" (pp. 1-6, my italics). Although there are some very reasonable maturational variables that one could say, with relative safety, must interact with children's concept learning (e.g., myelination of higher cortical centers, lateralization of hemispheric function), such variables are not explicitly discussed. Therefore, for all practical purposes, CLD defines development in terms of learning.

Another illustration of the primacy of learning in CLD is the fact that wide latitude is permitted when it comes to the movement of individual concepts through the concrete, identity, classificatory, and formal levels. Although a concept is said to move through these levels in an invariant order, largely for logical reasons having to do with increased complexity, it seems to be quite possible, within a given child, for some concepts to be at the identity level, others at the classificatory level, and others at the formal level. For example: "We are continuously attaining concepts of individual persons at the identity level; however, we already have the concept *person* at both the classificatory and formal levels . . ." (p. 13).

The possibility of large numbers of concepts coexisting at very different levels of sophistication is not a feature of Piagetian theory. On the contrary, we saw earlier that states that have this characteristic are described as unstable. The general picture that one derives from Piagetian theory is one in which large numbers of concepts move en masse through the progression of stages. Of course, the available data are not very consistent with this picture. Taking the concrete-operational level as a case in point, some concepts (e.g., transitivity) seem to reach this level during the preschool years, others (e.g., conservation) during the elementary school years, and still others (e.g., horizontality, class inclusion) during the adolescent years (see Brainerd, 1978b, Chapter 6). Facts such as these have prompted a good deal of tinkering with the idea of stage progression by neo-Piagetian writers (see Chapter 5 for some examples). But these ex post facto alterations do not change the fact that Genevan writers tend to portray concept development in the manner already described. To illustrate, in response to a question posed at a UNESCO conference by René Zazzo, Inhelder replied, "I am always struck by the fact that among developing children the dispersion within a stage is relatively small compared with the wide differences between behavior in one stage or another . . ." (1960, p. 88). Later, in response to a related question raised by Margaret Mead, Inhelder replied, "In one of the experiments on conservation solved by 75 per cent of the children at the age of six and a half, I did not find a single success until exactly five and a half and no failure after seven and half . . . For other experiments the dispersion may be slightly greater or slightly less" (1960, p. 92).

In CLD, then, the picture is one of individual concepts moving through the various levels, primarily as a result of learning experiences, at different rates and coexisting at different levels of sophistication. In Piagetian theory, on the other hand, there seems to be a uniform timetable—call it "cognitive development" for lack of a better phrase—that governs the progress of all concepts.

It should be clear by now that CLD does not impose the sort of developmental constraints on children's concept learning that Piagetian theory does. Since development is defined in terms of learning plus unspecified maturational factors, phrases such as "developmental constraints on learning," which play such a central role in any discussion of the Genevan view of concept learning, do not seem to make any sense in CLD. No global stages of cognitive development are posited. It is true that there are four levels of concept attainment, but these levels apply to individual concepts. They do not constitute grand templates to which all concepts must conform within given periods of development. Since no global stages are posited, CLD does not predict that concept learning and children's stage classifications will be positively related. Since the current level of any given concept is a consequence of learning, not development, the theory leads one to expect that the size of concept learning effects will be relatively independent of children's current levels of concept attainment. With conservation concepts, for example, CLD provides no grounds for expecting that Stage II children will be more susceptible to training than Stage I children.

THE DATA

Before we see what the literature has to say on these matters, it will be useful to introduce some notational shorthand. In "development versus learning" experiments of the type described above, one can speak of at least three dependent variables, namely, pretest performance, learning, and posttest performance. Let X denote pretest performance, let Y denote posttest performance, and let Z denote learning. It is clear that X is our measure of development, because the pretests are administered before any training takes place. It is also clear that Y is some function of X and Z, that is, it consists of what the subjects already know about the concept (development or X), what they learn during training (Z), and an error term of some sort. Piagetian theory says that X and Z are positively related. CLD, on the other hand, seems to say that X and Z are unrelated.

Several experiments have appeared in which investigators have attempted to test the prediction that children's concept learning is positively related to their pretest performance. Although a few experiments with classificatory concepts and relational concepts have been reported, nearly all of the relevant literature deals with conservation. The earliest conservation experiment in the "learning versus development" literature appears to be Beilin's (1965) classic study. Beilin administered pretests for number, length, and area conservation. Children who passed both the number and length pretests were classified as Stage III. Children who passed neither the number nor length pretest were classified as Stage I, and those who passed one of these pretests were classified as Stage II. Stage I and Stage II children proceeded to training. Three weeks after training, posttests for number, length, and area conservation were administered. The pretest criteria were applied to the posttest data, and the children were reclassified as Stage I, Stage II, or Stage III.

Beilin concluded that his data were consistent with Piaget's view that concept learning is positively related to pretest knowledge: "The group most likely to profit from training is that group whose members have at least some (but not the fullest) conservation ability, i.e., the group who passed one pretest as compared to the groups that passed no or two pretests" (1965, p. 335). He based this interpretation on the following evidence: "Of *S*s who passed one pretest, 67% (8 of 12) passed both posttests. Of those who passed neither pretest, 31% (36 of 117) passed both posttests. Of those *S*s who failed both number and length on pretest, 24% (28 of 117) passed one posttest and failed the other" (1965, p. 334).

Several other experiments are available in which Beilin's basic design was employed. In all of them, the prediction that pretest performance predicts learning has been examined by comparing children's pre- and posttest stage classifications. A positive relationship has been consistently observed. Children with higher stage classifications on the posttest tend to have higher stage classifications on the pretest. This finding has been interpreted by most investigators as confirming the hypothesis that learning is contrained by pretraining knowledge. But it is not hard to see that this finding has nothing to do with the hypothesis. The observed relationship between children's pre- and posttest stage classification shows that *X* and *Y* are positively related, not that *X* and *Z* are positively related. Recall that it is the latter prediction, not the former, that we are interested in. If the tests being administered are reliable and if the training procedure does not produce massive ceiling effects, then *X* and *Y* will always be positively related regardless of the true relationship between *X* and *Z*.

The last point, which is crucial, becomes obvious if one considers a pseudolearning experiment in which children do not receive any training on the target concept. Suppose that we conduct an experiment using the design already described except that we omit the training phase. That is, conservation pretests are administered, and the subjects are classified according to stage. Sometime later, conservation posttests are administered, and the subjects are reclassified. Will the two classifications be positively correlated? They will if the tests are reliable and if there is some variability in the subjects' performance. But this correlation tells us nothing about the relationship between *X* and *Z*, because no learning has occurred.

If we are going to examine the relationship between *X* and *Z*, it is clear that we need an estimate of *Z*. An obvious estimate is available in the form of pre- to posttest gain scores, that is, $Z = Y - X$. I have conducted a gain score analysis of most of the experiments that were available in early 1977 and reported the data elsewhere (Brainerd, 1977). A summary of this analysis appears in Table 8-1. The main finding is that the gain score analysis provides no support for the hypothesis that learning is constrained by pretraining performance. Generally speaking, the average gain in conservation performance for subjects classified as Stage I on the pretest was about the same as the average gain for subjects classified as Stage II on the pretest.

Some additional experiments have been reported since the analysis in Table 8-1 was completed. I have conducted a gain score analysis of these experiments, and the results are consistent with those in Table 8-1. The newer experiments also fail to show greater pre- to posttest improvement for Stage II subjects. Two experiments reported by Murray, Ames, and Botvin (1977) will serve as illustrations. In Experiment I, 96 kindergarten, first-, second-, and third-grade children were administered pretests for number, mass, quantity, and weight conservation. Stage I children failed all the pretest items while Stage II children passed up to 62.5 percent of these items. The pretests were readministered shortly after training. The average pre- to posttest improvement was 108 percent for Stage II subjects and 1450 percent for Stage I subjects. In Experiment II, 114 preschool, kindergarten, first-, and second-grade children were administered pretests for length, mass, quantity, and weight conservation. Classification criteria were the same as before. The pretests were readministered shortly after training and one week later. On the first posttest, the average improvement was 78 percent for Stage II subjects and 570 percent for Stage I subjects. On the second posttest, the average improvement was 88 percent for Stage II subjects and 590 percent for Stage I subjects.

Thus, when gain score analysis is applied to the extant data base, there does not appear to be any support for the hypothesis that development constrains concept learning. More specifically, there does not appear to be any support for the prediction that Stage II conservers should be more susceptible to training that Stage I conservers. However, extreme circumspection is called for in the interpretation of the gain score analysis due to the distinct possibility of ceiling effects in at least some of the experiments. It is obvious to common sense that ceiling effects would mask a positive X-Z relationship if it existed. If the training procedure used in any given experiment were so powerful that most subjects (regardless of their pretraining performance) performed perfectly on the posttest, then

Table 8-1. Difference Score Reanalysis of Conservation Training Experiments Which Tested the Hypothesis That Subjects' Stages Predict Learning

		Stage I				*Stage II*			
Experiment and types of tests administered	*High possible pre- and posttest score*	*Number of subjects*	*Mean pretest score*	*Mean posttest score*	*Mean improvement score*	*Number of subjects*	*Mean pretest score*	*Mean posttest score*	*Mean improvement score*
			Non-Genevan Experiments						
Beilin (1965)									
Number and length	2/2	117	0.00	0.85	0.85	12	1.00	1.67	0.67
Brainerd (1972b)									
Number	24/24	25	1.92	15.60	13.68	20	7.81	18.11	10.30
Langer and Strauss (1972)									
Number	2/2	25	0.00	0.56	0.56	30	1.00	1.33	0.33
Length	2/2	35	0.00	0.89	0.39	17	1.00	1.18	0.18
F.B. Murray (1972)									
Goldschmid-Bentler Form A	12/36	20	0.25	21.90	21.15	23	3.35	31.50	21.45
J.P. Murray (1974)									
Liquid quantity	6/6	12	0.00	2.70	2.70	22	2.28	5.10	2.82
Solid quantity	6/6	12	0.00	0.60	0.60	22	3.12	4.38	1.26
Rosenthal and Zimmerman									
Goldschmid-Bentler Form A									
Criterion #1	6/6	57	0.00	4.08	4.08	23	1.98	5.72	3.72
Criterion #2	6/6	66	0.00	2.58	2.58	14	1.77	4.84	3.07
Strauss and Langer (1970)									
Liquid Quantity									
Posttest 1	2/2	34	0.00	0.56	0.56	33	1.00	1.54	0.54
Posttest 2	2/2	34	0.00	0.74	0.74	33	1.00	1.61	0.61

Table 8-1. continued

		Stage I				*Stage II*			
Experiment and types of tests administered	*High possible pre- and posttest score*	*Number of subjects*	*Mean pretest score*	*Mean posttest score*	*Mean improvement score*	*Number of subjects*	*Mean pretest score*	*Mean posttest score*	*Mean improvement score*
Strauss and Rimalt (1974)									
Posttest 1									
Number	2/2	12	0.00	1.50	1.50	2	1.00	2.00	1.00
Length	2/2	27	0.00	0.30	0.30	19	1.00	0.95	-0.05
Liquid quantity	2/2	35	0.00	0.43	0.43	7	1.00	1.43	0.43
Posttest 2									
Number	2/2	12	0.00	1.17	1.17	2	1.00	2.00	1.00
Length	2/2	27	0.00	0.26	0.26	19	1.00	1.11	0.11
Liquid quantity	2/2	35	0.00	0.54	0.54	7	1.00	1.57	0.57
				Genevan Experiments					
Inhelder et al. (1974, Chapter 1)									
Liquid and solid quantity	2/2	15	0.00	0.13	0.13	19	0.95	1.42	0.47
Inhelder et al. (1974, Chapter 2)									
Liquid quantity	1/1	6	0.00	0.67	0.67	7	0.50	0.57	0.07
Solid quantity	1/1	3	0.00	0.33	0.33	6	0.50	0.67	0.17
Inhelder et al. (1974, Chapter 3)									
Liquid quantity	1/1	5	0.00	0.40	0.40	4	0.50	0.63	0.13
Solid quantity	1/1	10	0.00	0.45	0.45	4	0.50	1.00	0.50
Inhelder et al. (1974, Chapter 6)									
Length	1/1	12	0.00	0.42	0.42	4	0.50	0.63	0.13
Inhelder et al. (1974, Chapter 8)									
Liquid quantity	1/1	13	0.00	0.39	0.39	5	0.50	0.70	0.20

it would be impossible for Stage II subjects to show greater improvement than Stage I subjects. In fact, the reverse would be true. Since Stage I subjects begin at a lower performance level but reach the same ceiling as Stage II subjects, they must show greater pre- to posttest improvement. As an illustration, let us reconsider the experiments of Murray, Ames, and Botvin (1977). In Experiment I, the average posttest score of Stage II children was 15.5 out of a possible 16. On the first posttest in Experiment II, the average score of Stage II children was 7.56 out of a possible 8. The average score of the same children was 8 out of 8 on the second posttest. It is clear that ceiling effects were operating in these experiments and, hence, the gain score analysis reported in the preceding paragraph cannot be readily interpreted.

NEW EVIDENCE

Although the available literature on the relationship between prior knowledge of concepts and children's concept learning is substantial, it is clear from the preceding section that the experiements in this literature have not told us anything definite about learning-development. On the one hand, gain score analysis reveals no general tendency for children with higher pretraining performance to show more improvement than children with lower pretraining performance. On the other hand, any gain score analysis can be invalidated by ceiling effects, and effects of this sort seem to be present in at least some experiments. It appears, then, that additional data are called for.

The purpose of this section is to report five experiments, two on perspective-taking concepts and three on conservation concepts, that were designed to generate some less equivocal data on learning-development. The common features of the designs of these experiments are discussed in the first subsection below. The individual experiments, along with their main findings, are considered in the second subsection below.

Design

All five experiments relied on the same basic design as the studies reviewed in the preceding section. That is, children received pretests for certain concrete-operational concepts; children who performed less than perfectly on the pretests were given training; posttests were administered after training. The experiments differed from each other only in the specific concepts being measured and in the nature of the training procedure. The experiments differed from earlier studies in two major respects. First, some care was taken to avoid ceiling effects. Second, regression analysis was used to evaluate the relationship between concept learning and prior knowledge of the to-be-trained concepts. Let us briefly consider these two points before taking up the experiments themselves.

Ceiling Effects. We have already seen that ceiling effects render the results of a gain score analysis uninterpretable. They also invalidate any other analysis in which the learning-development question is investigated by examining the predictive relationship between pre- and posttest scores. If the training method is so powerful that there is no variation on the posttests, it is evident that there will be no predictive relationship between pre- and posttest scores.

There are at least three ways to reduce the possibility of ceiling effects: (a) constrain the power of the training treatment; (b) increase the number and variety of concept tests; (c) use younger subjects. Solution a is perhaps the most obvious one. Several highly effective procedures for training concrete-operational concepts are available in the literature (e.g., learning set, rule instruction, extinction of irrelevant cues, observation of a model, corrective feedback). If training is sufficiently extensive with any of these procedures, more or less perfect posttest performance results. However, training can be fine-tuned to avoid this outcome through pilot research. For a given procedure, one can ascertain through piloting just how much training is required to produce a *moderate* learning effect. This amount of training can then be used in the main experiment.

Solution b involves the administration of several parallel forms of the test for the to-be-trained concept. The assumption is that a given training procedure is unlikely to produce perfect performance in all the various situations in which the target concept can be measured. For example, suppose that we administer pre- and posttests for six types of conservation (e.g., number, length, weight, mass, area, quantity) and train subjects on two types of conservation (e.g., length and mass). Although subjects might perform perfectly on posttests for the last two concepts, they would be less likely to perform perfectly on posttests for the other four concepts. Solution b also has the desirable result of increasing the reliability of our measurements.

Finally, solution c takes advantage of the fact that, other things being equal, younger children are harder to train than older children. For example, suppose that we have a group of four-year-olds and a group of seven-year-olds. Suppose also that the subjects in both groups have failed a pretest for quantity conservation. There seems little doubt, regardless of the specific training method, that seven-year-olds will show more dramatic improvement in quantity conservation than four-year-olds will. Consequently, ceiling effects are less likely in the four-year-old group.

All three of these solutions have been employed in the five experiments reported below. In each experiment, the subjects were given enough training to produce a substantial learning effect but not enough to produce perfect posttest scores. The battery of pre- and posttests was always extensive. In the two experiments on perspective taking, three different types of perspective taking were measured (perceptual, cognitive, affective), and there was more than one test for each type. In the three experiments on conservation, four different concepts were measured (number, length, quantity, and weight) and there were multiple tests for each concept. Finally, the research was confined to children below first grade. The subjects were preschoolers in the perspective-taking experiments and kindergarteners in the conservation experiments.

Regression Analysis. At first glance, gain score analysis seems to be the logical way to determine whether or not pretest performance predicts learning. If these variables are positively related, gain scores should be greater for subjects with higher pretest scores than for subjects with lower pretest scores. Unfortunately, gain scores suffer from reliability problems (see Cronbach & Furby, 1970). Consequently, the use of gain scores, although intuitively plausible, tends to increase the chances that we will falsely accept the null hypothesis that learning is unrelated to pretest performance. This problem can be avoided by using regression analysis.

Consider the general linear equation

$$Y = a + bX, \qquad (8\text{-}1)$$

where X and Y are defined as before. If our concept tests are reliable, then Equation 8-1 gives a good account of the relationship between any two sets of scores on these tests. Equation 8-1 is related to ρ_{XY}, the test-retest reliability of our concept tests, by the expression

$$\rho_{XY} = b\,(\sigma_X/\sigma_Y). \qquad (8\text{-}2)$$

The tests are perfectly reliable if $b = 1$ and $\sigma_X = \sigma_Y$.

Now, consider a concept learning experiment in which children are divided into experimental and control groups following the administration of the pretest. For the subjects in the former group, the experiment consists of pretest items followed by training trials followed by a readministration of the pretest items (i.e., the posttests). For the controls, the experiment consists of two administrations of the pretest items separated by an interval of time. Consider the correlation between the two control group administrations. This correlation, r_{XY}, is an estimate of ρ_{XY}, the test-retest reliability. If the tests are reliable and if we have not committed serious sampling errors, r_{XY} should be large and positive. Now consider the same correlation for subjects in the experimental group. If the tests are reliable, if we have not committed serious sampling errors, *and* if ceiling effects are not operating, this correlation should also be large and positive.

Given the appropriate caveats about reliability, sampling, and ceiling effects, Equation 8-2 should describe the relationship between the two sets of scores in both experimental and control subjects. If it also transpires that pre- to posttest improvements in the former subjects are positively related to pretest scores, it follows that the XY regression line for experimentals will rise more steeply than the XY regression line for controls. More simply, the slope constant, b, will be larger for subjects who receive training. But if pre- to posttest improvements are unrelated to pretest scores, the magnitude of b will be the same for the two groups. These predictions are apparent from the following considerations.

The posttest score, y, for any control subject may be defined as

$$y = x + \text{error}. \tag{8-3}$$

The posttest score for any experimental subject may be defined as

$$y = x + z + \text{error}, \tag{8-4}$$

where z is the amount of improvement accruing from learning. Note that these two expressions differ only in the presence of z in Equation 8-4. If the value of z is unrelated to the value of x, the regression lines for experimentals and controls may differ in their respective intercepts, a, but they will not differ in their respective slopes; that is, the regression lines will be roughly parallel. But if z tends to increase as x increase, b must be larger for experimental subjects; that is, the regression lines will tend to diverge. In any given study, a straightforward significance test for differences in slope constants is available in the form of the F-test for homogeneity of regression coefficients in the analysis of covariance.

Everything that has been said in this subsection presupposes that the learning-development question is to be investigated by examining the relationship between pre- to posttest improvement and pretest scores. Other strategies are possible. In particular, the relationship between rate of improvement on the *training trials* and pretest scores might be investigated. For this particular strategy to be maximally effective, we require a precise mathematical model that describes how the probability of a correct response changes as a function of training. Although some preliminary work along these lines has been reported for conservation and class inclusion (Brainerd, 1979a, 1979b), the models developed in these papers have not yet been used to investigate the relationship between training trials and pretest scores.

The Experiments

The first two experiments were concerned with the learning of perspective-taking concepts. Pretests for perceptual perspective taking, cognitive perspective taking, and affective perspective taking were administered in both experiments. In Experiment I, 17 four-year-old children received a training procedure that consisted of teacher-guided experience in constructive play. These constructive play experiences were given in small groups, and there were a total of 10 training sessions in all. The pretests were readministered one day after completion of the tenth constructive play session. The procedure for Experiment II was similar. Following the pretests, the 17 four-year-olds in this experiment were also administer 10 sessions of teacher-guided play experience. However, the sessions involved dramatic play rather than constructive play. As was the case in Experiment I, the pretests were administered one day after completion of the tenth dramatic play session. Since the same test battery was used in both experiments, a single control group of 17 four-year-olds was run. This group received two administrations of the battery on the same days as the experimental subjects.

The findings from these two experiments are reported in detail by Burns and Brainerd (1979). For our purposes, the critical results are these. First, Equation 8-1 gave a good account of the data in all three groups. (See the scatter plots in Figure 8-1.) In the control and cooperative play groups, the regression lines in Figure 8-1 account for roughly 70 percent of the variance. In the dramatic play group, the regression line in Figure 8-1 accounts for roughly 35 percent of the variance. Thus, the fits were reasonably good. Second, the r_{XY} correlation for the control subjects, .85, was satisfactorily high; so the battery of perspective-taking tests was quite reliable. Third and most important, the regression lines for the three conditions were more or less parallel. Recall that if pre- to posttest improvement is positively related to pretest scores, the lines for the

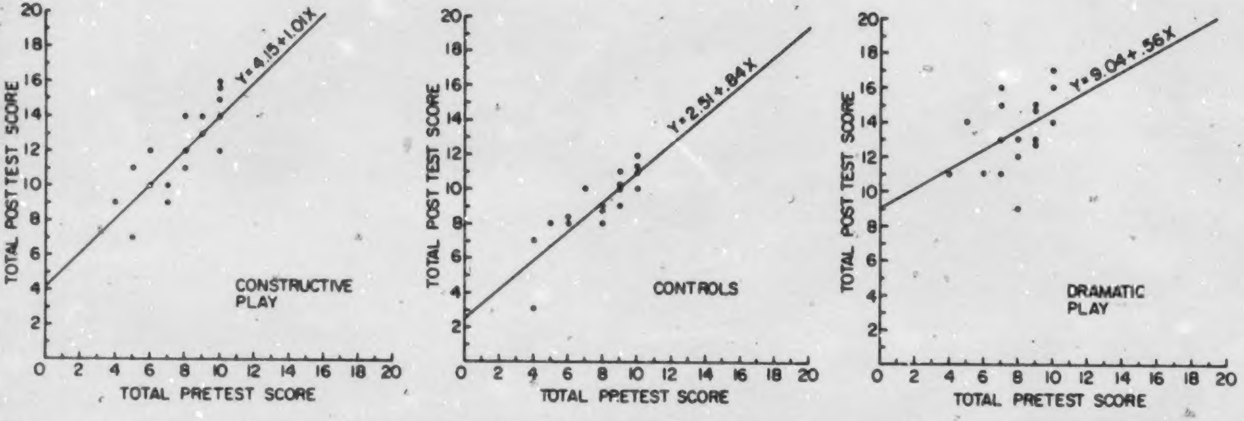

Figure 8-1. Pretest-Posttest Data for the Perspective-Taking Experiments Together with the Best-Fitting Linear Equations (Equations for all Three Groups Obtained by the Method of Least Squares)

constructive play and dramatic play groups should have risen more steeply than the line for the control group. However, the slopes were nearly identical and the slope for the dramatic play group was in the wrong direction. When the F-test for homogeneity of regression coefficients was computed, it was found that the three slopes did not differ significantly. Thus, experiments I and II did not produce any evidence that children's ability to learn perspective-taking concepts as a result of training is constrained by what they already know about these concepts.

The other three experiments were concerned with the learning of conservation concepts. The same battery of tests was administered as the pretest and the posttest in all three experiments. The battery consisted of two tests for number conservation, two tests for length conservation, two tests for quantity conservation, and two tests for weight conservation. There were nine items per concept, and the range of possible scores was 0–36. Each experiment spanned three days. The pretests were administered on Day 1, training was administered on Day 2, and the posttests were administered on Day 3. The only difference between the experiments was in the nature of the training trials on Day 2. In Experiment I, 30 kindergarten were given a series of training trials based on procedure developed by Gelman (1969). The procedure is basically a discrimination learning technique designed to induce children to ignore irrelevant perceptual cues. The training trials focused on number and length conservation but not on either quantity or weight conservation. In Experiment II, 30 kindergarten children were trained on number and length conservation (but not quantity or weight conservation) using a modeling technique based on a procedure developed by Rosenthal and Zimmerman (1972). Briefly, the treatment consisted of a 15-minute film strip in which an authoritative model (adult) gave correct judgments and logical explanations on a series of number and length tests. Finally, in Experiment III, 30 kindergarten children were trained on number and length conservation (but not quantity or weight conservation) using a corrective feedback procedure. This procedure was the same as the feedback technique used in several previous experiments that I have reported (e.g., Brainerd, 1972).

Two different control groups were run. The first control group was like the control group in the perspective-taking experiments; that is, they were simply given two administrations of the conservation battery, the first on Day 1 and the second on Day 3. The other group was run to control for repeated testing effects. In Experiments II and III, note that the subjects received additional exposure to number and length problems on Day 2. It may be that, quite apart from training per se, repeated conservation testing tends to produce some improvement in performance. Hence, the subjects in the second control group received a series of additional length and number problems, but without any actual training, on Day 2.

The pre-post data and the best-fitting regression lines appear in Figure 8-2. As was the case in the perspective-taking experiments, the fit of Equation 8-1 to the pre-post data was quite good for both control groups and for all three experimental groups. In fact, the fits were slightly better than they were in the perspective-taking experiments. None of the lines in Figure 8-2 accounts for less than 70 percent of the variance. These improved fits are probably a consequence of somewhat higher reliability for the conservation battery than for the perspective-taking battery. The value of r_{XY} was .92 for the pretest-posttest controls and .89 for the repeated testing controls. However, our main concern still lies with the question of relative slopes of the experimental and control group lines. Visual inspection of Figure 8-2 indicates that, like the perspective-taking experiments, the regression lines for these experiments are more or less parallel. When the three experimental groups are compared to the repeated testing controls, the direction of the slope difference is in the correct direction in two cases and the wrong direction in one case. When the experimental groups are compared to the pretest-posttest controls, the direction of the slope difference is in the correct direction in one case and the wrong direction in two cases. However, all of these differences are vanishingly small. When F-tests for homogeneity of regression coefficients were computed, there was no evidence of significant between group differences in slope.

Concluding Remark

The experiments whose findings I have just summarized all seem to show the same thing. Pre- to posttest improvements accruing from concept learning experiences are not related to pretest scores. Judging from the earlier discussion of Piagetian theory and CLD, this finding seems to be more consistent with CLD than it is with Piaget. However, the finding itself needs to be cautiously interpreted. Despite the fact that these experiments were designed to avoid the problems of ceiling effects and reliability that plague some other studies, it is quite possible that some extraneous source of Type II error is operating. There is the additional logical problem of attempting to interpret a negative finding.

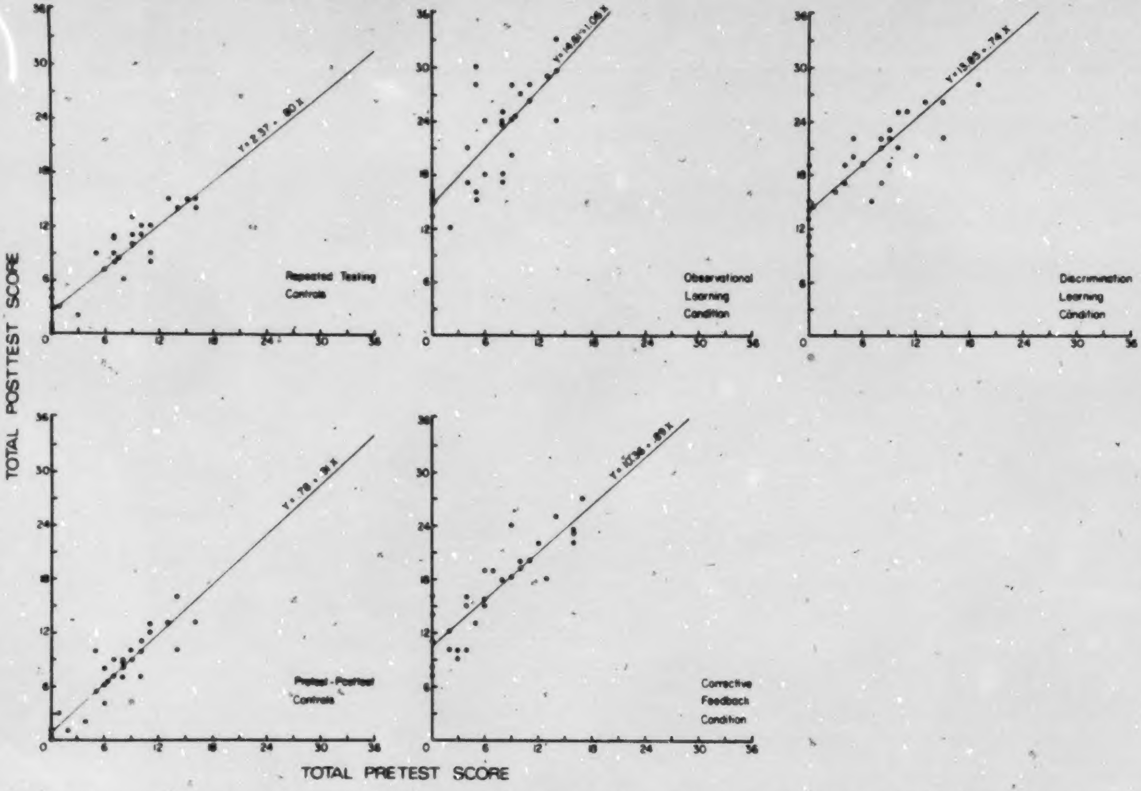

Figure 8-2. Pretest-Posttest Data for the Conservation Experiments Together with the Best-Fitting Linear Equations (Equations for all Five Groups Obtained by the Method of Least Squares)

Chapter 9

Educational Implications of Developmental Theory

Frank B. Murray

As early as 1825 James Carter was arguing that teaching was a science, or at least that it must be made a science, and that it must be based upon the study of psychology. The science of teaching was to be grounded in the study of

> . . . the philosophy of the infant mind, what powers are earliest developed, and what studies are best adapted to their development . . . the philosophy of the infant mind must be understood . . . before much progress can be made in the science of education; for a principal branch of the science consists in forming the mind . . . (Carter, 1926).

Such sentiments today drive educators to study Piagetian, CLD, and other theories of conceptual development with the same confident expectation that the art of teaching can be made more scientific by that contact with psychological science. The implications of Piagetian and CLD theories for educational practice can be examined profitably in terms of the universal relationship between all arts and sciences and the logical character of all implications.

Art and Science. Just as two points define a line, so do art and science serve as the endpoints of a linear dimension (see Table 9-1). Elements at one end are, by whatever criteria, more teachable than those at the other end because they are better known and formu-

Table 9-1. Relationships Between "Science" and "Art" Aspects of Performance as a Function of the Types of Rules Needed to Perform the Activity and Their Degrees of Formulation and Teachability

Dimension	*Levels of Rules*	*Levels of Teachability*	*Levels of Formulation*
"Art"	Rules for doing X brilliantly	Rarely teachable; learned perhaps by imitation or discovery	Rarely formulated or known
↕	Rules for doing X well	Sometimes learnable, usually with explicit instruction	Sometimes well-formulated but usually less explicit
"Science"	Rules for doing X	Learnable, sometimes with explicit instruction	Well formulated when known, explicit, and demonstrable

lated. The knowledge of anything in this sense can be viewed as composed of these teachable (scientific) and nonteachable (art) components. Plato in the *Philebus* made the same distinction as follows:

> Knowledge has two parts—the one productive, and the other educational . . . I mean to say if arithmetic, mensuration, weighing be taken away from any art, that which remains . . . will be only conjecture, and the better use of the senses which is given by experience and practice, in addition to a certain power of guessing, which is commonly called art . . . (Philebus 55).

Thus the rules that prescribe the doing of any activity can be thought of as of two sorts—rules that can be stated and taught (educational) and other rules that have not been formulated and of course cannot be taught but which guide performance nevertheless (productive). The linking dimension between sciences and arts is then a dimension composed of rules for doing them ordered by their degrees of clarity, specificity, teachability, and so on. At the "science" pole of the dimension the rules for doing an activity are well formulated and thereby demonstrable and teachable, while at the "art" pole the rules for doing it either do not exist yet or are so poorly formulated that they offer no real guidance for the performance of the activity. As academic disciplines have developed, the rules that describe and govern the performance of many of the "arts" have been discovered and formulated and to that extent these "arts" have become more "scientific" in the sense that larger components of their performance can be carried out by well-educated nonartists. Thus many aspects of musical composition can be carried out by people, even machines, schooled in well-formulated composition rules, which if followed yield acceptable musical compositions. At the same time musical composition remains closer to the art pole than the science pole because the most significant composition rules have not been formulated, although gifted composers follow them, almost surely without conscious awareness of them. When asked to describe the rules they use in the execution of their art, gifted composers (architects, teachers, physicians, engineers, performers, etc.) are often interesting, but uniformly unhelpful. The doing of science naturally is also governed by both types of rules, namely, procedures for doing science that are codified and teachable and other rules that if followed lead to scientific breakthrough, and so on, but which have not been codified or at least not in any useful way.

Related to this treatment of the art and science components of human activity as composed of teachable rules and nonteachable rules is a set of questions that bears upon the possible range of implications of psychology for teaching. The questions are (1) whether the skill is simply being performed, (2) whether it is being performed well, and (3) whether it is being performed superbly and brilliantly. It is one thing to follow rules that lead only to the doing of the thing, but it may be quite another to follow rules that lead to doing it well or even superbly. Thus, we wish to distinguish three levels of rules—(1) rules for doing the activity, (2) rules for doing it well, and (3) rules for doing it brilliantly. Consider the playing of the game of chess. The level 1 rules for playing the game are fairly easily taught and can be mastered by middle school children. If these level 1 rules are followed (viz., the rules of the game), one is playing chess; if not, some other game is played. Level 2 rules for chess have been formulated by chess theorists and can be learned, and when the player follows them, he or she could be said to be playing the game well (or at least better). These rules exist in various chess strategy and philosophy books and can provide useful guidance in mastering the game, although they are less directive than level 1 rules. Level 3 chess rules presumably exist but have not been formulated even though they are followed by chess masters whose behavior is of course assumed to be lawful and nonrandom. Thus only the masters and champions know these rules, and no doubt they know them in the unaware way most of us know, for example, the rules of the grammar that describe our speech and written composition. As one example, although no adult and few children fail to assign the appropriate plural allomorph to nonsense nouns, virtually no one knows the rule that describes and predicts his or her behavior in this language area. Should the artificial intelligence researchers be able to write chess playing programs that would best the grand masters, they would have formulated some level 3 rules.

Thus the scale of rules from the science pole to the art pole is in this analysis characterized by movement from the known to the unknown, the formulated to the unformulated, and from level 1 to level 3 rules. The distance each activity may be thought to be from the poles shifts, perhaps asymptotically, toward the science pole as research and scholarship advance in that field.

Now consider the activity of teaching and the sources of the rules for doing it, doing it well, and doing it brilliantly and the degree to which these have been formulated and are teachable. At what rule level do the various psychological theories provide information and guidance? To what extent can rules or practices be deduced from psychological theory that will distinguish teaching from nonteaching, teaching from teaching well, let alone teaching well from teaching brilliantly? What rules can be deduced from a theory such that, if they are followed, teaching as opposed to no teaching will result, or teaching well or optimally as opposed to simply teaching will result? This is the heart of the issue of the educational implications of psychological theory. No one claims that any contemporary psychological theory provides level 3 rules. While the derivation of level 2 rules from psychological theory has been claimed occasionally by theorists, some on the other hand, have argued the level 1 rules have not even been formulated let alone explained by or derived from any psychological or instructional theory.

The question of the derivation of rules and practices from a theory is not the same as the question of the initial formulation and development of rules and practices, unless of course the practice was discovered in the first place as an implication or deduction from a theory. This is a rare occurrence in the history of educational innovation and psychological theories as shall be examined in the next sections.

Deduction of Practice from Theory. William James in a series of talks to Cambridge teachers in 1892 articulated the link between theory and practice perfectly in his introductory lecture:

> . . . you make a great, a very great mistake, if you think that psychology, being the science of the mind's laws, is something from which you can deduce definite programmes and schemes and methods of instruction for immediate schoolroom use. Psychology is a science; and teaching is an art; and sciences never generate arts directly out of themselves. An intermediary inventive mind must make the application, by using its original-

> ity. . . . A science only lays down the lines within which the rules of art must fall, laws which the follower of the art must not transgress; but what particular thing he shall positively do within those lines is left exclusively to his own genius (James, 1958, pp. 22-23).

While James (1958), in presenting the matter to his Harvard students in his *Psychology* agreed that psychology was not yet a science, the point is made nevertheless that the generation of sound educational practices had its sources outside psychological theory. Psychology may eventually provide a rationale and explanation for the effectiveness of any educational practice even if, as James was quick to see, "many diverse methods of teaching may equally well agree with psychological laws" (James, 1958, p. 24).

"To know psychology, therefore, is absolutely no guarantee that we shall be good teachers," argued James (1958, p. 24) because the level 2 rules that guarantee good teaching have their sources elsewhere, namely, in "an additional endowment altogether, a happy tact and ingenuity to tell us what definite things to say and do when the pupil is before us . . . that tact for the concrete situation" (p. 24).

The validity of an educational practice is not guaranteed by its plausibility within nor its conformity to psychological theory even when the builders of the theory have created the educational technique. No better example of this fact exists than B.F. Skinner's programmed instruction innovation, that, although a proven effective teaching technique and often as effective as a classroom teacher, appears surprisingly not to lose that effectiveness when various theoretical requirements Skinner placed upon it are violated. Even though the theory that gave birth to programmed instruction insists upon a linear shaping sequence of items to which overt responses are made and immediately reinforced, programs that scramble frames randomly, delay or omit reinforcement, and require only covert responding are as effective as their theoretically sanctioned counterparts. The validity of educational practices, like engineering or medical practices, rests only in their solution to the problems to which they are addressed. Indeed very often the practice far outstrips an academic discipline's facility of explanation for it as witnessed by sound ancient engineering and medical practices that remain unexplained. Long before Euclid, for example, organized geometry as a coherent discipline, Greek surveyors had successfully employed the Pythagorean and other theorems in their trade. For the subject at hand it is clear that while the educational recommendations of Montessori are compatible with Piaget's theory or CLD theory, they were developed independently of it and are true or false quite apart from the truth or falsity of Piaget's theory or CLD theory.

This discussion of the relationship of theories and practices that may be deduced from them may be concluded with a brief treatment of the logical requirements of the relationship. These have significant implications for the establishment of the truth of theories. In the consideration of the truth of educational practices as they may be derived from a theory, we must be concerned with the stutus of these three factors in the relationship—(1) the correctness of the theory, (2) the correctness of the educational practices, and (3) the correctness of the deduction by which the first might generate the second. In practice we can determine only two of these (the practice and deduction) from which we can derive the third by the sets of logically permissable relationships of the truth of the parts of deduction to the truth of the deduction itself. These are portrayed in Table 9-2. The truth status of any two factors determines the status of the third. Thus it is clear from the table that the truth of a theory cannot be established by the demonstrated truth of an educational practice (or any fact for that matter) because both the practice and deduction can be true and the theory true (line 1) or false (line 2). In other words true educational practices may be deduced correctly from both true and false theories, which is another way of saying that the truth of an educational practice is not guaranteed by its place in a theory but by the careful and independent direct assessment of its effectiveness. A theory only can be shown to be false, never true, and then only when the deduction (line 3) is correct, but the practice claims that follow from it are false. As the correctness of theories is always in doubt, the business of science proceeds by the scientist's ability to think clearly, that is, formulate truthful deductions, and to establish the truth or falsity of practices and other facts. Even then all that can be accomplished is the falsification of a theory. The determination of the correctness of the theory is the overriding and inherently elusive goal of science since the scientist has control over just two factors in the truth table, namely, the deduction and the determination of the truth of the practice. Moreover, when theories are imprecise, proper deductions are not possible. In that case experimentation and educational innovations proceed as they usually do without the benefit of rigorous theoretical support. The foregoing considerations set the limits upon the educational implications of any theory, including, of course, any developmental theory.

A claim of this chapter is that Piagetian developmental theory, the preeminent prototype, is sufficiently imprecise in any case that no educational practices, however true and time honored they may

Table 9-2. Significance of the Possible Truth Relationships Among the Factors in the Deduction of Practice from Theory[a]

	Truth Status			*Interpretation*
Correct Reasoning Conditions	*"Theory*	*Practice"*	*Deduction*	
1.	True	True	True	Conditions 1 and 2 indicate that the truth of a theory cannot be established by correct reasoning and validated practice.
2.	False	True	True	
3.	False	False	True	Theory disconfirmed by failure of correctly deducted practice.
4.	True	False	True	This condition is not possible; false practice cannot be deduced from a true theory.
Incorrect Reasoning Conditions				
5.	True	False	False	Conditions 5, 6, and 7 are serendipitous in that true theories and true practices are arrived at by false argument. Nothing of value occurs in condition 8.
6.	True	True	False	
7.	False	True	False	
8.	False	False	False	

[a] Only practice, validated empirically (i.e. it works) and correct or true reasoning (deduction or implication) are under the scientist's control in the establishment theory. Progress in science rests solely in condition 3 since the outcomes of true reasoning and true practice are not theoretically unique (i.e. theory may be true or false in conditions 1 and 2).

be, can be derived uniquely from it. The theory does not permit exact deducible formulations for teaching, curriculum design, or educational assessment. That is, there are no educational implications that can be deduced from the theory that, if they failed as educational practices, would invalidate the theory. No contemporary psychological theories are stated with the precision needed to construct formal deductions in any case, and for that reason there are no educational recommendations derivable from any theory that, if proved false by empirical failure, would invalidate the theory (see Turner, 1965).

Since it does not seem possible to ground the statements of educational implications in formal deductive links to theory, a lower level criterion for an implication might be that it at least be compatible with the theory in the sense that it not be specifically proscribed by it.

While the criterion of compatibility with the theory is less stringent than deduction from the theory and while it consists of proscriptions rather than prescriptions for educational practice, theory still provides its traditional role as a heuristic for the generation of educational innovation and for prediction of likely educational outcomes.

THEORETICAL CLAIMS

It is clear that Piaget's account of intellectual development still progresses, and Piaget, now in his eighties, has modestly claimed that he has

> . . . laid bare a more or less evident general skeleton which remains full of gaps so that when these gaps will be filled, the articulations will have to be differentiated, but the general lines of the system will not be changed (Sinclair-deZwart, 1977, p. 1).

That the theory would be open-ended and tentative and subjected to major revisions by Piaget is in part a validation of it as a developmental theory that is developing. For example, his recent work, "The Equilibration of Cognitive Structures," in 1975 replaces and contradicts the earlier 1957 work, "Logic and Equilibrium," (Sinclair-deZwart, 1977). Equally forceful are the forthcoming modifications in the theory required by the research findings of hundreds of researchers with the result that if anything the surprising competence of the younger child and unevenness in adult thought has blunted the distinctions between the classic stages and softened the claims of the differences between children, adolescents, and adults.

In this sense CLD theory has anticipated these developments in Piagetian theory because it permits simultaneous development and growth on more than one conceptual level. Thus children may make simultaneous progress on identity and concrete conceptual levels and still preserve the theoretical hierarchy among the four levels of conceptual development. Moreover, the logical-structural account of the stages will need to give way to more powerful models (e.g., Osherson, 1974) and such work is underway (Sinclair-deZwart, 1977 and Chapter 5 of this volume).

Still the general lines of the system remain, and to understand the implications of Piaget's theory of intellectual development for educational practice, one must first understand that the theory is *not*, strictly speaking, a psychological or educational theory. It is not concerned primarily with the explanation and prediction of psychological or educational phenomena, although it may illuminate these phenomena. The theory has its origins in certain classic epistemological questions that it proposes to resolve through a treatment of how these philosophical concepts, dilemmas, and paradoxes have evolved and developed in thought. The point is that Piaget's work seeks to resolve through a method of "experimental philosophy," as Elkind (1976) has called it, certain philosophical issues, such as the reduction of causality to correlation, the priority or absoluteness of time or velocity, or the essence of number as a class or set or a relation.

The theory that results is not a theory of human behavior in exactly the same sense as grammar is not a theory of language behavior or the program of a chess-playing computer is not a theory of how humans play that game. The theory is, in other words, a competence model or theory and tells us only that we behave *as if* we were constructing the structures that Piaget claims define each of the stages. The properties of stages are in fact more properly properties of the theory than behavior and indicate only that the explanation for concrete operational thought (or period), for example, will not suffice for the kind of reasoning done later in life.

The basic theoretical constructs Piaget proposes for intellectual functioning are (1) a system of schemes or patterns of activity—not the activity or response itself—but rather its form, pattern, or structure, if you will, and (2) the balancing or equilibration of two mechanisms—one to conserve or preserve the system of schemes (assimilation) and the other to modify and transform the system (accommodation). It is a common mistake to think of these mechanisms, especially accommodation, as responses to "reality" as if there were some external reality independent of the reality embedded in the structural system and constructed by it. From this perspective, it may be clearer why or how Piaget claims that while maturation, learning, and imitation are necessary for cognitive change, they are insufficient for it; and why Piaget's constructs should not be reified. They do not really exist as material or efficient causes and have the same relation to intelligence as grammar does to speaking.

The appropriate model of equilibration is not the beam balance, but the analytical balance in chemistry. It is a balance of a self-regulated set of simultaneous interactions between the elements of the system in which, so to speak, the swings of balance continually increase in amplitude and power and do not dampen to a prior equilibrium. The theory suggests that the difficulties in school learning are more likely to be consequences of incomplete structure than faulty or distorted information. A *major* contribution of

Piaget's viewpoint to education is merely the provision for a way to conceptualize the instructional problem as the interaction and integration of perceptual and linguistic content (images, figures) with a system of cognitive or intellectual operations or structures. These seem to be best described as a mathematical group because of their ability to produce any member of the system from any two and to convert any member to an identity element.

Education or schooling is largely, and perhaps to some extent mistakenly, concerned with the acquisition of information (what is the country's population, capital?) and skills (3 Rs), while Piaget's theory is concerned with the development of knowledge and thinking, that is, things we know are true because they *must* be true, they have to be true, because they are consistent, have coherence, and can be deduced from other things. The relationship between information and knowledge, content and structure, is subtle and profound. Whatever implications there are from genetic epistemology for education, they deal only with the knowledge of things that are necessarily true, that are always true, that have to be true, and the process by which such knowledge develops. The relation and dependence of this knowledge upon information and fact, arbitrary as it might be, is the major research and theoretical problem of the decade for the behaviorial sciences and education. CLD theory does not have this restriction upon the concepts it treats and thus has the potential for providing wider ranges of information about concepts that in fact are in the curriculum. Still the range of concepts the Genevans have researched over the last 50 years has sampled the major curriculum areas well. Even though the Genevan intention was not to provide such a corpus, one by-product of their research of necessary knowledge is this corpus on the development over the school years of many important concepts in mathematics, logic, social studies, ethics, politics, physics, chemistry, biology, psychology, language, and fine art.

Another important claim in this theory, and perhaps its most controversial one, and one not shared by CLD theory, is that knowledge develops out of systems of action and not language. Language is taken to be a poor medium for developing knowledge, especially before adolescence, while for CLD theory it is the prime medium for the transition to the formal conceptual level. Still the two positions merge in the obvious, although not original or unique, educational implication that teaching by telling is inadequate unless a provision for the acquisition of the linguistic referents has been made previously.

In the main, the practical implications of Piaget's theory for education have already been made by others, and in fact what has really happened is that the theory has provided a theoretical justification for a number of long-standing educational innovations. For example, any instructional innovation, like progressive education, Montessori, open education, or discovery learning, for example, that makes a major provision for the self-initiated and self-regulated activity of the pupil can be justified in the theory. On the other hand, one overriding curricular recommendation from the theory is that the demands of the curriculum sequence should match the competencies of the pupil's stage of cognitive development. That the curriculum sequence should be based upon the psychology of the developing child is not a novel idea, of course, but what makes it interesting in this case is that Piaget and his colleagues have made specific claims about the order in which specific curricular concepts and competencies develop. While CLD theory also permits asynchronous development of concepts, it makes no prediction about the order even though it makes exact predictions about the order in which the various components of the concept are manifested.

PIAGETIAN CURRICULA

Piaget's acceptance in the American educational and intellectual establishment is marked by many events, not the least of which were the creation of the Jean Piaget Society in 1971; honorary degrees awarded at the Universities of Pennsylvania (1967), New York, Michigan (1968), Chicago (1974), and Clark (1968), Yale, Columbia, Catholic (1970), and Rockefeller (1975) Universities; and prizes from the American Educational Research Association (1967), the American Psychological Association (1972), and the Kittay Foundation (1973).

Two social crises in the United States and psychology's preoccupation with S-R and S-S animal learning and psychometry made the adoption of Piagetian theory virtually inevitable. The first was the post-1957 Sputnik concern of a nation preoccupied with its diminished role as a scientific and engineering leader, and the second, 10 years later, was the civil rights crisis in which a nation grappled with the need for compensatory education for its poor and other minority citizens. The need to educate a new generation of scientifically literate and productive citizens led to the initiation of the "structure of the discipline" curriculum projects of New Math (Beeberman), Minimath (Rosenbloom), Science Curriculum Improvement Study (Karplus), Project Physics (Zacharias), and Man: A Course of Study (Bruner), all of which were based upon Piaget's notions of structure and stages of intellectual development. Given that the content of Piaget's theory and research was largely children's reasoning in mathematics, science, and logic and the lack of similar content anywhere else in psychology, it was all but inevitable that Piaget would dominate these curricular reforms.

Similarly, given that Piaget's theory of intelligence claims an hierarchically ordered sequence of acquisitions from birth onward that places great stress on sensorimotor roots of all intelligence, it was also inevitable that day-care and other preschool planners would turn to this theory (e.g., summarized in Maccoby & Zellner, 1970; Hooper & DeFrain, 1974; Schwebel & Raph, 1973). The results were principally the Early Childhood Curriculum (Lavatelli), the Cognitively Oriented Curriculum (Weikart), Piaget for Early Education (Kamii & Devries), Thinking Goes to School (Furth & Wachs), and the Piagetian Preschool Educational Program (Wisconsin). In searching for sets of teachable skills for Headstart and compensatory education projects that would provide maximal positive generalized transfer to school learning tasks, many planners selected basic cognitive and intellectual skills. Here again Piaget's tasks and theory provided an attractive alternative to pragmatic, atheoretical IQ skills of the prevailing psychometric approach to understanding intelligence.

As the British Infant School movement had already been rationalized in terms of Piagetian theory, Silberman's insistent call for its transplant to America in *Crises in the Classroom* (1970) only reinforced Piagetian reform in American primary education.

It is ironic that the way for Piaget in education was paved by the demise of the progressive education movement with its preparation for life and social adaptation goals (Cremin, 1961). Its emphasis on the social studies perhaps contributed to its unpopularity during the post-Sputnik attacks on American schools. The irony is that Dewey's and Piaget's theories are remarkably similar (e.g., Seltzer, 1977), and the ostensive educational recommendations of child-centeredness, prerequisite of interest as condition of learning and development, and active discovery in place of passive listening are shared by both.

Perhaps the most ironic aspect of Piaget's acceptance by the educational reformers of the last 20 years was the vehicle by which he became widely known, namely, Bruner's *The Process of Education* (1961). The book contained a famous hypothesis that, although offered as compatible with Piaget in theory, clearly could be seen as contradictory to it. "We begin," writes Bruner (1961, p. 32), "with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development" and with the corollary notion of a spiral curriculum that holds as its criterion for a subject matter "whether, when fully developed, it is worth an adult's knowing" (p. 52). The issue is begged somewhat by the phrase "some intellectually honest form," but one clear feature of Piaget's account of the child's conceptual development is that the qualitative leaps in development make many later ideas inherently unavailable to the younger child. Bruner's hypothesis ignores the epigenic emergent assumption of Piagetian and other organismic developmental accounts. On the other hand, there is a plausibility in Bruner's hypothesis that only points up again the difficulty in deriving educational practices from theory. Neither Bruner's hypothesis nor its opposite (viz., that by viruie of epigenesis some subjects cannot be taught with integrity) is proscribed by the theory. As Elkind (1976) sensibly observes, the issue is not whether the geoboard activity is an intellectually honest form of geometry but whether it is interesting and contributes to the development of spatial concepts or whatever.

Not to put a fine point on the matter, there are at least two other somewhat contradictory recommendations from the theory that are consistent with it. One recommendation is that instruction should "match" (Hunt, 1961) in some artful way the level of cognitive development the child has attained. Bruner's (1961) hypothesis explicitly provided for a match between the child's cognitive structure and the character of the instructional effort. In the early grades, for example, instruction and curriculum would recognize and push the limitations of preoperational and highly egocentric thought and would not presuppose or demand, for example, hypothetical and inferential thought, or awareness of proportionality. The theory predicted that not until early adolescence would cognitive structure have developed to assimilate less concrete and more logically complex forms of information. Moreover, instruction in concepts before the prerequisite schemata have developed could result in serious "misacquisitions" (Flavell, 1963, p. 365) often indicated by a superficial verbal fluency such as "counting" without number conservation.

The second recommendation requires that instruction *not* match the level of the child's intellectual development at first, but rather that it begin at the most primitive and basic levels of cognitive development and move quickly through the stages of the pupil's intellectual history. This recommendation has been made by Piaget's colleague, Hnas Aebli, and is reminiscent of Werner's orthogenic and microgenic principles applied to the learning act in which the acquisition of a subject matter would recapitulate the stages in the development of the intellect. Since the principal concept in Piaget's theory is that mental operations are the result of the gradual transformation of overt actions, Aebli has suggested that instruction

> might begin by having the child operate directly on physical entities, then on pictorial representations of these entities, then have him proceed to cognitive anticipation and retrospections of operations not actually performed at the moment, and so on, until the originally external actions can take place internally and in complete autonomy from the environment (Flavell, 1963, pp. 368-369).

The difficulty, not only of deriving nonconflicting and specific educational practices from the theory, but also of discovering specifically prohibited educational practices, is apparent. Other than absurd practices that no theory recommends, specific proscriptions are more difficult to derive than might be imagined. Teaching by telling cannot be clearly proscribed, for example, because of the effectiveness of verbal rule conservation training techniques (Beilin, 1971c). In a recent paper Beilin (1977) has argued for the compatability of verbal rule instruction and Piaget's theory. Ironically Brainerd (1973a and 1974) argued the Genevans violate their own theory by insisting, as they do, that the child support his or her conservation judgment with a reason, which appears to be the final testament in the difficulty in deriving straightforward practices from Piagetian theory.

One might imagine an instructional recommendation, like programmed instruction, even if it is not as secure in operant theory as was once thought, would be sufficiently incompatible with Piagetian theory as to be not recommended. Indeed Inhelder, Sinclair, and Bovet (1974) have concluded that "programmed learning runs counter to the idea that for true learning to occur the child must be intellectually active" (p. 26). Let us consider this question of compatibility of programmed instruction and Piagetian theory.

ON THE PEDAGOGICAL SIMILARITY OF OPERANT AND PIAGETIAN THEORY

Are there instructional recommendations from the operant model, for example, that are inherently different or incompatible with those from the Piagetian model? One would think so, but the differences operationally between these models' pedagogical recommendations could be very small indeed. As an example, in the *1942 NSSE Yearbook* some leading learning theorists of the era were invited to discuss the implications of their theories of learning for educational practice. Guthrie had the lead article, which he called "Conditioning" and in which he described learning as only the contiguity of stimulus and response and cited many clever educational applications of his position. His paper was followed by Clark Hull's whose theory of learning was antithetical to Guthrie's in many important respects. After about 30 pages, Hull concluded:

> Because of the severe limitations on the space available for the present chapter no attempt will be made to draw detailed educational maxims from the preceding theoretical outline. Despite certain sharp differences between the primary assumptions of Professor Guthrie's chapter and this one, the two systems have a strong kinship. If practical morals or recommendations were to be drawn from the present system, they would agree almost in detail with those put forward by Professor Guthrie (Hull, 1942, p. 93).

The clearest educational recommendations from the operant theory for education are programmed instruction, computer system instruction (which is the technological extension of the other), contingency managed classrooms, and token economies. A most often cited educational recommendation from Piagetian theory (other than the previously mentioned curriculum innovations) is discovery learning. The notion is that the information the child discovers for himself or herself will be remembered longer and learned easier than information acquired in some other way. However, when one teaches by the discovery method, one does very much the same kind of thing one could be doing when one teaches by the operant method. What one does in both cases is to arrange the environment in some way so that some responses are more likely than others to occur. By careful arrangement of the materials and directions in either model the child moves from his or her position to some position that is the goal of instruction. Finally, upon arrival, some means for informing the child that he or she is there is provided in both models either by an extrinsic device or by an intrinsic feedback or reinforcement from the materials when he or she arrives at a position that fits in some coherent way with what he or she already knew. That feeling of coherence is presumably gratifying, and this occurs in both pedagogical procedures and is not a peculiar feature of only one of them, although only one dares to describe its cause.

In a very real sense the rat in the Skinner box discovers the solution to its thirst as the laboratory scientist is programmed by his or her laboratory, and so on.

What of the case where the goal of instruction is really unanticipated, where not enough is known to predict what the child is going to do, and where the teacher may not even know what the child is to do as in a creativity lesson where the goal is for some creative or novel behavior to occur. In this case the teacher probably should be a human being because it will take a human being to recognize the validity of a particular response, unanticipated as it was at the beginning, and evaluate it, or praise it, or shape it in some way. That not enough is known about what someone will do in a situation to anticipate and program for all the possible outcomes is a limitation in both the pedagogy of discovery and programming and does not provide grounds for choosing one over the other. If a thing can be taught, it can be programmed, and it can be programmed in such a way as to be indistinguisable from a discovery learning exercise. The procedural operations of discovery and programmed instruction can be identical as may be the thoeretical operations Piaget hypothesizes to explain the child's responses to both situations.

The differences between theories may not lead to different pedagogies but surely to different explanations of teaching. Both Piaget and Skinner, for example, have different but adequate explanations for Montessori teaching. Neither explanation need influence Montessori practice, and Montessori practice is compatible with both theories. This should not be a particularly surprising state of affairs as there are many instances in science and other disciplines where different theoretical positions yield the same practical recommendations, but of course for different reasons. Both Ptolemaic and Copernican astronomies yield the same navigational practices, for example. All learning theories recommend the level 1 rules that teachers make provisions for presentation of the item to be learned, the pupil's response to it, and reinforcement of that response.

There is nothing in the operant model that requires that the teacher be *the* source of knowledge nor is there a requirement that the child not construct knowledge from his or her own actions. Teachers, especially in the arts or athletics, may be able to "teach" their pupils behaviors they themselves do not have, nor may never have had, and pupils may be guided by the teacher to ideas or insights that the teacher has never had.

That various models result in different or identical educational practices is based partly on the fact that the connections between theory and practice are not deductions, but vague and imprecise analogies. However, it is possible that the teacher's *belief* in a model may influence the style of teaching, and in this regard the different theories may lead to different pedagogical results (Stephens 1967).

If one believes that a person is merely a conditionable organism, then one may treat people as though they were only conditionable, thereby influencing unanticipated features of the teaching situation. Similarly, if a teacher were to adopt a Piagetian perspective, he or she could become sensitized to different things in the classroom from the operant teacher. That there may be important differences in the behavior of operant and Piagetian teachers as their behavior is directed by their beliefs in the two theories may be as much a testimony of the restrictive interpretations each makes of the possibilities inherent in and compatible with the theories as it is of true differences in the models' consequences.

LEGACY OF PIAGETIAN THEORY FOR CURRICULUM

A major issue in curriculum design is the sequence of subject matters since, if for no other reason, everything cannot be learned at once. On this issue Piagetian theory would appear to be most useful, containing, as it does, strong claims about the invariant sequence of intellectual development. While the dual recommendations of matching curricular demand and cognitive development or recapitulating that development in each learning episode have been mentioned, the long span of time in the major stages can of necessity give little guidance to sequences within grade levels in any case. The well-known horizontal decalage phenomena, on the other hand, do mark sequences within the stages and could be of some use, but they cannot be derived from the theory because the theory does not explain them. The horizontal decalage phenomena are theoretical anomalies and threats to the theory as it is presently formulated. The theory is by this fact of little use in the sequencing of the parts of new subject matters except to caution the educational planner to expect these structural anomalies.

These within stage lags or sequences of conceptual attainment are reliable but not predictable *a priori*. They are chiefly testaments to a well-known instructional phenomenon in which students' mastery of a principle in one area of the subject fails to transfer to other areas, even to those strikingly similar to the mastered area. While Piagetian research has delineated a number of such arresting sequences, which in fact define the concrete operational period, without a theory to link them, their significance for education remains clouded.

While the sequencing of curricular content is an inescapable matter, the prior issue of curriculum development is, of course, the nature of the content itself. No psychological theory has statements in it that prescribes curricular content. To the extent that the Piagetian tasks themselves become curricular elements, the theory contributes, along with the results of correlated research, to an understanding of the factors that are related to the child's performance on the task. At the very least the theory describes the intellectual problems and constructions on which the child is working at various ages. Should the school view its role as an assistant to natural conceptual development, the theory provides guidance for that role (See Varma & Williams, 1976) by identifying the contents and course of that development. To make the Piagetian tasks part of the curriculum is probably misguided as these tasks are mere diagnostic techniques for revealing the schema or operational structures that give the child's thought coherence. It is analogous to teaching IQ items in an attempt to develop intelligence. To teach the properties of the logical models that describe intellectual operations as an attempt to foster intellectual development is equally wrongheaded in that it confuses competence and performance models.

No doubt much of the attention Piaget has received has been due to the surprising errors he discovered in children's conceptions. Presumably equally arresting errors remain to be discovered in children's understanding of other concepts in the traditional curriculum. The continued diagnosis of these peculiarities is recommended along the same clinical and critical exploration method lines that Piaget and his collaborators initiated and employ. However, the research program of CLD yields equivalent information about these concepts.

The analysis of curriculum concepts from this perspective might result in models of the concept such as one proposed by Murray and Johnson (1975) for the concept of weight or those advocated by CLD theory. The model may be extended to any curricular concept and describes the relevant (or "logical") conditions under which an attribute changes, and some of the important irrelevant conditions under which it does not change. The source of the relevant conditions is the appropriate academic discipline. The source of the irrelevant conditions is not clear, but what makes these factors important is that they are taken as relevant conditions by students. Why they are is a matter of psychological research.

At present there is no adequate description or theory of these conditions. Of all the transformations that can be performed on an object, why are some taken as actions that change weight and some taken as actions that do not change the weight of an object? It has been speculated by Nummedal and Murray (1969) and Haldas and

Murray (1974) that the source of these important irrelevant conditions or transformations is the connotative meaning of the concept in question. Such was the case in the Nummedal and Murray (1969) investigation in which changes in a clay ball's hardness, size, roughness, and strength were all thought by young children to affect the ball's weight and were all connotatively related to "heavy" and "light."

This approach provides no prescription to the curriculum writer for correcting these misconceptions; rather, it merely points out that these errors exist and that perhaps they should be treated.

LEGACY OF PIAGETIAN THEORY FOR INSTRUCTION

Between 1961 and 1978 about 150 research studies were published that were designed specifically in one way or another to train young children between four and seven years old to conserve. Quite apart from the theoretical reasons that may have motivated and legitimized the attention scientific journals gave to this issue, the thesis of this concluding section is that in these studies a number of precise teaching techniques were created and more importantly were evaluated. Moreover, the extensive experimental literature on conservation, of which the training literature is a small part, provides serendipitously a model for programmatic research in educational psychology. From its inception at the turn of the century, educational psychology was planned as the discipline of the psychology of the various subject matters in which variables that predicted and controlled the acquisition of subject matter would be charted and explained. The claim that teaching could be a science was to be based upon this new discipline. For various reasons, the discipline wandered in the following decades from this central purpose; but the developmental psychologists have, perhaps unwittingly, followed the program faithfully in their evaluation of the conservation phenomenon.

While conservation researchers have limited themselves generally to the concepts Piaget researched originally (e.g., number, amount, length, area, weight, time, and volume), this limitation has been self-imposed because the conservation paradigm can be applied without exception to any concept. When it is, it provides a diagnostic technique for measuring a person's understanding of a concept that is as old as Platonic dialectic in which various concepts such as justice, virtue, and goodness were each subjected to a series of relevant and irrelevant transformations by which their essences were finally apprehended. The point is that there is nothing in principle to distinguish the traditionally researched conservation concept from any other that might be of interest to the curriculum developer.

In addition, researchers have restricted themselves generally to the types of transformations used by Piaget and his colleagues (viz., shape and position). When this limitation, too, has been discarded, it becomes possible to chart other important and unsuspected—yet arresting—errors in children's thinking (e.g., fathers that are drunks are no longer fathers, Saltz & Hamilton, 1968; making a clay ball colder makes it heavier, and warmer makes it lighter, Murray & Johnson, 1975).

A number of reviews of the conservation training literature exist (e.g., Beilin, 1971c and 1977; Brainerd & Allen, 1971; Glaser & Resnick, 1972; Goldschmidt, 1971; Peill, 1975; and Strauss, 1972, 1974 to 1975). Because Piagetian theory had claimed that conceptual development was under the sufficient control of unique and largely unmodifiable structural mechanisms, the intent of the early studies was to demonstrate that conservation could be trained and was amenable to conventional learning procedures. The failure of the first dozen training attempts (Flavell, 1963) undoubtedly motivated the subsequent attempts with the result that there is no longer doubt that conservation can be taught (e.g., Beilin, 1977). Still there is an overwhelming and somewhat pessimistic result that training—even highly individualized training—is only successful (by whatever criteria) with about half the children in the sample, or more precisely that the children make about half the gains in conservation performance that could be made, although the gains are stable for as long as a month and are significantly different from pretest or control group subjects' performance (Murray, 1978). This result seems to be as true for procedures that take as little as five minutes as it is for those that have taken an hour.

It also seems to be a well-supported, but not surprising, principle in this literature that the training effort needed to bring a behavior to criterion is most successful with behaviors that were initially closest to the criterion. The evidence for aptitude treatment interactions in this literature, like all others, is scant, but there remains the theoretical issue of whether training is constrained by the child's developmental level. The Genevans (Inhelder, Sinclair, & Bovet, 1974) claim training is only successful with transitional children who give prior evidence of some conservation—either with a *simpler* concept, content, or transformation. Murray (1972) and Murray, Ames, & Botvin (1977), however, have reported no significant differences in the training gains between nonconservers who had exhibited no pretest conservation and those who had exhibited some. Brainerd (1977) concluded that to date there is no compelling evidence that training is constrained by preoperational or transitional cognitive level in that each receives equal benefit from training.

It should be noted that many of these training strategies have face validity as classroom techniques and in many instances would seem to be a teacher's first response to a child who thought, for example, that the number of objects in a patterned array changed when the pattern changed.

The chief legacy of Piagetian theory for instruction has been and will continue to be the development and evaluation of specific training techniques for various school concepts even if the success of the technique may be more parsimoniously explained by another theory. The value of a theory, and ultimately its truth, is in the uses to which it is put. Piagetian theory's value to education is by this criterion not in dispute if even specific educational practices are not deducible from it. Such logical entailment is rare in science, and virtually absent in the behavioral sciences, which leaves the discipline of education with a number of practices that are, while not derivable, at least consistent with the theory insofar as their not being specifically proscribed by it.

In summary, the only certain educational recommendation from a theory such as Piaget's is that schools should simulate "natural" human development; not that schooling is unnatural but rather that it promotes intellectual growth best when it is based upon natural mechanisms of intellectual development. These Piaget has tried to describe, and they suggest that schools are good when they place a high instructional premium upon self-initiated and self-regulated "discovery" learning activities in situations that demand social interactions and a higher curricular premium on thinking and knowledge than on learning information and skills.

However, these summary recommendations have only the form of level 2 or 3 rules without their guidance. "Nothing is simpler or more definite than the principles of (pedagogy)" argued James in this context.

> . . . in teaching, you must simply work your pupil into such a state of interest in what you are going to teach him that every other object of attention is banished from his mind; then reveal it to him so impressively that he will remember the occasion to his dying day; and finally fill him with devouring curiousity to know what the next steps in connection with the subject are (James, 1958, p. 25).

While James concluded that "divination and perception" were the only helpers in the application of these principles, it is clear that the needed assistance will not come from theoretically derived or compatible application rules. An analysis of the specific conditions of development of curriculum concepts and their instructional modification is more likely to be a richer source of the higher level rules. CLD theory has promise in this regard since it focuses upon specific levels of curriculum conceptual development (Klausmeier & Hooper, 1974; Klausmeier & Allen, 1978).

The implications of CLD theory for instructional and curricular design have been developed elsewhere (viz. Klausmeier & Allen, 1978) and have not been treated in any detail here for that reason. The strategy of the theory's proponents, however, "has not terminated with stating implications for improving instruction" (Klausmeier & Allen, 1978, p. 276), but has focused upon the implementation of the implications as controlled experiments in school settings. The strategy entails a description of conceptual development over the school years as described by CLD theory, the design of curriculum materials and instructional procedures, and the reform of the schools that avoids the excesses of open education, the prescriptions of programmed instruction based upon operant theory, and the traditional age-graded classroom group instructional format.

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Appendix

Table A-2-1. Longitudinal Block and Grade at Which the Test for Each Level and Each Use of Each Concept Was and Was Not Administered

Longitudinal block and grade	*Battery*	*Test*^{a}						
		CO	*ID*	*CL*	*FO*	*PR*	*TAX*	*PS*
A1	CT	A	A	A	A	A	A	A
	TR	A	A	A	A	*	A	A
	ET	A	A	A	A	*	A	*
	NN	A	A	A	*	*	*	*
A2	CT	A	A	A	A	A	A	A
	TR	A	A	A	A	A	A	A
	ET	A	A	A	A	*	A	*
	NN	A	A	A	A	*	*	*
A3	CT	A	A	A	A	A	A	A
	TR	A	A	A	A	A	A	A
	ET	A	A	A	A	A	A	A
	NN	A	A	A	A	A	A	A
B4	CT	A	A	A	A	A	A	A
	TR	A	A	A	A	A	A	A
	ET	A	A	A	A	A	A	A
	NN	A	A	A	A	A	A	A
B5	CT	A	A	A	A	A	A	A
	TR	A	A	A	A	A	A	A
	ET	A	A	A	A	A	A	A
	NN	A	A	A	A	A	A	A
B6	CT	A	A	A	A	A	A	A
	TR	A	A	A	A	A	A	A
	ET	A	A	A	A	A	A	A
	NN	A	A	A	A	A	A	A
C7	CT	**	**	A	A	A	A	A
	TR	**	**	A	A	A	A	A
	ET	**	**	A	A	A	A	A
	NN	A	A	A	A	A	A	A
C8	CT	**	**	A	A	A	A	A
	TR	**	**	A	A	A	A	A
	ET	**	**	A	A	A	A	A
	NN	**	**	A	A	A	A	A
C9	CT	**	**	A	A	A	A	A
	TR	**	**	A	A	A	A	A
	ET	**	**	A	A	A	A	A
	NN	**	**	A	A	A	A	A
D10	CT	**	**	A	A	A	A	A
	TR	**	**	A	A	A	A	A
	ET	**	**	A	A	A	A	A
	NN	**	**	A	A	A	A	A
D11	CT	**	**	A	A	A	A	A
	TR	**	**	A	A	A	A	A
	ET	**	**	A	A	A	A	A
	NN	**	**	A	A	A	A	A
D12	CT	**	**	A	A	A	A	A
	TR	**	**	A	A	A	A	A
	ET	**	**	A	A	A	A	A
	NN	**	**	A	A	A	A	A

[a] Single asterisk indicates test not administered; previous research showed that children at this grade responded at a chance level. Double asterisk indicates test not administered; previous research showed 100% attainment at this grade.

Table A-2-2. Hoyt Reliability Coefficients and Standard Errors of Measurement for Grades 1, 4, 7, and 10 Combined in 1974

Test	*Cutting tool (N = 349)*		*Tree (N = 354)*		*Equilateral triangle (N = 351)*		*Noun (N = 362)*	
	Hoyt	SE_M	*Hoyt*	SE_M	*Hoyt*	SE_M	*Hoyt*	SE_M
Concrete	.51	.19	.16	.29	.46	.28	.63	.45
Identity	.53	.22	.15	.36	.26	.28	.85	.59
Classificatory	.63	.52	.84	.63	.71	.30	.91	.85
Formal	.77	1.39	.91	2.10	.83	1.64	.92	1.87
Combined levels	.78	1.62	.92	2.39	.83	1.88	.95	2.38
Principle	.73	1.26	.93	1.62	.91	1.12	.83	1.15
Taxonomic Relations	.58	.91	.72	1.01	.69	1.10	.78	.99
Problem Solving	.58	.80	.67	1.24	.79	.83	.67	.75
Combined uses	.82	1.87	.94	2.38	.93	1.91	.91	1.78

Table A-4-1. Mean and Standard Deviation for Each Level and Each Use of Each Concept for Combined Longitudinal and Control Group Students, Grades 1-12

Battery and Test	*1 (N = 126)*		*2 (N = 84)*		*3 (N = 82)*		*4 (N = 151)*		*5 (N = 113)*		*6 (N = 77)*	
	Mean	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*
CTco	7.90	.35	7.90	.37	8.00	.00	7.97	.21	7.91	.76	8.00	.00
CTid	7.87	.38	7.88	.45	8.00	.00	7.93	.25	7.91	.76	8.00	.00
CTcl	7.46	.97	7.64	.90	7.84	.58	7.70	.85	7.81	.82	7.99	.11
CTfo	12.67	2.94	13.12	2.49	14.47	2.13	14.39	2.59	15.22	2.12	16.49	2.74
CTpr	4.65	1.68	3.98	1.69	5.27	1.63	5.82	2.38	5.99	2.37	7.51	2.32
CTtax	4.85	1.94	5.11	1.88	5.71	2.02	6.47	1.31	6.45	1.33	7.13	1.12
CTps	2.65	1.13	2.46	.95	3.31	.90	3.58	1.10	3.70	1.05	3.78	1.03
TRco	7.57	.69	7.60	.62	7.97	.25	7.79	.49	7.83	.44	8.00	.00
TRid	7.63	.68	7.62	.77	7.87	.50	7.80	.40	7.78	.55	8.00	.00
TRcl	6.40	1.92	7.35	1.30	7.61	1.31	7.35	1.40	7.61	1.01	7.82	.81
TRfo	16.99	4.66	20.07	3.98	18.53	5.74	21.76	5.43	23.48	6.38	23.55	7.29
TRpr	NA[a]		.83	1.04	5.61	3.96	7.79	3.38	8.84	3.68	9.29	5.08
TRtax	1.83	1.02	2.45	1.12	3.73	1.83	4.28	1.53	4.63	1.69	5.69	1.76
TRps	2.63	1.87	3.57	2.08	4.32	1.84	3.26	1.75	3.96	1.83	5.32	2.21
ETco	7.86	.47	7.94	.24	8.00	.00	7.89	.42	8.00	.00	8.00	.00
ETid	7.82	.54	7.98	.22	8.00	.00	7.86	.43	8.00	.00	8.00	.00
ETcl	2.46	.94	2.82	.58	2.95	.28	2.61	.86	2.80	.48	2.96	.25
ETfo	6.61	2.65	7.98	2.60	9.94	2.81	10.73	2.92	11.96	2.71	14.26	2.40
ETpr	NA[a]		NA[a]		4.85	2.07	5.21	2.60	6.26	2.26	8.51	1.82
ETtax	2.29	1.19	2.77	1.19	3.63	1.94	4.52	1.73	4.95	1.54	5.96	1.87
ETps	NA[a]		NA[a]		3.27	1.45	2.58	1.56	2.91	1.57	4.18	1.45
NNco	7.00	1.04	7.77	.61	7.77	.69	7.92	.29	7.99	.09	8.00	.00
NNid	5.28	1.97	7.43	1.03	7.63	.91	7.86	.40	7.96	.21	8.00	.00
NNcl	.22	.70	4.01	2.02	5.79	2.01	4.99	1.92	6.00	1.75	6.26	1.73
NNfo	NA[a]		1.63	.83	8.06	2.92	8.94	3.05	11.04	3.39	12.60	3.35
NNpr	NA[a]		NA[a]		2.73	1.81	3.13	1.81	3.58	1.81	4.34	1.98
NNtax	NA[a]		NA[a]		2.92	1.53	3.08	1.62	3.79	1.25	4.30	1.27
NNps	NA[a]		NA[a]		1.19	.96	1.28	1.03	1.65	1.16	2.08	1.18

	7 (N = 148)		8 (N = 112)		9 (N = 80)		10 (N = 145)		11 (N = 105)		12 (N = 73)	
	Mean	*S.D*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D*
CTco	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
CTid	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
CTcl	7.59	1.00	7.72	.76	8.00	.00	7.80	.49	7.74	.64	7.75	.76
CTfo	14.57	3.11	15.22	2.89	16.55	2.44	16.50	1.90	16.41	1.89	17.06	1.97
CTpr	6.28	2.48	6.64	2.30	7.90	2.16	7.94	2.57	7.98	2.44	8.14	1.98
CTtax	6.59	1.50	6.62	1.53	7.35	1.15	7.29	1.09	7.36	.91	7.59	.85
CTps	3.19	1.53	3.66	1.31	3.75	.83	4.10	1.21	3.83	1.36	3.96	1.21
TRco	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
TRid	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
TRcl	7.49	1.35	7.79	.79	8.00	.00	7.74	.87	7.69	.98	8.00	.00
TRfo	25.95	5.34	26.10	5.42	30.39	4.74	29.41	4.28	30.01	4.22	31.73	4.11
TRpr	10.76	3.97	11.27	4.24	14.94	3.88	14.51	4.40	15.58	3.94	15.99	3.71
TRtax	4.97	1.45	5.60	1.43	7.15	1.34	5.76	1.62	6.21	1.59	6.85	1.58
TRps	4.70	2.29	5.19	2.32	7.95	1.66	5.83	2.32	6.82	2.24	8.53	1.82
ETco	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
ETid	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
ETcl	2.84	.48	2.88	.42	3.00	.00	2.94	.36	2.86	.54	2.93	.30
ETfo	13.24	2.99	14.05	2.62	15.53	2.27	14.57	2.76	15.18	2.62	15.84	2.71
ETpr	7.47	2.45	7.89	2.27	8.55	1.97	9.11	2.15	9.13	2.50	9.38	2.45
ETtax	4.76	1.80	5.39	1.66	6.74	1.70	6.13	2.29	6.59	1.66	6.33	2.11
ETps	3.38	1.50	3.45	1.41	4.46	1.55	4.50	1.48	4.49	1.61	4.79	1.36
NNco	7.99	.08	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
NNid	7.99	.12	NA[a]		NA[a]		NA[a]		NA[a]		NA[a]	
NNcl	6.14	2.06	6.69	1.80	7.61	1.02	7.11	1.44	7.16	1.36	7.84	.58
NNfo	12.44	4.61	14.34	5.19	16.31	5.30	16.10	5.02	18.31	4.11	19.26	5.16
NNpr	5.14	2.06	5.73	2.33	5.83	2.52	6.56	1.94	7.05	1.95	7.74	2.25
NNtax	3.91	1.50	4.86	1.97	5.64	1.97	5.48	1.69	6.03	1.59	6.11	1.61
NNps	2.37	1.24	2.47	1.17	3.35	1.17	2.97	1.34	3.33	1.14	3.88	1.13

[a] NA = not administered.

Table A-4-2. Summary of Correlations Within Batteries and Across Batteries Based on Annual Measurements for Each Grade 1-12

Longitudinal Block and Grade	*Direction and Size of rs*	*Concept Battery*										*Total N and %*	
		CT				*TR*			*ET*		*NN*		
		CT	*TR*	*ET*	*NN*	*TR*	*ET*	*NN*	*ET*	*NN*	*NN*	*Within Battery*	*Across Battery*
A1 (*N* = 126)	(.-01) - (-.24)	1	21	10	8	2	8	2	0	3	0	3 (6%)	52 (32%)
	+.00 - .19	10	16	23	11	9	17	10	6	10	2	27 (55%)	87 (54%)
	+.20 - .39	10	5	2	2	2	5	6	2	1	0	14 (29%)	21 (13%)
	+.40 - .72	0	0	0	0	2	0	0	2	1	1	5 (10%)	1 (1%)
A2 (*N* = 84)	(-.01) - (-.34)	1	15	14	13	5	13	17	2	8	0	8 (14%)	80 (41%)
	+.00 - .19	14	26	19	13	8	20	11	5	5	0	27 (47%)	94 (48%)
	+.20 - .39	5	7	2	2	6	2	0	2	6	4	17 (29%)	19 (10%)
	+.40 - .58	1	1	0	0	2	0	0	1	1	2	6 (10%)	2 (1%)
A3 (*N* = 62)	(-.01) - (-.36)	2	14	17	11	7	25	16	3	23	0	12 (14%)	106 (36%)
	+.00 - .19	15	32	32	35	9	22	23	16	24	4	44 (52%)	168 (57%)
	+.20 - .39	3	2	0	3	4	2	9	2	1	10	19 (23%)	17 (6%)
	+.40 - .79	1	1	0	0	1	0	1	0	1	7	9 (11%)	3 (1%)
B4 (*N* = 151)	(-.01) - (-.20)	4	15	9	22	5	14	21	0	18	4	13 (15%)	99 (34%)
	+.00 - .19	9	30	26	27	13	30	25	10	28	6	38 (45%)	166 (56%)
	+.20 - .39	5	4	13	0	2	5	3	5	3	10	22 (26%)	28 (10%)
	+.40 - .67	3	0	1	0	1	0	0	6	0	1	11 (13%)	1 (1%)
B5 (*N* = 113)	(.-01) - (-.29)	8	12	11	24	2	6	19	1	16	4	15 (18%)	88 (30%)
	+.00 - .19	5	25	30	19	15	36	26	16	33	13	49 (58%)	169 (57%)
	+.20 - .39	3	12	7	6	3	7	4	2	0	3	11 (13%)	36 (12%)
	+.40 - .97	5	0	1	0	1	0	0	2	0	1	9 (11%)	1 (1%)
B6 (*N* = 77)	(-.01) (-.16)	2	4	8	2	0	5	4	2	5	0	4 (5%)	28 (10%)
	+.00 - .19	14	31	40	34	12	41	30	12	40	14	52 (62%)	216 (73%)
	+.20 - .39	3	12	1	12	3	3	14	5	4	3	14 (17%)	46 (16%)
	+.40 - .69	2	2	0	1	6	0	1	2	0	4	14 (17%)	4 (1%)

	(-.01) - (-.15)	0	0	0	6	0	2	10	0	13	10	10 (20%)	31 (17%)
	+.00 - .19	1	5	8	19	0	12	11	4	12	2	7 (14%)	67 (37%)
148)	+.20 - .39	2	19	10	9	7	11	13	3	10	5	17 (33%)	72 (40%)
	+.40 - .70	7	1	7	1	3	0	1	3	0	4	17 (33%)	10 (6%)
	(-.01) - (-.15)	0	3	0	2	0	8	0	1	6	0	1 (3%)	19 (13%)
	+.00 - .19	0	12	7	7	2	7	3	2	9	0	4 (10%)	45 (30%)
112)	+.20 - .39	5	10	12	14	4	9	13	4	6	1	14 (35%)	64 (43%)
	+.40 - .74	5	0	6	2	4	1	9	3	4	9	21 (53%)	22 (15%)
	(-.01) - (-.03)	0	0	0	1	0	0	0	0	0	0	0 (0%)	1 (1%)
	+.00 - .19	4	14	11	17	4	22	14	4	14	0	12 (30%)	92 (61%)
80)	+.20 - .39	0	11	8	6	2	3	11	3	11	4	9 (23%)	50 (33%)
	+.40 - .72	6	0	6	1	4	0	0	3	0	6	19 (48%)	7 (5%)
	(-.01) - (-.12)	0	0	1	0	0	3	2	0	5	0	0 (0%)	11 (7%)
	+.00 - .19	0	17	8	13	4	9	13	4	4	0	8 (20%)	64 (43%)
145)	+.20 - .39	4	8	12	12	1	12	10	0	16	5	10 (25%)	70 (47%)
	+.40 - .77	6	0	4	0	5	1	0	6	0	5	22 (55%)	5 (3%)
	(-.01) - (-.11)	2	1	3	2	2	3	3	1	0	0	5 (13%)	12 (8%)
	+.00 - .19	2	6	5	5	1	8	5	2	13	0	5 (13%)	42 (28%)
105)	+.20 - .39	2	14	13	18	3	11	9	2	9	2	9(23%)	74 (49%)
	+.40 - .69	4	4	4	0	4	3	8	5	3	8	21 (53%)	22 (15%)
	(-.01) - (-.12)	2	1	5	4	0	1	4	2	1	0	4 (10%)	16 (11%)
	+.00 - .19	3	13	6	11	4	12	14	2	5	0	9 (23%)	61 (41%)
73)	+.20 - .39	1	10	8	10	0	10	7	0	16	4	5 (13%)	61 (41%)
	+.40 - .71	4	1	6	0	6	2	0	6	3	6	22 (55%)	12 (8%)
	(-.01) - (-.36)	22 12%	86 20%	78 19%	95 23%	23 13%	88 21%	98 24%	12 7%	98 25%	18 11%	75 11%	543 22%
	.00 - .19	77 41%	227 52%	215 52%	211 52%	81 45%	236 57%	185 46%	83 51%	197 50%	41 25%	282 41%	1271 52%
es	+.20 - .39	43 23%	114 26%	88 21%	94 23%	37 21%	80 19%	99 25%	30 18%	83 21%	51 31%	161 23%	558 23%
	+.40 - .97	44 24%	10 2%	35 8%	5 1%	39 22%	7 2%	20 5%	39 24%	13 3%	54 33%	176 25%	90 4%

Table A-4-3. Comparable Common Factors for Each Grade 1-12[a]

	Orthogonal			Oblique		
	P	*M*	*H*[b]	*P*	*M*	*H*
Grade 1						
Comparable Common Factor 1						
CTco	73	59	50	70		
CTid	77	63	56	77		
Comparable Common Factor 2						
CTps	63	55	55	66	52	53
CTtax	63	55	46	53	50	55
CTfo	74	56	44	59	42	45
CTpr	52	47	44	61	38	41
Comparable Common Factor 3						
TRco	78	68			65	64
TRid	76	65			59	53
Comparable Common Factor 4						
ETco	89	98	84	91	58	85
ETid	84	73	78	85	71	79
ETcl	66	45	47	63	48	45
Comparable Common Factor 5						
NNco	83	71	62	79	75	73
NNid	75	71	69	82	63	61
Comparable Common Factor 6						
NNcl			68	79	100	75
NNtax			60	78	50	63
Comparable Specific Factor 7						
TRtax	72	44	50	48		
ETfo			73	76	79	71
Grade 2						
Comparable Common Factor 1						
CTco	81	98	67	86	100	70
CTid	71	52	62	72	43	61
Comparable Common Factor 2						
TRcl	81	93	65	68	34	72
TRid	60	36		53		33
Comparable Common Factor 3						
ETid	80	87	73		98	79
ETcl	80	59	64		51	69
NNid	55	31	55			50
Comparable Common Factor 4						
NNco	77	46	48	41	52	45
NNid	47	78	39	76	80	30
NNcl	74	49	51	56	58	47
Comparable Common Factor 5						
CTpr	32	39	34	51	37	35
CTps	53	48	33	35		
ETfo			-68	-78	-82	-72
Comparable Common Factor 6						
TRpr	80	73	63	60	52	63
TRtax	71	58	38	68	75	48
TRps			70	78	67	72
CTtax	66	52		46	34	
CTpr			46	53	44	40

Table A-4-3 continued

	Orthogonal			*Oblique*		
	P	*M*	*H*	*P*	*M*	*H*
Comparable Common Factor 7						
CTcl	-51	-32	-41	-60	-53	-51
ETtax	70	60	59	69	38	58
Grade 3						
Comparable Common Factor 1						
CTcl			85	89	78	87
TRid			87	93	100	89
Comparable Common Factor 2						
CTfo	77	83	67	84	100	66
TRco			59	68	39	62
Comparable Common Factor 3						
CTps	75	61	46	59	38	55
CTtax			64	77	74	62
CTpr			55	68	54	48
Comparable Common Factor 4						
TRpr	45	32	57	66	81	70
ETcl	71	51	45	66		42
TRcl	78	64		44		31
TRps			32	51	36	47
CTps			-44	-38	-51	-37
Comparable Common Factor 5						
NNcl	35	31	49	55	97	79
ETps	53	52	52	72		47
ETfo	35	35	41	50		34
ETtax			30	36	51	44
Comparable Common Factor 6						
NNco			78	83	84	84
NNid			82	82	86	82
ETfo			-40	-49	-39	-43
ETpr			-30	-50	-39	-35
Comparable Common Factor 7						
NNfo	81	86	62	64	100	84
NNps	58	31	58	63	45	64
NNtax	77	59	65	65		33
NNpr	63	48	65	77		39
NNcl	66	70	63	57		
TRpr	48	35	37	37		
Grade 4						
Comparable Common Factor 1						
CTps	71	58	61	68	75	70
CTfo	74	73	59	73	58	60
CTpr	68	65	50	63	46	49
CTtax	59	48	72	67	72	70
CTcl	42	39	40	34	30	32
Comparable Common Factor 2						
TRcl	72	58	52	69	68	59
ETco	71	47	49	76	47	54
TRco	52	41	39	49	48	45

Table A-4-3 continued

	Orthogonal			Oblique		
	P	*M*	*H*	*P*	*M*	*H*
Comparable Common Factor 3						
TRtax	55	35	56	73	82	66
TRpr	69	60	39	52	35	37
TRps	51		58	76	64	64
Comparable Common Factor 4						
ETps	69	70	71	73	82	78
ETfo	46	45	68	74	79	76
ETpr	70	74	52	51	35	37
ETtax	58	48	44	46	38	37
ETcl	53	42	44	47	34	35
ETid	56	40	42	55		34
CTid	45	33	34	42		31
TRfo	37			54	39	43
Comparable Common Factor 5						
NNco	68	52	47	70	52	48
NNid	63	43	47	62	36	42
TRid	55	40	43	49	30	35
Comparable Common Factor 6						
NNps	66	61	61	69	63	61
NNpr	68	63	54	67	56	57
NNfo	67	55	58	64	56	54
NNtax	64	53	57	65	52	54
NNcl	51	42	44	46	35	37
Grade 5						
Comparable Common Factor 1						
CTco	97	98	97	98	99	98
CTid	96	96	96	96	97	97
CTcl	94	91	92	95	91	93
Comparable Common Factor 2						
CTps	35	59	54		77	59
ETcl	70		35	79	33	45
CTfo		51	58		69	67
Comparable Common Factor 3						
TRtax	68	43	73	76	87	80
TRpr	78	76	52	71		35
TRps			77	83	71	78
ETtax			45	49	35	39
Comparable Common Factor 4						
ETps	75	76	70	80	70	69
ETfo	47	35	72	81	79	76
ETpr	77	72	45	50	41	45
ETtax	51	42	39	46	30	35
Comparable Common Factor 5						
NNcl	66	55	52	[illegible]	59	57
TRid	72	40	43	73	48	50
Comparable Common Factor 6						
NNtax	48	50	60	69	42	55
NNpr	61	38	43	60	49	42
NNfo	73	48	40	49		39
NNps	58		51	60	30	45
TRpr			-38	-49	-69	-51

Table A-4-3 continued

	Orthogonal			*Oblique*		
	P	*M*	*H*	*P*	*M*	*H*
Grade 6						
Comparable Common Factor 1						
CTtax	57	33	63	62	64	73
CTpr	79	89	35	55	33	37
CTps			73	80	100	85
Comparable Common Factor 2						
TRtax	79	72	68	69	84	74
TRpr	64	60	72	81	75	75
TRps	50	42	61	63	69	72
TRcl	65.	51	52	74	53	56
TRfo	71	72	45	45	45	47
Comparable Common Factor 3						
ETtax	73	72	68	78	63	67
ETfo	77	66	63	74	69	67
ETpr	62	45	71	73	72	72
ETps	70	62	45	62	43	44
Comparable Common Factor 4						
NNfo	56	69	64	69	65	66
NNps	61	63	61	43	63	57
NNcl	69	47	56	74	44	55
NNtax	44		67	60	69	65
TRcl	-36			-56	-45	-47
Grade 7						
Comparable Common Factor 1						
CTfo	72	47	49	58	62	63
CTpr	83	57	31	38	52	47
CTps	67		71	92	88	83
CTtax	49		62	72	67	70
ETfo	64	56	67	74		32
ETps	71	68	68	78		37
ETpr	70	62	50	32		
ETtax	37	49	61	57		
Comparable Common Factor 2						
TRps	62	55	60	73	80	66
TRtax	75	51	56	71	60	60
TRpr	64	62	57	79	53	64
TRfo	66	55	45	61	52	63
TRcl	45	32	35	40		
Comparable Common Factor 3						
NNcl	77	47	58	85	43	68
NNtax	59	39	73	67	87	73
NNps	34	44	66	64	79	71
NNfo	72	83	41		38	31
NNpr	64	69	43	55		39
Comparable Common Factor 4						
ETcl	74	55	52	75	62	59
TRcl	65	66	59	59	69	58
CTcl	68	46	54	72	61	55
CTpr			53	50	42	43

Table A-4-3 continued

	Orthogonal			*Oblique*		
	P	*M*	*H*	*P*	*M*	*H*
Grade 8						
Comparable Common Factor 1						
CTps	60	61	72	72	82	82
CTtax	61	52	72	71	82	77
CTfo	70	76	60	61	68	68
CTpr	69	74	54	62	48	47
CTcl	56	44	45	62	43	41
ETfo	58	50	64	82	37	34
ETps	74	61	55	58		
ETpr	62	50	44	58		
ETtax	57	40	54	51		
Comparable Common Factor 2						
TRps	62	43	69	91	80	81
TRtax	52	43	67	72	83	75
TRfo	74	77	45	65	40	46
TRpr	82	71	41	46	32	36
Comparable Common Factor 3						
TRcl	83	36	39	88	46	53
ETpr	35	60		33	37	44
ETps			46	34	89	83
ETfo	31	54			46	50
Comparable Common Factor 4						
NNfo	77	80	63	92	74	77
NNps	76	62	77	79	82	79
NNtax	67	58	80	75	85	78
NNpr	76	76	63	68	58	60
NNcl	64	53	60	65	52	57
TRpr			47	40	39	37
Grade 9						
Comparable Common Factor 1						
CTps	64	33	77	74	100	89
CTtax	84	34	68	58	44	68
CTpr	74	95	60	96		68
CTfo	67	54	48	61		43
Comparable Common Factor 2						
TRpr	88	89	79	85	76	81
TRtax	84	76	78	81	81	80
TRps	58	47	83	89	94	87
TRfo	80	78	46	57	47	45
Comparable Common Factor 3						
ETtax	75	69	68	62	87	68
ETpr	70	37	72	83	80	85
ETfo	64	81	55	60	64	62
ETps	58	56	42	84	37	48
NNcl	51	43	44	52	46	47
CTfo	43	54	30		36	
CTtax		40	41	32	42	
Comparable Common Factor 4						
NNps	70	64	74	84	81	79
NNfo	77	75	65	86	69	71

Table A-4-3 continued

	Orthogonal			Oblique		
	P	*M*	*H*	*P*	*M*	*H*
NNtax	67	64	73	78	78	76
NNpr	81	77	63	62	61	61
NNcl	45	37	37	41	33	33
Grade 10						
Comparable Common Factor 1						
CTps	76	82	81	80	87	84
CTfo	79	73	80	77	88	85
CTtax	69	52	74	83	77	78
CTpr	79	83	55	73	53	56
CTcl	56	33	36	64	36	40
Comparable Common Factor 2						
TRps	66	59	81	90	92	83
TRtax	69	52	70	81	70	72
TRpr	87	90	58	65	47	54
TRfo	79	69	61	62	53	56
Comparable Common Factor 3						
ETtax	74	67	74	88	91	86
ETpr	75	70	66	78	74	74
ETps	68	53	69	79	78	76
ETfo	80	79	56	73	58	62
Comparable Common Factor 4						
NNtax	69	56	74	82	88	81
NNps	71	59	69	74	76	74
NNfo	78	80	61	72	60	63
NNpr	72	71	58	71	58	61
NNcl	57	41	43	59	40	44
Grade 11						
Comparable Common Factor 1						
CTps	53	72	72	52	88	76
CTfo	67	50	69	47	82	76
TRcl	76		37	86	32	48
CTtax			58	63	49	59
Comparable Common Factor 2						
TRfo	76	33	70	84	99	80
TRps	68	94	66	38	32	42
TRpr	81	47	44	38	32	34
Comparable Common Factor 3						
ETcl	66	33	40	46	32	39
TRtax			55	90	99	84
TRps	51			71	48	56
Comparable Common Factor 4						
ETps	80	73	80	95	94	88
ETfo	65	57	71	91	82	82
ETpr	86	85	62	75	57	61
ETtax	72	59	58	64	53	54
Comparable Common Factor 5						
NNpr	64	68	73	92	83	85
NNtax	78	72	72	73	87	79
NNfo	70	76	64	85	67	70

Table A-4-3 continued

	Orthogonal			*Oblique*		
	P	*M*	*H*	*P*	*M*	*H*
NNps	75	61	66	67	80	74
NNcl	68	53	56	76	54	58
Grade 12						
Comparable Common Factor 1						
CTps	89	95	70	78	100	62
CTpr	78	71			45	83
CTfo	57	35	80	96		
CTtax			45	50	36	62
Comparable Common Factor 2						
TRfo	85	79	76	89	84	84
TRtax	80	72	79	87	84	81
TRps	83	77	79	80	79	77
TRpr	82	80	72	82	74	73
Comparable Common Factor 3						
ETcl	87	71	69	91	75	74
NNcl	78	80	68	78	84	69
Comparable Common Factor 4						
ETps	65	58	78	95	100	94
ETpr	82	85	68	90	73	70
ETtax	81	69	69	80	76	78
ETfo	72	69	56	69	59	57
Comparable Common Factor 5						
NNtax	75	69	76	93	97	86
NNfo	84	82	64	73	58	63
NNpr	70	57	70	77	64	68
NNps	70	60	61	70	57	56

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[a] Includes values greater than .30. Decimals have been omitted.

[b] P = Principal Components, M = UMLFA, H = Harris R-S^2

Table A-4-4. Mean and Standard Deviation for Each Level and Each Use of Each Concept for Each Longitudinal Block

Longitudinal Block and Grade	*Battery*	*Test*													
		CO		*ID*		*CL*		*FO*		*PR*		*TAX*		*PS*	
		Mean	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*
A1	CT	7.92	.27	7.89	.32	7.57	.80	13.71	3.02	5.32	1.52	5.87	1.57	3.13	1.06
	TR	7.79	.41	7.66	.54	6.34	2.01	14.94	4.20	[a]		1.79	1.06	1.65	1.24
	ET	7.86	.44	7.81	.51	2.50	.84	6.47	2.67	[a]		2.52	1.08	[a]	
	NN	6.94	1.08	4.86	1.87	.45	.95	[a]		[a]		[a]		[a]	
A2	CT	7.90	.39	7.89	.37	7.65	.85	13.13	2.51	3.81	1.64	5.65	1.65	2.44	1.00
	TR	7.47	.67	7.55	.74	7.36	1.16	19.84	3.76	1.11	1.07	2.65	1.10	2.92	1.91
	ET	7.92	.27	7.97	.25	2.84	.58	8.02	2.79	[a]		2.68	1.21	[a]	
	NN	7.84	.45	7.39	1.15	4.24	2.00	1.68	.84	[a]		[a]		[a]	
A3	CT	8.00	.00	8.00	.00	7.84	.58	14.47	2.13	5.27	1.63	5.71	2.02	3.31	.90
	TR	7.97	.25	7.87	.49	7.61	1.31	18.53	5.74	5.61	3.96	3.73	1.83	4.32	1.84
	ET	8.00	.00	8.00	.00	2.95	.28	9.94	2.81	4.85	2.07	3.63	1.94	3.27	1.45
	NN	7.77	.69	7.63	.91	5.79	2.01	8.06	2.92	2.73	1.81	2.92	1.53	1.19	.95
B4	CT	7.94	.29	7.92	.27	7.65	.98	14.16	2.73	5.34	2.33	6.79	1.30	3.61	1.22
	TR	7.86	.39	7.79	.41	7.35	1.37	22.05	5.09	8.26	3.41	4.30	1.33	3.01	1.61
	ET	7.82	.55	7.83	.41	2.70	.74	10.42	2.85	4.53	2.17	4.38	1.62	2.10	1.24
	NN	7.97	.16	7.88	.40	4.62	1.86	7.83	2.89	2.69	1.64	2.60	1.36	1.04	1.01
B5	CT	7.97	.16	7.97	.16	7.83	.41	15.01	2.08	5.55	2.50	6.61	1.23	3.81	1.12
	TR	7.78	.48	7.70	.61	7.60	.95	25.68	4.67	9.96	3.39	4.88	1.20	3.70	1.68
	ET	8.00	.00	8.00	.00	2.79	.52	12.05	2.90	5.68	2.35	4.60	1.51	2.23	1.30
	NN	7.99	.11	7.94	.25	6.05	1.72	11.25	3.20	3.73	1.77	3.92	1.00	1.58	1.20
B6	CT	8.00	.00	8.00	.00	7.99	.11	16.49	2.74	7.51	2.32	7.13	1.12	3.78	1.03
	TR	8.00	.00	8.00	.00	7.82	.81	23.55	7.29	9.29	5.08	5.69	1.76	5.33	2.21
	ET	8.00	.00	8.00	.00	2.96	.25	14.26	2.40	8.51	1.82	5.96	1.87	4.18	1.45
	NN	8.00	.00	8.00	.00	6.26	1.73	12.60	3.35	4.34	1.98	4.30	1.27	2.08	1.18

Table A-4-4 continued

Longitudinal Block and Grade	Battery	Test													
		CO		*ID*		*CL*		*FO*		*PR*		*TAX*		*PS*	
		Mean	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*	*Mean*	*S.D.*
C7	CT	b		b		7.58	1.15	14.14	3.38	5.80	2.56	6.44	1.63	3.13	1.64
	TR	b		b		7.25	1.70	26.76	5.30	11.69	3.80	4.96	1.53	4.16	1.95
	ET	b		b		2.80	.56	12.95	3.18	7.23	2.53	4.48	1.68	3.10	1.38
	NN	7.99	.11	7.98	.15	6.55	1.70	12.43	4.65	4.91	2.10	4.09	1.55	2.28	1.23
C8	CT	b		b		7.23	.78	14.92	3.17	6.50	2.39	6.65	1.53	3.64	1.41
	TR	b		b		7.81	.73	27.06	4.89	12.29	3.67	5.71	1.39	4.70	2.20
	ET	b		b		2.88	.37	14.14	2.63	7.91	2.04	5.23	1.67	3.24	1.23
	NN	b		b		6.98	1.34	14.88	5.19	5.90	2.30	5.30	1.95	2.48	1.15
C9	CT	b		b		8.00	.00	16.55	2.44	7.90	2.16	7.35	1.15	3.75	.83
	TR	b		b		8.00	.00	30.39	4.72	14.94	3.88	7.15	1.34	7.95	1.66
	ET	b		b		3.00	.00	15.53	2.27	8.55	1.97	6.74	1.70	4.46	1.55
	NN	b		b		7.61	1.02	16.31	5.30	5.83	2.52	5.64	1.97	3.35	1.17
D10	CT	b		b		7.71	.59	16.36	1.89	7.73	2.59	7.32	.97	3.95	1.33
	TR	b		b		7.74	.75	30.71	2.56	15.90	3.66	6.11	1.48	5.73	2.20
	ET	b		b		2.88	.50	15.11	2.57	9.29	2.16	6.14	1.95	4.52	1.60
	NN	b		b		7.14	1.37	16.48	4.76	6.81	1.85	5.25	1.68	2.55	1.34
D11	CT	b		b		7.74	.71	16.33	2.06	7.93	2.45	7.38	.97	3.80	1.36
	TR	b		b		7.70	1.08	30.90	3.14	15.90	3.91	6.10	1.61	6.37	2.21
	ET	b		b		2.89	.51	15.29	2.88	9.14	2.56	6.60	1.72	4.42	1.61
	NN	b		b		7.07	1.53	18.27	4.36	7.04	1.95	6.10	1.54	3.00	1.05
D12	CT	b		b		7.75	.76	17.05	1.97	8.14	1.98	7.59	.85	3.96	1.21
	TR	b		b		8.00	.00	31.73	4.11	15.99	3.71	6.85	1.58	8.53	1.82
	ET	b		b		2.93	.30	15.84	2.71	9.38	2.45	6.33	2.11	4.79	1.36
	NN	b		b		7.84	.58	19.26	5.16	7.74	2.25	6.11	1.61	3.88	1.13

[a] Test not administered; previous research showed that children at this grade responded at a chance level.

[b] Test not administered; previous research showed 100 percent attainment at this grade.

Table A-4-5. Summary of Correlations Within Each Battery and Across Batteries for the 12- and 24-Month Intervals of Testing

Longitudinal Block and Grades	*Direction and Size of rs*	*Concept Batteries*										*Total Within Battery*		*Total Across Battery*	
		CT				*TR*			*ET*		*NN*				
		CT	*TR*	*ET*	*NN*	*TR*	*ET*	*NN*	*ET*	*NN*	*NN*	*N*	*%*	*N*	*%*
A1 and 2 (*N* = 62)	(-.01) - (-.29)	29	29	29	10	16	21	11	6	6	0	51	43	106	33
	.00 - .19	14	40	34	23	19	26	19	12	13	3	48	40	155	48
	+.20 - .39	6	15	6	8	1	11	5	7	9	4	18	15	54	17
	+.40 - .48	0	0	1	1	0	2	1	0	2	2	2	2	7	2
A2 and 3 (*N* = 62)	(-.01) - (-.44)	16	48	22	15	16	26	19	6	11	0	38	27	141	36
	.00 - .19	31	46	41	31	24	41	28	16	21	4	75	54	208	53
	+.20 - .39	2	3	7	10	9	3	9	2	8	9	22	16	40	10
	+.40 - .50	0	1	0	0	0	0	0	1	0	3	4	3	1	1
B4 and 5 (*N* = 77)	(-.01) - (-.27)	14	18	29	28	9	20	36	9	26	11	43	22	157	27
	.00 - .19	24	45	49	57	26	63	47	32	64	26	108	55	325	55
	+.20 - .39	11	26	17	13	10	13	15	7	8	11	39	20	92	16
	+.40 - .53	0	9	3	0	4	2	0	1	0	1	6	3	14	2
B5 and 6 (*N* = 77)	(-.01) - (-.22)	7	12	15	15	2	16	9	3	14	5	17	9	81	14
	.00 - .19	24	62	69	59	30	71	70	34	73	34	122	62	404	69
	+.20 - .39	12	22	14	19	13	10	16	10	10	9	44	22	91	15
	+.40 - .67	6	2	0	5	4	1	3	2	1	1	13	7	13	2
C7 and 8 (*N* = 80)	(-.01) - (-.20)	1	1	6	9	0	4	9	0	7	6	7	6	36	11
	.00 - .19	6	12	19	11	1	19	11	14	25	7	28	25	97	29
	+.20 - .39	9	20	15	25	7	17	23	7	19	14	37	34	119	36
	+.40 - .71	9	17	10	15	17	10	17	4	9	8	38	35	78	24
C8 and 9 (*N* = 80)	(-.01) - (-.19)	0	5	1	6	0	4	2	0	7	0	0	0	25	8
	.00 - .19	6	23	20	19	9	27	12	10	13	1	26	26	114	38
	+.20 - .39	8	20	23	18	6	18	28	8	27	10	32	32	134	45
	+.40 - .72	11	2	6	7	10	1	8	7	3	14	42	42	27	9

Table A-4-5 continued

Longitudinal Block and Grades	Direction and Size of *rs*	*Concept Batteries*										*Total Within Battery*		*Total Across Battery*	
		CT				*TR*			*ET*		*NN*				
		CT	*TR*	*ET*	*NN*	*TR*	*ET*	*NN*	*ET*	*NN*	*NN*	*N*	%	*N*	%
D10 and 11 (*N* = 73)	(-.01) - (-.20)	0	2	8	5	2	9	3	0	4	0	2	2	31	10
	.00 - .19	10	11	10	10	4	8	9	9	11	3	26	26	59	20
	+.20 - .39	3	25	20	25	2	21	20	1	32	6	12	12	143	48
	+.40 - .77	12	12	12	10	17	12	18	15	3	16	60	60	63	21
D11 and 12 (*N* = 73)	(-.01) - (-.17)	5	3	8	5	0	5	3	3	2	0	8	8	26	9
	.00 - .19	6	15	11	24	7	14	17	5	16	1	19	19	97	32
	+.20 - .39	8	13	25	16	2	18	26	3	25	15	28	28	123	41
	+.40 - .67	6	19	6	5	16	13	4	14	7	9	45	45	54	18
Total	(-.01) - (-.44)	72	118	118	93	45	105	92	27	72	22	166	16	603	19
	Percent =	24	20	22	17	15	20	18	11	15	7				
	.00 - .19	121	254	253	234	120	269	213	132	230	79	452	43	1459	47
	Percent =	41	44	47	47	43	51	43	53	49	34				
	+.20 - .39	59	144	127	134	50	111	142	45	140	78	232	22	796	26
	Percent =	20	25	24	27	18	21	29	18	30	35				
	+.40 - .77	44	62	38	43	68	41	51	44	24	54	210	20	257	8
	Percent =	15	11	7	9	24	8	10	18	5	24				

A1 and 3 (*N* = 62)	(-.01) - (-.37)	12	28	20	11	17	24	13	6	8	1	36	30	104	32
	.00 - .19	37	51	44	28	15	31	10	18	16	2	72	61	186	58
	+.20 - .39	0	5	6	3	4	5	6	1	6	4	9	6	31	10
	+.40 - .49	0	0	0	0	0	0	1	0	0	2	2	2	1	1
B4 and 6 (*N* = 77)	(-.01) - (-.27)	16	18	20	24	8	16	15	15	14	3	42	21	107	18
	.00 - .19	29	63	65	64	29	72	64	33	64	33	124	63	392	67
	+.20 - .39	4	16	13	10	9	10	18	1	19	13	27	14	86	15
	+.40 - .51	0	1	0	0	3	0	1	0	1	0	3	2	3	1
C7 and 9 (*N* = 80)	(-.01) - (-.19)	0	2	2	7	0	2	9	4	6	7	11	10	28	8
	.00 - .19	8	19	26	26	5	25	19	13	36	13	39	36	151	46
	+.20 - .39	11	25	21	26	15	20	25	6	15	13	45	41	132	40
	+.40 - .51	6	4	1	1	5	3	7	2	3	2	15	14	19	6
D10 and 12 (*N* = 73)	(-.01) - (-.13)	2	3	6	0	2	4	1	1	2	0	5	5	16	5
	.00 - .19	5	15	15	29	7	18	28	6	17	8	26	26	122	41
	+.20 - .39	13	22	23	18	10	18	18	3	26	14	40	40	125	42
	+.40 - .61	5	10	6	3	6	10	3	15	5	3	29	29	37	12
Total	(-.01) - (-.37)	30	51	48	42	27	46	38	26	30	11	94	18	255	17
	Percent =	24	21	19	16	20	18	13	21	12	4				
	.00 - .19	79	148	150	141	56	146	127	70	133	56	261	50	851	55
	Percent =	48	49	54	58	41	57	53	56	55	50				
	+.20 - .39	28	68	63	51	38	53	67	11	66	44	121	23	374	24
	Percent =	20	25	24	24	28	21	29	9	29	40				
	+.40 - .51	11	15	7	4	14	13	12	17	9	7	49	9	60	4
	Percent =	7	5	3	2	10	5	5	14	4	6				

Table A-4-6. Interbattery Factors for 12- and 24-Month Testing Intervals, Longitudinal Block A[a]

		Factors								Factors								Factors						
Test	*Grades*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Grades*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Grades*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
CTCO	1				37			-13	2								1							
CTID	1								2								1							
CTCL	1		07				49	07	2								1	65						-10
CTFO	1	-17	06	63	-04		03		2			05		-03	66		1		-07					
CTPR	1		50		-04				2				62		10	07	1		37				04	
CTTAX	1					-34			2		40	13					1		-10	-38				
CTPS	1				-26			-52	2	17			60				1					-14		
TRCO	1		21			51			2					15		-31	1					-58		10
TRID	1			-33	36	02		-39	2			-69					1					23	-37	
TRCL	1	53			-03		39		2					49		13	1	44		-66				
TRFO	1					-10		35	2				06	37		-42	1				-01		40	
TRPR	1								2	11					26		1							
TRTAX	1		52		33			-01	2				-34		04		1							42
TRPS	1	33		20				36	2		32	-08	05			-39	1		-40	-28				
ETCO	1			25		38			2								1							
ETID	1					35			2								1							
ETCL	1		-24		53				2	-02	68						1				63			
ETFO	1	62							2				-04		31		1			-12		37		
ETTAX	1	35							2	47							1		-35			11	33	
NNCO	1						57		2	30	-10	-30			46		1	55		-30				
NNID	1	70							2	48	15	-32			37		1	24						57
NNCL	1		38	-22					2	38	32				04		1		46	-29			31	
NNFO	1								2	46				11	17		1							
CTCO	2				-10			37	3								3							
CTID	2								3								3							
CTCL	2		30				-04	-43	3								3	17						-43
CTFO	2	39	35	-01	36		36		3			-44		-36	09		3		44					
CTPR	2		08		31				3				-16		-44		3		16				-54	
CTTAX	2					50			3		-16	-36				-36	3		66	16				
CTPS	2				55			19	3	43			-19				3					-46		
TRCO	2		51			26			3					40		06	3					-10		55
TRID	2			-01	-07	-45		09	3			30					3					-42	01	
TRCL	2	12			42		-02		3					-22		-41	3	30		13				
TRFO	2					33		-19	3				61	02		01	3				-34		-09	
TRPR	2								3	33					33		3							
TRTAX	2		05		-04			-36	3				-03		64		3							-15
TRPS	2	15		52				-13	3		04	48	-53			-04	3		06	45				
ETCO	2			53		15			3								3							
ETID	2					-04			3								3							
ETCL	2		48		-16				3	62	-09						3				40			
ETFO	2	66							3				35		13		3			-59		02		
ETTAX	2	43							3	-31							3		-10			-31	27	
NNCO	2						77		3	-11	47	-05			07		3	61		07				
NNID	2	62							3	06	65	-12			08		3	65						13
NNCL	2		01	47					3	11	56				35		3		07	-62			13	
NNFO	2								3	32				30	33		3							
Tests																								
30 or higher both years		3	0	0	0	0	1	0		2	1	1	0	0	0	0		2	0	0	1	0	0	0
30 or higher one year only		3	7	5	8	6	3	7		6	5	5	6	5	9	5		2	6	6	1	5	5	4
Below 30 both years		15	14	15	13	15	17	14		11	13	13	13	14	10	14		13	11	11	15	12	12	13

[a] Decimals have been omitted.

Table A-4-7. Interbattery Factors for 12- and 24-Month Testing Intervals, Longitudinal Block B[a]

Test	*Grade*	*Factors*								*Grade*	*Factors*					*Grade*	*Factors*				
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*		*1*	*2*	*3*	*4*	*5*		*1*	*2*	*3*	*4*	*5*
CTCO	4					48		-37	11	5						4					
CTID	4							-15		5						4					
CTCL	4	69				06				5			03	50		4			13	38	
CTFO	4	45				38				5	71					4				53	-05
CTPR	4	22			-01	-12				5	56				-06	4		33	15		-46
CTTAX	4				56			05		5	52			30		4			32		-06
CTPS	4	38		-22	13					5	77					4		-01	58		03
TRCO	4		43				-04			5						4					
TRID	4	-01		51						5						4					
TRCL	4		-03				-36			5	31		-57		-01	4	40				
TRFO	4	62				44				5	52		-32			4	38			43	28
TRPR	4				07	55			26	5	53	08			33	4		06		73	
TRTAX	4			35		26				5	42		02			4				12	31
TRPS	4			-04						5	07					4	-34	32		19	
ETCL	4			-13				-34		5		36				4	-30				
ETFO	4	43				28				5	45	45				4					43
ETPR	4	09				52				5	33	49				4	-10				53
ETTAX	4				62					5		-01		10		4		15			
ETPS	4	14				39				5		64				4		23			
NNCO	4								-34	5						4					
NNID	4						-06			5						4					
NNCL	4		40	07		37				5				46		4	40				
NNFO	4			-38	27	34				5	27				46	4	13	39			23
NNPR	4							03		5	08			33	46	4		44			
NNTAX	4		39							5						4		32	-32	-10	
NNPS	4	07	40				44			5	10			39		4	18	49			21
CTCO	5					-04		-04	36	6						6					
CTID	5							-47		6						6					
CTCL	5	21				42				6			54	-08		6			55	-16	
CTFO	5	08				65				6	39					6				09	40
CTPR	5	31			33	39				6	61				41	6		10	30		37
CTTAX	5				03			48		6	35			39		6			10		56
CTPS	5	15		39	39					6	34					6		32	-09		37
TRCO	5		-01				50			6						6					
TRID	5	58		29						6						6					
TRCL	5		41				40			6	35		-28		-33	6	04				
TRFO	5	53				22				6	64		-06			6	-12			56	31
TRPR	5				33	41			37	6	51	30			-30	6		31		58	
TRTAX	5			11		48				6	59		-39			6				46	08
TRPS	5			-37						6	44					6	-04	13		37	
ETCL	5			33				-06		6		15				6	28				
ETFO	5	24				50				6	21	59				6					-01
ETPR	5	44				36				6	08	48				6	38				13
ETTAX	5				-01					6		50		35		6		60			
ETPS	5	33				-12				6		39				6		42			
NNCO	5								27	6						6					
NNID	5						48			6						6					
NNCL	5		-02	47		18				6				56		6	52				
NNFO	5			05	54	01				6	65				05	6	48	19			31
NNPR	5							-38		6	46			18	29	6		47			
NNTAX	5		-40							6						6		06	-14	33	
NNPS	5	42	-48				21			6	35			17		6	32	23			33
Tests 30 or higher both years		1	2	0	0	3	1	0	0		8	3	0	2	1		1	1	0	2	1
30 or higher one year only		9	3	7	6	9	3	5	3		6	3	4	4	4		7	9	5	5	9
Below 30 both years		16	21	19	20	14	22	21	23		6	14	16	14	15		12	10	15	13	10

[a] Decimals have been omitted.

Table A-4-8. Interbattery Factors for 12- and 24-Month Testing Intervals, Longitudinal Block C[a]

Test	*Grade*	*Factor*	*Grade*	*Factor*	*Grade*	*Factor*
		I		*I*		*I*
CTCL	7	42	8		7	
CTFO	7	75	8	69	7	60
CTPR	7	62	8	55	7	51
CTTAX	7	63	8	34	7	50
CTPS	7	67	8	53	7	58
TRCL	7	56	8		7	
TRFO	7	75	8	66	7	64
TRPR	7	77	8	56	7	72
TRTAX	7	46	8	49	7	43
TRPS	7	55	8	56	7	46
ETCL	7		8		7	
ETFO	7	75	8	70	7	67
ETPR	7	59	8	47	7	54
ETTAX	7		8	31	7	20
ETPS	7	35	8	40	7	34
NNCL	7	40	8	53	7	44
NNFO	7	76	8	73	7	60
NNPR	7	50	8	63	7	40
NNTAX	7	49	8	66	7	47
NNPS	7	43	8	46	7	37
CTCL	8	13	9		9	
CTFO	8	68	9	66	9	52
CTPR	8	52	9	62	9	58
CTTAX	8	29	9	40	9	31
CTPS	8	54	9	66	9	66
TRCL	8	29	9		9	
TRFO	8	81	9	52	9	60
TRPR	8	68	9	58	9	61
TRTAX	8	63	9	47	9	54
TRPS	8	72	9	53	9	58
ETCL	8		9		9	
ETFO	8	73	9	61	9	52
ETPR	8	63	9	36	9	27
ETTAX	8		9	53	9	47
ETPS	8	28	9	53	9	42
NNCL	8	59	9	41	9	27
NNFO	8	80	9	67	9	64
NNPR	8	74	9	66	9	51
NNTAX	8	67	9	75	9	67
NNPS	8	41	9	35	9	23
Tests:						
30 or higher both years		14		17		13
30 or higher one year only		4		0		4
Below 30 both years		2		3		3

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[a] Decimals have been omitted.

Table A-4-9. Interbattery Factors for 12- and 24-Month Testing Intervals, Longitudinal Block D[a]

Test	*Grade*	*Factor*	*Grade*	*Factor*	*Grade*	*Factor*
		1		*1*		*1*
CTCL	10	30	11		10	34
CTFO	10	73	11	58	10	67
CTPR	10	69	11	52	10	61
CTTAX	10	55	11	61	10	43
CTPS	10	60	11	45	10	55
TRCL	10		11		10	
TRFO	10	68	11	75	10	60
TRPR	10	67	11	67	10	58
TRTAX	10	63	11	81	10	58
TRPS	10	68	11	61	10	51
ETCL	10		11		10	
ETFO	10	56	11	58	10	55
ETPR	10	64	11	63	10	57
ETTAX	10	56	11	57	10	51
ETPS	10	56	11	58	10	58
NNCL	10	32	11	47	10	
NNFO	10	81	11	64	10	72
NNPR	10	65	11	53	10	57
NNTAX	10	51	11	50	10	41
NNPS	10	42	11	49	10	36
CTCL	11	07	12		12	15
CTFO	11	61	12	72	12	59
CTPR	11	60	12	63	12	56
CTTAX	11	55	12	63	12	49
CTPS	11	44	12	39	12	39
TRCL	11		12		12	
TRFO	11	68	12	62	12	53
TRPR	11	62	12	75	12	76
TRTAX	11	79	12	65	12	51
TRPS	11	61	12	49	12	43
ETCL	11		12		12	
ETFO	11	65	12	71	12	66
ETPR	11	64	12	64	12	68
ETTAX	11	68	12	75	12	77
ETPS	11	60	12	65	12	66
NNCL	11	38	12	19	12	
NNFO	11	74	12	49	12	44
NNPR	11	65	12	47	12	44
NNTAX	11	58	12	51	12	41
NNPS	11	56	12	52	12	50
Tests:						
30 or higher both years		17		16		16
30 or higher one year only		1		1		1
Below 30 both years		2		3		3

[a] Decimals have been omitted.

Table A-4-10. Correlations Among CLD Tests and Educational Achievement Tests, Grade 6[a]

	CTCL	*CTFO*	*CTPR*	*CTTAX*	*CTPS*	*TRCL*	*TRFO*	*TRPR*	*TRTAX*	*TRPS*	*ETCL*	*ETFO*	*ETPR*	*ETTAX*	*ETPS*	*NNCL*	*NNFO*	*NNPR*	*NNTAX*
CTCL																			
CTFO	-05																		
CTPR	11	69																	
CTTAX	12	20	44																
CTPS	-21	07	29	17															
TRCL	-04	-01	15	16	06														
TRFO	-15	46	52	30	34	25													
TRPR	-14	31	34	21	34	18	64												
TRTAX	-05	34	47	27	32	33	53	53											
TRPS	15	41	46	20	33	25	49	60	51										
ETCL	-02	14	24	23	-06	-04	10	-14	29	01									
ETFO	-05	18	20	21	17	04	30	27	13	25	16								
ETPR	-13	07	21	07	12	38	22	09	21	13	28	35							
ETTAX	-17	19	13	05	26	-06	20	42	25	23	-17	48	18						
ETPS	22	-23	-03	-16	-01	14	-04	18	07	23	-20	16	26	37					
NNCL	-06	17	19	24	12	-02	37	48	46	21	-14	03	09	20	-06				
NNFO	05	24	50	39	33	-01	39	41	23	31	26	46	23	36	-09	34			
NNPR	01	39	59	39	23	-05	43	30	21	34	14	44	21	25	-03	32	68		
NNTAX	-19	-01	12	07	22	-05	25	24	37	27	22	15	19	04	02	21	16	06	
NNPS	-01	45	55	34	41	04	58	34	46	32	24	37	17	19	-12	45	46	60	22

Total		
r	*N*	*Percent*
+.28 - (-.23)	125	66
+.29 - .36 ($p < .05$)	24	13
+.37 - .69 ($p < .01$)	41	22

	Concepts	*Arithmetic Computation*	*Application*	*Vocabulary*	*Reading*	*Word Meaning*	*Spelling*	*Language*	*Listening Comprehension*	*Science*
Arithmetic Concepts										
Arithmetic Computation	68									
Arithmetic Application	76	62								
Vocabulary	63	46	51							
Reading	77	54	70	62						
Word Meaning	76	53	66	52	70					
Spelling	67	61	54	48	78	67				
Language	71	63	70	54	74	61	64			
Listening Comprehension	67	51	53	72	55	56	46	49		
Science	73	54	73	71	84	74	64	77	61	
Social Studies	75	55	71	72	78	67	60	71	68	91

Total		
r	*N*	*Percent*
+.28 - ( )	0	0
+.29 - .36 ($p < .05$)	0	0
+.37 - .91 ($p < .01$)	55	100

[a] Decimals have been omitted.

Table A-4-11. Correlations Among CLD Tests, Grade 9[a]

	CTFO	*CTPR*	*CTTAX*	*CTPS*	*TRFO*	*TRPR*	*TRTAX*	*TRPS*	*ETFO*	*ETPR*	*ETTAX*	*ETPS*	*NNCL*	*NNFO*	*NNPR*	*NNTAX*
CTFO																
CTPR	71															
CTTAX	47	47														
CTPS	41	52	45													
TRFO	15	22	19	19												
TRPR	24	17	17	33	72											
TRTAX	07	27	08	31	68	66										
TRPS	29	19	20	34	33	49	37									
ETFO	57	51	46	43	13	18	07	23								
ETPR	35	28	01	16	12	29	12	18	37							
ETTAX	46	33	19	39	01	19	14	21	61	35						
ETPS	32	25	26	22	04	01	07	09	32	19	45					
NNCL	38	16	13	08	09	16	04	06	41	22	36	29				
NNFO	23	16	01	19	27	30	34	34	14	09	21	11	27			
NNPR	38	30	23	20	18	17	15	15	39	23	20	30	34	60		
NNTAX	46	34	14	31	31	40	34	31	37	31	29	24	36	55	55	
NNPS	13	01	11	-02	22	22	19	03	11	05	08	07	24	50	48	43

Total		
r	*N*	*Percent*
.22 - (-.02)	63	46
.23 - .29 ($p < .05$)	16	12
.30 - .72 ($p < .01$)	57	42

[a] Decimals have been omitted.

Table A-4-12. Correlations Among Project Talent Tests, Grade 9[a]

	Introductory High School Math	*Math Information*	*Mechanical Reasoning*	*Visualization in 3D*	*Capitalization*	*English Usage*	*Biological Science Information*
Introductory High School Math							
Math Information	72						
Mechanical Reasoning	49	59					
Visualization in 3D	39	50	56				
Capitalization	47	61	44	47			
English Usage	50	57	52	38	49		
Biological Science Information	40	60	51	38	49	39	
Physical Science Information	51	63	60	48	47	33	70

Total		
r	*N*	*Percent*
.22 - ( )	0	0
.23 - .29 ($p < .05$)	0	0
.30 - .72 ($p < .01$)	28	100

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[a] Decimals have been omitted.

Table A-4-13. Correlations Among CLD Tests, Grade 12[a]

	CTFO	*CTPR*	*CTTAX*	*CTPS*	*TRFO*	*TRPR*	*TRTAX*	*TRPS*	*ETFO*	*ETPR*	*ETTAX*	*ETPS*	*NNCL*	*NNFO*	*NNPR*	*NNTAX*
CTFO																
CTPR	52															
CTTAX	22	17														
CTPS	40	72	-14													
TRFO	25	12	25	-03												
TRPR	19	18	39	17	61											
TRTAX	24	13	14	20	72	57										
TRPS	08	03	31	06	52	61	48									
ETFO	20	19	19	14	26	46	25	27								
ETPR	51	33	15	37	28	36	32	20	69							
ETTAX	41	37	30	10	35	41	23	15	57	55						
ETPS	50	43	09	40	17	33	31	22	43	55	55					
NNCL	20	05	-05	05	01	03	01	01	09	32	24	24				
NNFO	29	19	19	10	05	23	19	17	33	30	27	35	23			
NNPR	30	13	29	09	24	27	27	31	34	44	38	23	39	58		
NNTAX	41	39	15	18	08	20	14	15	43	41	47	43	38	62	53	
NNPS	30	22	31	08	05	35	08	15	37	34	48	35	36	54	49	45

Total		
r	*N*	*Percent*
.22 - (-.14)	53	39
.23 - .29 ($p < .05$)	19	14
.30 - .72 ($p < .01$)	64	47

[a] Decimals have been omitted.

Table A-4-14. Correlations Among Project Talent Tests, Grade 12[a]

	Introductory High School Math	*Math Information*	*Mechanical Reasoning*	*Visualization in 3D*	*Capitalization*	*English Usage*	*Biological Science Information*
Introductory High School Math							
Math Information	85						
Mechanical Reasoning	61	54					
Visualization in 3D	49	49	66				
Capitalization	41	36	29	25			
English Usage	54	57	63	58	36		
Biological Science Information	61	68	51	52	23	63	
Physical Science Information	57	65	49	52	24	49	69

Total

r	*N*	*Percent*
.22 - ( )	0	0
.23 - .29 ($p < .05$)	4	14
.30 - .85 ($p < .01$)	24	86

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[a] Decimals have been omitted.

Table A-5-1. Analysis of Variance Summary for the Groupement Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (First, Fourth, and Seventh), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*	*F value*	*MS*
Groupement I	3.34	1.46	34.10	14.94^{c}	1.01	.44	1.53	.67	1.30	.57	.01	.00	.53	.23	2.28
Groupement II	6.52	2.69	32.58	13.43^{c}	.74	.31	.41	.17	.09	.04	3.18	1.31	1.38	.57	2.42
Groupement III	8.76	1.78	14.20	2.88	4.25	.86	1.96	.40	.49	.10	9.00	1.82	.20	.04	4.93
Groupement IV	17.20	3.34	56.50	10.99^{c}	2.83	.55	5.18	1.01	1.67	.32	16.13	3.14^{b}	4.51	.88	5.14
Combined Groupements I-IV	131.98	3.89^{b}	497.56	14.65^{c}	.27	.01	20.41	.60	2.03	.06	70.01	2.06	11.15	.33	33.97
Groupement V	.28	.23	4.49	3.69^{b}	2.29	1.88	1.21	.99	.02	.02	.23	.19	1.75	1.44	1.22
Groupement VI	.23	.17	15.68	11.48^{c}	.03	.02	.56	.41	4.52	3.31	.46	.33	.78	.57	1.37
Groupement VII	2.52	1.08	14.74	6.33^{c}	.22	.09	.31	.14	.01	.00	1.13	.49	2.34	1.01	2.33
Groupement VIII	.05	.03	33.18	17.20^{c}	.03	.02	.85	.44	4.49	2.33	.02	.01	.76	.39	1.93
Combined Groupements V-VIII	3.47	.31	235.22	20.70^{c}	5.41	.48	9.67	.85	19.97	1.76	.07	.01	14.79	1.30	11.36

[a] Degrees of Freedom: Assessment Condition = 1, Grade = 2, Sex = 1, Condition × Grade = 2, Condition × Sex = 1, Grade × Sex = 2, Condition × Grade × Sex = 2, Error = 216.

[b] $p < .05$

[c] $p < .01$

Table A-5-2. Analyses of Variance Summary for Class Inclusion, Combinatorial Reasoning Task, Auditory Memory Span, Auditory Sequential Memory, and Visual Orientation Memory with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (First, Fourth, and Seventh), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Class Inclusion	5.99	3.59	20.86	12.51^{c}	.00	.00	.05	.03	.87	.52	.39	.23	.02	.01	1.67
Combinatorial Reasoning	50.32	1.07	1310.45	27.98^{c}	.35	.01	31.55	.67	2.41	.05	37.00	.79	1.28	.03	46.84
Auditory Memory Span	41.93	.42	1951.77	19.69^{c}	3.78	.04	108.69	1.10	34.11	.34	214.92	2.17	51.32	.52	99.11
Auditory Sequential Memory	58.39	.32	1770.04	9.71^{c}	101.96	.56	243.56	1.34	.18	.00	20.90	.11	55.59	.30	182.35
Visual Orientation Memory	1.27	.14	551.13	61.12^{c}	.22	.02	11.25	1.25	9.22	1.02	27.60	3.06^{b}	13.89	1.54	9.02

[a] Degrees of freedom: Assessment condition = 1, Grade = 2, Sex = 1, Condition × Grade = 2, Condition × Sex = 1, Grade × Sex = 2, Condition × Grade × Sex = 2, Error = 216.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-3. Analyses of Variance Summary for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (First and Fourth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
	MS	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Some-All	.06	.24	3.05	12.86[b]	.01	.05	.16	.66	.12	.50	.01	.03	.10	.44	.24
Dichotomies	.07	.12	20.14	35.77[b]	1.13	2.00	.85	1.51	1.16	2.07	.25	.45	.04	.08	.56
Cardinality	.48	.39	52.37	43.21[b]	.10	.08	3.23	2.67	1.61	1.32	.46	.38	.46	.38	1.21
Arithmetic Addition	.02	.00	119.04	18.82[b]	13.59	2.15	.04	.01	3.17	.50	5.69	.90	2.74	.43	6.32

[a] Degrees of freedom: Assessment condition, Grade, Sex, Condition × Grade, Condition × Sex, Grade × Sex, Condition × Grade × Sex = 1; Error = 142.

[b] $p < .01$.

Table A-5-4. Analyses of Variance Summary for the Seriation, Length Transitivity, and Weight Transitivity Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level, and Sex (First and Fourth)

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Seriation	1.82	2.11	55.88	64.87^{c}	1.22	1.41	1.17	1.36	1.22	1.41	1.60	1.86	.40	.47	.86
Transitivity of length	1.39	.58	56.25	23.64^{c}	.02	.01	1.12	.47	.63	.27	.30	.13	.16	.07	2.38
Transitivity of weight	.00	.00	4.32	4.50^{b}	1.54	1.60	.09	.09	.08	.09	4.25	4.42^{b}	.03	.03	.96

[a] Degrees of freedom: Assessment condition, Grade, Sex, Condition × Grade, Condition × Sex, Grade × Sex, Condition × Grade × Sex = 1; Error = 142.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-5. Analyses of Variance Summary for the Number Conservation, Length Conservation, and Weight Conservation Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (First and Fourth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Conservation of Number	3.32	1.17	96.80	34.26^c	.02	.01	3.28	1.16	1.10	.39	.28	.10	.03	.01	2.83
Identity Conservation of Length	.78	.33	75.87	32.51^c	1.58	.68	15.13	6.48^b	.00	.00	1.53	.66	.28	.12	2.33
Equivalence Conservation of Length	11.09	2.18	117.03	23.01^c	3.90	.77	28.61	5.63^b	3.90	.77	.15	.03	2.28	.45	5.09
Identity Conservation of Weight	7.75	2.79	46.91	16.87^c	.49	.18	.99	.36	8.22	2.96	1.96	.70	.04	.02	2.78
Equivalence Conservation of Weight	12.76	2.32	63.98	11.61^c	1.27	.23	.31	.06	.42	.08	4.95	.90	3.06	.56	5.51

[a] Degrees of freedom: Assessment condition, Grade, Sex, Condition × Grade, Condition × Sex, Grade × Sex, Condition × Grade × Sex = 1; Error = 142.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-6. Analyses of Variance Summary for the Groupment Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (Second, Fifth, and Eighth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Groupment I	.24	.09	58.73	23.09[c]	4.31	1.69	4.75	1.87	2.89	1.14	.83	.33	2.82	1.11	2.54
Groupment II	.12	.05	51.95	21.32[c]	10.22	4.19[b]	1.17	.48	7.55	3.10	.23	.10	2.58	1.06	2.44
Groupment III	3.03	.60	86.35	17.00[c]	1.58	.31	6.65	1.31	5.61	1.10	.42	.08	13.76	2.71	5.08
Groupment IV	34.31	8.03[c]	71.36	16.70[c]	1.87	.44	3.35	.78	3.41	.80	9.40	2.20	19.34	4.52[b]	4.27
Combined Groupments I-IV	59.89	1.82	1034.63	31.43[c]	26.64	.81	45.01	1.37	24.69	.75	12.49	.38	121.04	3.68[b]	32.92
Groupment V	1.63	1.74	8.09	8.61[c]	.00	.00	1.02	1.09	1.32	1.41	1.48	1.58	1.74	1.85	.94
Groupment VI	.60	.49	2.69	2.19	1.11	.90	.37	.30	.01	.01	.19	.15	.49	.40	1.23
Groupment VII	9.68	5.12[b]	9.54	5.05[c]	5.76	3.04	2.15	1.14	1.79	.95	12.70	6.72[c]	3.67	1.94	1.89
Groupment VIII	16.58	10.02[c]	33.43	20.20[c]	8.65	5.23[b]	1.33	.80	1.07	.65	2.67	1.61	1.74	1.05	1.65
Combined Groupments V-VIII	59.01	6.07[b]	166.24	17.11[c]	41.73	4.29[b]	7.93	.82	.93	.10	13.24	1.36	5.41	.35	9.72

[a] Degrees of freedom: Assessment condition = 1, Grade = 2, Sex = 1, Condition × Grade = 2, Condition × Sex = 1, Grade × Sex = 2, Condition × Grade × Sex = 2, Error = 202.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-7. Analyses of Variance Summary for Class Inclusion, Combinatorial Reasoning Task, Auditory Memory Span, Auditory Sequential Memory, and Visual Orientation Memory with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (Second, Fifth, and Eighth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Class Inclusion	15.14	15.69^{c}	19.45	20.16^{c}	.16	.17	3.12	3.24^{b}	.64	.66	.04	.05	6.47	6.71^{c}	.96
Combinatorial Reasoning	24.52	.62	1563.72	39.31^{c}	72.34	1.82	169.35	4.26^{b}	3.16	.08	70.12	1.76	49.55	1.25	39.76
Auditory Memory Span	59.29	.66	3591.05	40.22^{c}	62.50	.70	54.34	.61	5.46	.06	323.49	3.62^{b}	13.71	.15	89.28
Auditory Sequential Memory	51.10	.31	4824.40	29.56^{c}	52.12	.32	147.88	.91	11.10	.07	318.98	1.95	6.02	.04	163.20
Visual Orientation Memory	17.58	2.22	244.63	30.93^{c}	69.21	8.75^{c}	27.50	3.48^{b}	2.52	.32	19.79	2.50	6.23	.79	7.91

[a] Degrees of freedon: Assessment condition = 1, Grade = 2, Sex = 1, Condition × Grade = 2, Condition × Sex = 1, Grade × Sex = 2, Condition × Grade × Sex = 2; Error = 202.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-8. Analyses of Variance Summary for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (Second and Fifth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Some-All	.32	2.81	.02	.16	.32	2.85	.10	.87	.02	.20	.30	2.63	.13	1.11	.11
Dichotomies	2.28	4.75^{b}	16.89	35.17^{c}	.08	.16	1.02	2.12	2.58	5.37^{b}	.45	.93	.02	.04	.48
Cardinality	7.70	7.30^{c}	12.30	11.67^{c}	.04	.04	1.92	1.82	.70	.66	.27	.25	.22	.21	1.05
Arithmetic Addition	2.95	.88	33.69	10.05^{c}	2.66	.79	4.04	1.20	1.77	.53	1.79	.54	4.68	1.40	3.35

[a] Degrees of freedom: Assessment condition, Grade, Sex, Condition × Grade, Condition × Sex, Grade × Sex, Condition × Grade × Sex = 1; Error = 130.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-9. Analyses of Variance Summary for the Seriation, Length Transitivity, and Weight Transitivity Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (Second and Fifth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Seriation	4.27	5.51[b]	14.92	19.23[c]	.49	.63	.65	.84	.31	.40	.00	.00	.04	.05	.78
Transitivity of Length	2.08	.77	7.76	2.88	5.58	2.08	.20	.08	2.52	.94	7.19	2.67	9.40	3.49	2.69
Transitivity of Weight	.76	.51	10.30	6.84[c]	5.57	3.70	3.65	2.42	.00	.00	1.75	1.16	1.21	.81	1.51

[a] Degrees of freedom: Assessment of condition, Grade, Sex, Condition × Grade, Condition × Sex, Condition × Grade × Sex = 1; Error = 130.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-10. Analyses of Variance Summary for the Number Conservation, Length Conservation, and Weight Conservation Tasks with Three Factors: Assessment Condition (Longitudinal vs. Testing Control Subjects), Grade Level (Second and Fifth), and Sex

	Sources of Variance[a]														
	Condition		*Grade*		*Sex*		*Condition × Grade*		*Condition × Sex*		*Grade × Sex*		*Condition × Grade × Sex*		*Error*
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*
Conservation of Number	3.91	2.81	5.44	3.92^{b}	1.50	1.08	.00	.00	8.72	6.27^{b}	.64	.46	.87	.62	1.39
Identity Conservation of Length	9.52	5.49^{b}	19.63	11.32^{c}	3.99	2.30	.06	.03	1.12	.64	.13	.08	7.23	4.17^{b}	1.73
Equivalence Conservation of Length	34.30	11.04^{c}	20.24	6.51^{b}	1.62	.52	2.30	.74	.01	.00	.89	.29	.00	.00	3.11
Identity Conservation of Weight	6.53	2.47	22.92	8.66^{c}	6.48	2.45	.00	.00	4.21	1.59	.35	.13	.30	.11	2.65
Equivalence Conservation of Weight	7.91	1.75	3.99	.88	8.23	1.82	14.73	3.27	6.52	1.45	6.10	1.35	.20	.04	4.51

[a] Degrees of freedom: Assessment condition, Grade, Sex, Condition × Grade, Condition × Sex, Grade × Sex, Condition × Grade × Sex = 1; Error = 130.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-11. Means and Standard Deviations for the Four Class Groupement Tasks for Subject Assessed in All Four Years

Assessment Year and Grade	*Groupements*								*Total Class*	
	I		*II*		*III*		*IV*			
	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
First Year										
Kindergarten										
Males (*n* = 21)	3.76	1.37	4.43	1.69	3.05	1.07	3.67	1.39	14.90	2.95
Females (*n* = 20)	4.70	1.42	4.80	1.94	3.35	1.87	3.85	2.18	16.70	5.22
Total (*n* = 41)	4.22	1.46	4.61	1.80	3.20	1.50	3.76	1.80	15.78	4.26
Third										
Males (*n* = 22)	4.46	1.06	4.68	1.36	3.18	2.26	4.14	1.94	16.45	4.65
Females (*n* = 25)	4.48	1.12	4.52	1.01	3.32	2.46	5.08	2.22	17.40	4.07
Total (*n* = 47)	4.47	1.08	4.60	1.17	3.26	2.35	4.64	2.12	16.96	4.33
Sixth										
Males (*n* = 21)	5.86	1.42	5.91	1.55	4.29	2.05	6.43	1.47	22.48	4.97
Females (*n* = 25)	6.36	1.22	6.56	1.36	4.76	2.55	6.32	2.38	24.00	5.52
Total (*n* = 46)	6.13	1.33	6.26	1.47	4.54	2.33	6.37	1.99	23.30	5.27
Combined Grades										
Males (*n* = 64)	4.69	1.54	5.00	1.64	3.50	1.93	4.73	2.00	17.92	5.34
Females (*n* = 70)	5.21	1.50	5.33	1.70	3.84	2.41	5.17	2.44	19.56	5.92
Total (*n* = 134)	4.96	1.54	5.17	1.68	3.68	2.19	4.96	2.25	18.78	5.69
Second Year										
First										
Males (*n* = 21)	4.62	.92	4.95	1.36	3.38	2.31	4.43	2.48	17.32	5.32
Females (*n* = 20)	4.95	1.19	5.10	1.52	3.90	2.15	4.15	2.25	18.10	5.19
Total (*n* = 41)	4.78	1.06	5.02	1.42	3.63	2.22	4.29	2.35	17.73	5.20
Fourth										
Males (*n* = 22)	5.68	1.73	5.50	1.63	4.18	2.09	6.41	1.76	21.77	6.06
Females (*n* = 25)	5.56	1.69	5.56	1.56	3.96	2.07	5.60	2.50	20.68	6.14
Total (*n* = 47)	5.62	1.69	5.53	1.57	4.06	2.06	5.98	2.20	21.19	6.06

Table A-5-11 continued

Assessment Year and Grade	*Groupements*								*Total Class*	
	I		*II*		*III*		*IV*			
	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
Seventh										
Males ($n = 21$)	6.19	1.60	6.38	1.60	4.19	2.06	6.76	1.92	23.52	5.60
Females ($n = 25$)	6.36	1.66	6.60	1.63	5.52	2.18	6.92	1.91	25.40	5.99
Total ($n = 46$)	6.28	1.62	6.50	1.60	4.91	2.21	6.85	1.90	24.54	5.83
Combined Grades										
Males ($n = 64$)	5.50	1.58	5.61	1.62	3.92	2.16	5.88	2.29	20.91	6.15
Females ($n = 70$)	5.67	1.63	5.80	1.67	4.50	2.24	5.66	2.47	21.63	6.49
Total ($n = 134$)	5.59	1.61	5.71	1.64	4.22	2.21	5.76	2.38	21.28	6.32
Third Year										
Second										
Males ($n = 21$)	5.24	1.48	4.86	1.11	3.38	1.96	5.14	2.63	18.62	4.97
Females ($n = 20$)	5.35	1.57	4.60	1.39	4.05	2.26	5.35	2.30	19.35	4.92
Total ($n = 41$)	5.29	1.50	4.73	1.25	3.71	2.11	5.24	2.45	18.98	4.90
Fifth										
Males ($n = 22$)	5.82	1.71	5.82	1.59	4.46	2.09	7.05	1.21	23.14	5.38
Females ($n = 25$)	5.52	1.81	5.72	1.75	3.96	2.41	6.72	2.39	21.92	6.79
Total ($n = 47$)	5.66	1.75	5.77	1.66	4.19	2.25	6.87	1.92	22.49	6.14
Eighth										
Males ($n = 21$)	6.38	1.69	6.38	1.80	5.19	2.80	7.05	1.66	25.00	6.70
Females ($n = 25$)	6.72	1.49	6.68	1.57	5.96	2.35	7.24	1.42	26.60	5.93
Total ($n = 46$)	6.57	1.57	6.54	1.67	5.61	2.57	7.15	1.52	25.87	6.27
Combined Grades										
Males ($n = 64$)	5.81	1.67	5.69	1.63	4.34	2.39	6.42	2.09	22.27	6.24
Females ($n = 70$)	5.90	1.72	5.74	1.78	4.70	2.50	6.51	2.18	22.86	6.63
Total ($n = 134$)	5.86	1.69	5.72	1.70	4.53	2.45	6.47	2.13	22.57	6.43

Fourth Year											
Third											
Males	(*n* = 21)	5.29	1.27	5.57	1.54	4.52	1.99	6.48	2.23	21.86	4.75
Females	(*n* = 20)	5.50	1.61	5.85	1.69	4.20	2.46	6.20	2.40	21.75	6.00
Total	(*n* = 41)	5.39	1.43	5.71	1.60	4.37	2.21	6.34	2.29	21.80	5.33
Sixth											
Males	(*n* = 22)	7.00	1.45	7.09	1.45	4.27	2.12	7.59	1.05	25.95	4.42
Females	(*n* = 25)	6.32	1.87	6.32	1.89	5.20	2.18	7.00	2.02	24.84	6.41
Total	(*n* = 47)	6.64	1.70	6.68	1.72	4.77	2.18	7.28	1.65	25.36	5.54
Ninth											
Males	(*n* = 21)	6.67	1.49	6.71	1.52	5.57	2.34	7.52	1.25	26.48	5.55
Females	(*n* = 25)	7.44	1.12	7.36	1.41	6.16	2.01	7.60	1.12	28.56	4.84
Total	(*n* = 46)	7.09	1.35	7.07	1.48	5.89	2.16	7.57	1.17	27.61	5.22
Combined Grades											
Males	(*n* = 64)	6.33	1.57	6.47	1.61	4.78	2.19	7.20	1.64	24.78	5.27
Females	(*n* = 70)	6.49	1.73	6.56	1.77	5.26	2.31	6.99	1.94	25.29	6.32
Total	(*n* = 134)	6.41	1.65	6.52	1.69	5.03	2.26	7.09	1.80	25.04	5.82

Table A-5-12. Means and Standard Deviations for the Four Relations Groupement Tasks for Subjects Assessed in All Four Years

	Groupements									
	V		*VI*		*VII*		*VIII*		*Total Relations*	
Assessment Year and Grade	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
First Year										
Kindergarten										
Males (*n* = 21)	6.24	1.64	6.38	1.88	5.24	2.21	5.05	1.94	22.90	5.30
Females (*n* = 20)	6.40	1.60	6.10	2.05	5.35	2.32	5.05	1.73	22.90	5.47
Total (*n* = 41)	6.32	1.60	6.24	1.95	5.29	2.24	5.05	1.82	22.90	5.31
Third										
Males (*n* = 22)	6.82	1.56	7.86	.47	6.32	2.06	6.27	1.52	27.27	4.10
Females (*n* = 25)	7.16	1.25	7.64	1.11	5.80	2.45	6.04	1.90	26.64	3.86
Total (*n* = 47)	7.00	1.40	7.75	.87	6.04	2.27	6.15	1.72	26.94	3.94
Sixth										
Males (*n* = 21)	7.76	.62	7.57	1.21	6.71	1.27	6.91	1.34	28.95	2.96
Females (*n* = 25)	7.08	1.80	7.56	1.26	6.76	1.51	6.76	1.48	28.16	4.11
Total (*n* = 46)	7.39	1.42	7.57	1.22	6.74	1.39	6.83	1.40	28.52	3.61
Combined Grades										
Males (*n* = 64)	6.94	1.48	7.28	1.44	6.09	1.97	6.08	1.77	26.39	4.88
Females (*n* = 70)	6.91	1.58	7.17	1.61	6.01	2.17	6.01	1.82	26.11	4.89
Total (*n* = 134)	6.93	1.52	7.22	1.53	6.05	2.07	6.05	1.79	26.25	4.87
Second Year										
First										
Males (*n* = 21)	7.48	.87	7.24	1.97	6.52	1.72	6.14	1.85	27.38	4.41
Females (*n* = 20)	6.80	1.79	6.65	1.76	6.35	2.03	6.00	1.69	25.80	4.95
Total (*n* = 41)	7.15	1.42	6.95	1.87	6.44	1.86	6.07	1.75	26.61	4.69
Fourth										
Males (*n* = 22)	7.55	1.10	7.86	.35	7.32	1.04	6.77	1.31	29.50	2.60
Females (*n* = 25)	7.28	1.02	7.36	1.32	7.16	1.21	6.64	1.50	28.44	2.79
Total (*n* = 47)	7.40	1.06	7.60	1.01	7.23	1.13	6.70	1.40	28.94	2.72

Seventh											
Males	(*n* = 21)	7.43	1.08	7.91	.44	6.86	2.08	7.48	.87	29.67	3.29
Females	(*n* = 25)	7.72	.68	7.92	.40	7.16	1.49	7.28	1.17	30.08	2.74
Total	(*n* = 46)	7.59	.88	7.91	.41	7.02	1.77	7.37	1.04	29.89	2.98
Combined Grades											
Males	(*n* = 64)	7.48	1.01	7.67	1.20	6.91	1.67	6.80	1.48	28.86	3.60
Females	(*n* = 70)	7.30	1.24	7.36	1.33	6.93	1.60	6.69	1.52	28.27	3.88
Total	(*n* = 134)	7.39	1.14	7.51	1.27	6.92	1.63	6.74	1.50	28.55	3.74
Third Year											
Second											
Males	(*n* = 21)	7.33	1.15	7.33	1.35	7.33	1.28	7.00	.89	29.00	2.63
Females	(*n* = 20)	7.20	1.51	7.30	1.30	6.60	1.57	5.75	1.97	26.85	4.42
Total	(*n* = 41)	7.27	1.33	7.32	1.31	6.98	1.46	6.39	1.63	27.95	3.73
Fifth											
Males	(*n* = 22)	7.86	.47	7.82	.85	7.36	1.36	7.68	.57	30.73	2.03
Females	(*n* = 25)	7.52	1.05	7.48	1.61	7.76	.60	7.48	.96	30.24	2.20
Total	(*n* = 47)	7.68	.84	7.64	1.31	7.57	1.04	7.57	.80	30.47	2.12
Eighth											
Males	(*n* = 21)	7.81	.51	7.81	.68	7.48	1.12	7.48	1.17	30.57	2.68
Females	(*n* = 25)	7.84	.47	7.68	1.25	7.64	1.04	7.52	1.16	30.68	3.30
Total	(*n* = 46)	7.83	.49	7.74	1.02	7.57	1.07	7.50	1.15	30.63	3.00
Combined Grades											
Males	(*n* = 64)	7.67	.80	7.66	1.01	7.39	1.24	7.39	.94	30.11	2.54
Females	(*n* = 70)	7.54	1.07	7.50	1.39	7.39	1.20	7.00	1.58	29.43	3.69
Total	(*n* = 134)	7.60	.95	7.58	1.22	7.39	1.21	7.19	1.32	29.75	3.20
Fourth Year											
Third											
Males	(*n* = 21)	7.24	1.22	7.95	.22	6.91	1.79	6.86	1.49	28.95	3.43
Females	(*n* = 20)	7.60	.88	7.80	.70	7.25	1.48	6.45	1.70	29.10	3.52
Total	(*n* = 41)	7.42	1.07	7.88	.51	7.07	1.63	6.66	1.59	29.02	3.43
Sixth											
Males	(*n* = 22)	7.91	.43	8.00	.00	7.50	1.19	7.59	.85	31.00	2.05
Females	(*n* = 25)	7.36	1.08	8.00	.00	7.40	.96	7.00	1.44	29.76	2.63
Total	(*n* = 47)	7.62	.87	8.00	.00	7.45	1.06	7.28	1.23	30.34	2.43

Table A-5-12 continued

Assessment Year and Grade		*Groupements*								*Total Relations*	
		V		*VI*		*VII*		*VIII*			
		$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	X	*SD*	$\bar{X}$	*SD*
Ninth											
Males	(*n* = 21)	**7.76**	**.62**	**7.86**	**.65**	**7.57**	**1.25**	**7.24**	**1.22**	**30.43**	**2.84**
Females	(*n* = 25)	**7.60**	**1.00**	**8.00**	**.00**	**7.64**	**1.11**	**7.40**	**1.76**	**30.64**	**3.63**
Total	(*n* = 46)	**7.67**	**.84**	**7.94**	**.44**	**7.61**	**1.16**	**7.33**	**1.52**	**30.54**	**3.26**
Combined Grades											
Males	(*n* = 64)	**7.64**	**.86**	**7.94**	**.39**	**7.33**	**1.44**	**7.23**	**1.23**	**30.14**	**2.91**
Females	(*n* = 70)	**7.51**	**.99**	**7.94**	**.38**	**7.44**	**1.18**	**6.99**	**1.66**	**29.89**	**3.29**
Total	(*n* = 134)	**7.58**	**.93**	**7.94**	**.38**	**7.39**	**1.30**	**7.10**	**1.47**	**30.01**	**3.10**

Table A-5-13. Means and Standard Deviations for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span Test, Auditory Sequential Memory Test, and Visual Orientation Memory Tests for Subjects Assessed in All Four Years

Assessment Year and Grade	*CI*		*CR*		*AMS*		*ASM*		*VOM*	
	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
First Year										
Kindergarten										
Males (*n* = 21)	2.10	1.45	6.81	6.38	23.76	4.73	22.38	10.19	9.33	3.31
Females (*n* = 20)	3.15	1.42	9.00	8.02	24.90	10.10	24.30	13.98	8.50	2.24
Total (*n* = 41)	2.61	1.51	7.88	7.22	24.32	7.74	23.32	12.07	8.93	2.83
Third Grade										
Males (*n* = 22)	3.68	.99	18.41	8.20	30.59	7.99	29.91	9.88	13.41	3.08
Females (*n* = 25)	3.72	1.24	19.52	8.82	30.04	10.54	26.92	10.64	13.24	3.64
Total (*n* = 47)	3.70	1.12	19.00	8.46	30.30	9.34	28.32	10.29	13.32	3.36
Sixth Grade										
Males (*n* = 21)	4.10	1.18	23.24	7.00	33.57	10.41	34.48	10.36	16.95	1.83
Females (*n* = 25)	4.44	.92	24.76	6.98	35.44	9.31	35.60	11.73	17.76	2.09
Total (*n* = 46)	4.28	1.05	24.07	6.96	34.59	9.76	35.09	11.02	17.39	1.99
Combined Grades										
Males (*n* = 64)	3.30	1.48	16.19	9.93	29.33	8.93	28.94	11.15	13.23	4.17
Females (*n* = 70)	3.81	1.29	18.39	10.12	30.50	10.72	29.27	12.84	13.50	4.62
Total (*n* = 134)	3.57	1.40	17.34	10.05	29.94	9.88	29.11	12.02	13.37	4.40
Second Year										
First Grade										
Males (*n* = 21)	3.10	1.41	15.95	8.44	28.57	8.67	27.05	11.58	12.52	4.17
Females (*n* = 20)	3.45	1.19	16.65	9.23	34.65	10.39	32.70	17.00	12.45	3.72
Total (*n* = 41)	3.27	1.30	16.29	8.73	31.54	9.92	29.80	14.58	12.49	3.91
Fourth Grade										
Males (*n* = 22)	4.00	1.20	23.64	5.84	36.64	10.98	35.73	16.07	17.05	2.24
Females (*n* = 25)	4.16	1.11	22.24	7.52	33.68	11.78	33.40	14.45	14.64	3.29
Total (*n* = 47)	4.09	1.14	22.89	6.75	35.06	11.39	34.49	15.10	15.77	3.07

Table A-5-13 continued

Assessment Year and Grade		CI		CR		AMS		ASM		VOM	
		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
Seventh Grade											
Males	(n = 21)	4.43	1.08	24.57	5.08	41.14	9.08	37.43	12.98	17.29	2.37
Females	(n = 25)	4.56	.82	25.08	5.60	41.48	8.30	35.60	10.08	18.00	1.50
Total	(n = 46)	4.50	.94	24.85	5.32	41.33	8.57	36.43	11.40	17.67	1.96
Combined Grades											
Males	(n = 64)	3.84	1.34	21.42	7.56	35.47	10.83	33.44	14.24	15.64	3.71
Females	(n = 70)	4.10	1.12	21.66	8.10	36.74	10.72	33.99	13.75	15.21	3.68
Total	(n = 134)	3.98	1.23	21.55	7.82	36.13	10.75	33.72	13.94	15.42	3.69
Third Year											
Second Grade											
Males	(n = 21)	3.86	1.01	19.81	7.65	31.52	9.27	28.71	10.15	15.33	2.87
Females	(n = 20)	3.95	1.15	17.75	9.72	32.05	9.21	31.65	15.51	13.70	3.23
Total	(n = 41)	3.90	1.07	18.81	8.67	31.78	9.13	30.15	12.96	14.54	3.12
Fifth Grade											
Males	(n = 22)	4.64	.79	25.68	3.82	37.91	10.90	34.23	13.42	17.91	1.69
Females	(n = 25)	4.12	1.20	23.04	6.45	32.96	10.16	30.36	13.15	16.28	4.03
Total	(n = 47)	4.36	1.05	24.28	5.49	35.28	10.69	32.17	13.28	17.04	3.23
Eighth Grade											
Males	(n = 21)	4.24	.94	24.91	6.77	46.05	9.94	46.19	16.15	17.62	1.75
Females	(n = 25)	4.92	.28	26.52	2.86	46.36	8.31	45.48	11.15	18.52	1.87
Total	(n = 46)	4.61	.74	25.78	5.04	46.22	8.99	45.80	13.50	18.11	1.85
Combined Grades											
Males	(n = 64)	4.25	.96	23.50	6.70	38.48	11.56	36.34	15.13	16.97	2.42
Females	(n = 70)	4.36	1.04	22.77	7.47	37.49	11.30	36.13	14.80	16.34	3.67
Total	(n = 134)	4.31	1.00	23.12	7.09	37.96	11.39	36.23	14.90	16.64	3.14

Fourth Year

Third Grade											
Males	(*n* = 21)	4.19	1.21	23.10	6.40	32.90	7.38	32.43	10.06	16.57	2.14
Females	(*n* = 20)	4.45	.83	20.45	8.91	36.30	9.59	31.60	9.06	16.75	2.94
Total	(*n* = 41)	4.32	1.04	21.81	7.74	34.56	8.59	32.02	9.48	16.66	2.53
Sixth Grade											
Males	(*n* = 22)	4.68	.72	26.59	2.13	41.91	11.19	42.32	12.52	18.64	1.09
Females	(*n* = 25)	4.24	1.09	24.48	5.55	36.48	10.18	35.68	13.59	17.96	2.26
Total	(*n* = 47)	4.45	.95	25.47	4.39	39.02	10.90	38.79	13.39	18.28	1.83
Ninth Grade											
Males	(*n* = 21)	4.62	.74	27.57	1.54	40.14	7.29	48.81	13.97	18.76	1.14
Females	(*n* = 25)	4.88	.33	28.00	.00	43.80	8.44	51.00	11.25	19.24	1.09
Total	(*n* = 46)	4.76	.57	27.80	1.05	42.13	8.06	50.00	12.47	19.02	1.13
Combined Grades											
Males	(*n* = 64)	4.50	.93	25.77	4.35	38.37	9.55	41.20	13.85	18.00	1.81
Females	(*n* = 70)	4.53	.85	24.59	6.46	39.04	9.94	39.99	14.21	18.07	2.36
Total	(*n* = 134)	4.52	.88	25.15	5.57	38.72	9.73	40.57	14.00	18.04	2.11

Table A-5-14. Means and Standard Deviations for the Some-all, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks for Subjects Assessed in All Four Years

Assessment Year and Grade		*Some-All*		*Dichotomies*		*Cardinality*		*Arithmetic addition*	
		$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
First Year									
Kindergarten									
Males	(*n* = 21)	3.52	.60	1.14	.73	1.67	1.24	4.67	5.69
Females	(*n* = 20)	3.75	.64	.90	.97	1.80	1.24	6.85	5.52
Total	(*n* = 41)	3.63	.62	1.02	.85	1.73	1.23	5.73	5.64
Third Grade									
Males	(*n* = 22)	3.77	.53	2.00	.76	4.00	.82	15.77	.53
Females	(*n* = 25)	3.88	.44	2.04	.79	3.56	1.12	15.48	.96
Total	(*n* = 47)	3.83	.48	2.02	.77	3.77	1.00	15.62	.80
Combined Grades									
Males	(*n* = 43)	3.65	.57	1.58	.85	2.86	1.57	10.35	6.86
Females	(*n* = 45)	3.82	.53	1.53	1.04	2.78	1.46	11.64	5.70
Total	(*n* = 88)	3.74	.56	1.56	.95	2.82	1.51	11.01	6.29
Second Year									
First Grade									
Males	(*n* = 21)	3.62	.59	1.71	.90	3.10	1.09	13.67	3.58
Females	(*n* = 20)	3.70	.66	1.80	1.11	2.95	1.54	13.85	3.38
Total	(*n* = 41)	3.66	.62	1.76	.99	3.02	1.31	13.76	3.44
Fourth Grade									
Males	(*n* = 22)	3.86	.35	2.73	.55	4.18	.96	15.68	1.29
Females	(*n* = 25)	3.92	.28	2.72	.54	3.64	.86	15.80	.65
Total	(*n* = 47)	3.89	.31	2.72	.54	3.89	.94	15.75	.99
Combined Grades									
Males	(*n* = 43)	3.74	.49	2.23	.90	3.65	1.15	14.70	2.82
Females	(*n* = 45)	3.82	.49	2.31	.95	3.33	1.24	14.93	2.47
Total	(*n* = 88)	3.78	.49	2.27	.92	3.49	1.20	14.82	2.64

Third Year									
Second Grade									
Males	(*n* = 21)	3.81	.40	2.38	.92	3.62	1.07	15.10	2.19
Females	(*n* = 20)	3.95	.22	2.05	.89	3.95	.94	15.15	2.01
Total	(*n* = 41)	3.88	.33	2.22	.91	3.78	1.01	15.12	2.08
Fifth Grade									
Males	(*n* = 22)	3.96	.21	2.91	.29	4.41	.80	15.77	.69
Females	(*n* = 25)	4.00	.00	2.76	.52	4.12	.83	16.00	.00
Total	(*n* = 47)	3.98	.15	2.83	.43	4.26	.82	15.89	.48
Combined Grades									
Males	(*n* = 43)	3.88	.32	2.65	.72	4.02	1.01	15.44	1.62
Females	(*n* = 45)	3.98	.15	2.44	.79	4.04	.88	15.62	1.39
Total	(*n* = 88)	3.93	.25	2.55	.76	4.03	.94	15.53	1.50
Fourth Year									
Third Grade									
Males	(*n* = 21)	3.76	.54	2.71	.72	4.33	.91	15.86	.48
Females	(*n* = 20)	3.85	.49	2.55	.60	3.85	1.09	15.80	.41
Total	(*n* = 41)	3.81	.51	2.63	.66	4.10	1.02	15.83	.44
Sixth Grade									
Males	(*n* = 22)	3.91	.29	2.77	.53	4.32	.84	15.46	1.53
Females	(*n* = 25)	3.96	.20	2.80	.50	3.84	.99	15.92	.28
Total	(*n* = 47)	3.94	.25	2.79	.51	4.06	.94	15.70	1.08
Combined Grades									
Males	(*n* = 43)	3.84	.43	2.74	.62	4.33	.87	15.65	1.15
Females	(*n* = 45)	3.91	.36	2.69	.56	3.84	1.02	15.87	.34
Total	(*n* = 88)	3.88	.40	2.72	.59	4.08	.97	15.76	.84

Table A-5-15. Means and Standard Deviations for the Seriation, Length Transitivity, and Weight Transitivity Tasks for Subjects Assessed in All Four Years

Assessment Year and Grade		*Seriation*		*Length Transitivity*		*Weight Transitivity*	
		$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
First Year							
Kindergarten							
Males	**(*n* = 21)**	**1.71**	**1.06**	**3.57**	**2.01**	**4.38**	**1.07**
Females	**(*n* = 20)**	**1.95**	**1.00**	**3.90**	**1.83**	**4.50**	**1.32**
Total	**(*n* = 41)**	**1.83**	**1.02**	**3.73**	**1.91**	**4.44**	**1.18**
Third Grade							
Males	**(*n* = 22)**	**3.73**	**.83**	**4.50**	**1.47**	**4.77**	**1.07**
Females	**(*n* = 25)**	**3.60**	**.65**	**4.40**	**1.44**	**4.92**	**.40**
Total	**(*n* = 47)**	**3.66**	**.73**	**4.45**	**1.44**	**4.85**	**.78**
Combined Grades							
Males	**(*n* = 43)**	**2.74**	**1.38**	**4.05**	**1.80**	**4.58**	**1.07**
Females	**(*n* = 45)**	**2.87**	**1.16**	**4.18**	**1.63**	**4.73**	**.94**
Total	**(*n* = 88)**	**2.81**	**1.27**	**4.11**	**1.70**	**4.66**	**1.00**
Second Year							
First Grade							
Males	**(*n* = 21)**	**3.10**	**.83**	**3.71**	**2.15**	**4.86**	**.65**
Females	**(*n* = 20)**	**2.90**	**1.25**	**3.80**	**2.02**	**4.40**	**1.57**
Total	**(*n* = 41)**	**3.00**	**1.05**	**3.76**	**2.06**	**4.63**	**1.20**
Fourth Grade							
Males	**(*n* = 22)**	**4.41**	**.67**	**5.00**	**.00**	**5.00**	**.00**
Females	**(*n* = 25)**	**3.92**	**.76**	**4.84**	**.55**	**4.92**	**.40**
Total	**(*n* = 47)**	**4.15**	**.75**	**4.92**	**.41**	**4.96**	**.29**
Combined Grades							
Males	**(*n* = 43)**	**3.77**	**1.00**	**4.37**	**1.62**	**4.93**	**.46**
Females	**(*n* = 45)**	**3.47**	**1.12**	**4.38**	**1.48**	**4.69**	**1.10**
Total	**(*n* = 88)**	**3.61**	**1.07**	**4.38**	**1.54**	**4.81**	**.86**
Third Year							
Second Grade							
Males	**(*n* = 21)**	**3.48**	**.93**	**3.95**	**2.01**	**4.76**	**1.09**
Females	**(*n* = 20)**	**3.70**	**1.03**	**4.25**	**1.83**	**4.30**	**1.72**
Total	**(*n* = 41)**	**3.59**	**.97**	**4.10**	**1.91**	**4.54**	**1.43**
Fifth Grade							
Males	**(*n* = 22)**	**4.14**	**.77**	**4.77**	**1.07**	**5.00**	**.00**
Females	**(*n* = 25)**	**4.16**	**.80**	**4.76**	**.66**	**4.60**	**1.38**
Total	**(*n* = 47)**	**4.15**	**.78**	**4.77**	**.87**	**4.79**	**1.02**
Combined Grades							
Males	**(*n* = 43)**	**3.81**	**.91**	**4.37**	**1.63**	**4.88**	**.76**
Females	**(*n* = 45)**	**3.96**	**.93**	**4.53**	**1.32**	**4.47**	**1.53**
Total	**(*n* = 88)**	**3.89**	**.92**	**4.46**	**1.48**	**4.67**	**1.23**
Fourth Year							
Third Grade							
Males	**(*n* = 21)**	**4.00**	**.84**	**4.76**	**.77**	**4.76**	**1.09**
Females	**(*n* = 20)**	**4.20**	**.83**	**4.50**	**1.40**	**4.65**	**1.18**
Total	**(*n* = 41)**	**4.10**	**.83**	**4.63**	**1.11**	**4.71**	**1.12**

Table A-5-15 continued

Assessment Year and Grade		*Seriation*		*Length Transitivity*		*Weight Transitivity*	
		$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
Sixth Grade							
Males	**(*n* = 22)**	**4.64**	**.49**	**5.00**	**.00**	**5.00**	**.00**
Females	**(*n* = 25)**	**3.96**	**.79**	**4.56**	**1.39**	**4.72**	**1.06**
Total	**(*n* = 47)**	**4.28**	**.74**	**4.77**	**1.03**	**4.85**	**.78**
Combined Grades							
Males	**(*n* = 43)**	**4.33**	**.75**	**4.88**	**.54**	**4.88**	**.76**
Females	**(*n* = 45)**	**4.07**	**.81**	**4.53**	**1.38**	**4.69**	**1.10**
Total	**(*n* = 88)**	**4.19**	**.79**	**4.71**	**1.06**	**4.78**	**.95**

Table A-5-16. Means and Standard Deviations for the Number Conservation, Length Conservation, and Weight Conservation Tasks for Subjects Assessed in All Four Years

Assessment Year and Grade	*Number Conservation*		*Length Conservation*				*Weight Conservation*			
			Identity		*Equivalence*		*Identity*		*Equivalence*	
	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*	$\bar{X}$	*SD*
First Year										
Kindergarten										
Males (*n* = 21)	3.91	2.14	3.05	1.86	1.81	2.02	3.19	1.66	1.33	1.93
Females (*n* = 20)	3.95	2.39	3.70	1.63	2.50	2.33	3.90	2.08	2.50	2.40
Total (*n* = 41)	3.93	2.24	3.37	1.76	2.15	2.17	3.54	1.89	1.90	2.22
Third Grade										
Males (*n* = 22)	6.82	.50	5.32	1.43	5.27	1.91	5.32	1.55	5.05	2.15
Females (*n* = 25)	6.52	1.12	4.60	1.80	3.64	2.83	4.60	1.83	4.08	2.66
Total (*n* = 47)	6.66	.89	4.94	1.66	4.40	2.55	4.94	1.72	4.53	2.46
Combined Grades										
Males (*n* = 43)	5.40	2.12	4.21	1.99	3.58	2.61	4.28	1.92	3.23	2.76
Females (*n* = 45)	5.38	2.20	4.20	1.77	3.13	2.65	4.29	1.95	3.38	2.64
Total (*n* = 88)	5.39	2.15	4.21	1.87	3.35	2.63	4.28	1.92	3.31	2.68
Second Year										
First Grade										
Males (*n* = 21)	5.71	2.03	4.19	1.94	4.00	2.49	4.33	1.96	3.19	2.70
Females (*n* = 20)	5.30	2.27	4.55	1.73	3.85	2.52	4.40	1.82	4.10	2.47
Total (*n* = 41)	5.51	2.13	4.37	1.83	3.93	2.47	4.37	1.87	3.63	2.60
Fourth Grade										
Males (*n* = 22)	6.86	.35	5.50	1.22	5.32	1.84	5.73	.94	5.55	1.50
Females (*n* = 25)	6.76	.52	5.00	1.68	4.36	2.55	5.16	1.63	4.44	2.52
Total (*n* = 47)	6.81	.45	5.23	1.49	4.81	2.27	5.43	1.36	4.96	2.16
Combined Grades										
Males (*n* = 43)	6.30	1.54	4.86	1.73	4.67	2.25	5.05	1.66	4.40	2.45
Females (*n* = 45)	6.11	1.71	4.80	1.70	4.13	2.52	4.82	1.74	4.29	2.47
Total (*n* = 88)	6.21	1.62	4.83	1.70	4.40	2.40	4.93	1.69	4.34	2.45

Third Year										
Second Grade										
Males (*n* = 21)	6.86	.65	5.19	1.54	5.14	1.88	5.00	1.67	4.81	2.11
Females (*n* = 20)	6.20	2.02	4.65	1.63	4.80	1.79	4.25	2.00	3.80	2.59
Total (*n* = 41)	6.54	1.50	4.93	1.59	4.98	1.82	4.63	1.85	4.32	2.38
Fifth Grade										
Males (*n* = 22)	7.00	.00	5.64	1.18	5.64	1.18	5.82	.85	5.68	1.13
Females (*n* = 25)	7.00	.00	5.92	.40	5.56	1.08	5.16	1.57	5.04	2.01
Total (*n* = 47)	7.00	.00	5.79	.86	5.60	1.12	5.47	1.32	5.34	1.67
Combined Grades										
Males (*n* = 43)	6.93	.46	5.42	1.37	5.40	1.56	5.42	1.37	5.26	1.72
Females (*n* = 45)	6.64	1.38	5.36	1.28	5.22	1.48	4.76	1.81	4.49	2.34
Total (*n* = 88)	6.78	1.04	5.39	1.32	5.31	1.51	5.08	1.63	4.86	2.09
Fourth Year										
Third Grade										
Males (*n* = 21)	6.34	2.11	5.81	.60	5.71	1.31	5.57	1.21	4.91	2.12
Females (*n* = 20)	6.60	1.39	5.00	1.65	5.10	1.74	4.95	1.82	4.65	2.06
Total (*n* = 41)	6.46	1.78	5.42	1.28	5.42	1.55	5.27	1.55	4.78	2.07
Sixth Grade										
Males (*n* = 22)	7.00	.00	5.82	.85	5.73	1.28	6.00	.00	6.00	.00
Females (*n* = 25)	7.00	.00	5.72	.98	5.72	.84	5.48	1.33	5.44	1.33
Total (*n* = 47)	7.00	.00	5.77	.91	5.72	1.06	5.72	.99	5.70	1.00
Combined Grades										
Males (*n* = 43)	6.67	1.49	5.81	.73	5.72	1.28	5.79	.86	5.47	1.56
Females (*n* = 45)	6.82	.94	5.40	1.36	5.44	1.34	5.24	1.57	5.09	1.72
Total (*n* = 88)	6.75	1.23	5.60	1.11	5.58	1.31	5.51	1.30	5.27	1.65

Table A-5-17. Trend Analyses Summary for the Groupement Tasks for the Four Assessment Years ($N = 134$)

	Sources of Variance and Trend Components[a]															
	Sex		*Grade*		*Sex × Grade*			*Linear*		*Linear × Sex*		*Linear × Grade*		*Linear × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Groupement I	29.75	1.70	442.31	25.33[c]	29.24	1.68	17.46	2850.18	79.25[c]	47.47	1.32	145.04	4.03[b]	41.39	1.15	35.96
Groupement II	14.69	.73	436.16	21.78[c]	22.55	1.13	20.03	2184.19	55.60[c]	24.49	.62	221.76	5.65[c]	12.30	.31	39.28
Groupement III	102.75	2.70	437.54	11.49[c]	24.02	.63	38.07	2545.19	34.17[c]	1.05	.01	17.47	.23	40.96	.55	74.49
Groupement IV	.30	.009	755.87	23.17[c]	3.51	.11	32.63	6735.07	103.52[c]	91.37	1.40	351.11	5.40[c]	66.94	1.03	65.06
Groupement V	7.17	.97	62.04	8.37[c]	.78	.10	7.41	627.61	19.90[c]	2.16	.07	58.18	1.84	60.12	1.91	31.54
Groupement VI	11.07	1.23	104.55	11.58[c]	4.36	.48	9.03	658.28	32.34[c]	8.49	.42	279.02	13.71[c]	1.24	.06	20.36
Groupement VII	.09	.005	118.90	6.94[c]	3.72	.22	17.13	2686.57	55.11[c]	10.31	.21	82.72	1.70	12.15	.25	48.75
Groupement VIII	22.17	1.49	275.07	18.46[c]	7.67	.51	14.90	1762.66	50.27[c]	23.25	.66	145.34	4.15[c]	34.59	.99	35.06
Class Composition	202.22	1.13	3974.14	22.27[c]	206.19	1.16	178.44	26488.48	94.00[c]	187.52	.67	1988.53	7.06[c]	62.64	.22	281.76
Class Negation	33.05	.93	796.12	22.40[c]	25.10	.71	35.54	4884.19	69.06[c]	44.59	.63	550.79	7.79[c]	68.67	.97	70.73
Combined Classes	398.76	1.34	7929.67	26.59[c]	266.98	.90	298.23	54121.26	141.90[c]	415.00	1.09	1762.85	4.62[c]	207.89	.55	381.42
Relations Composition	67.93	1.84	783.24	21.22[c]	26.04	.71	36.91	7021.64	101.04[c]	.09	.001	576.54	8.30[c]	24.06	.35	69.49
Relations Reciprocity	4.69	.17	306.50	11.31[c]	2.47	.09	27.10	3688.13	75.16[c]	.02	.0004	373.08	7.60[c]	11.96	.24	49.07
Combined Relations	108.32	.93	2058.46	17.58[c]	44.49	.38	117.08	20887.53	117.12[c]	.03	.0002	1827.82	10.25[c]	47.57	.27	178.35

Table A-5-17 continued

	Sources of variance and trend components[a]																		
	Quadratic		*Quadratic × Sex*		*Quadratic × Grade*		*Quadratic × Sex × Grade*			*Cubic*		*Cubic × Sex*		*Cubic × Grade*		*Cubic × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Groupement I	.75	.14	6.05	1.12	7.47	1.39	3.62	.67	5.38	55.19	2.21	.46	.02	73.68	2.94	.08	.003	25.03
Groupement II	9.14	1.56	.98	.17	3.82	.65	10.55	1.80	5.86	233.80	9.97[c]	.92	.04	18.80	.80	6.15	.26	[illegible].44
Groupement III	.27	.02	.45	.03	2.26	.17	32.03	2.42	13.26	25.10	.45	21.33	.38	49.75	.89	27.45	.4[illegible]	55.63
Groupement IV	4.30	.43	3.98	.40	25.47	2.57	11.32	1.14	9.91	.00	.00	84.02	2.39	4.46	.13	27.89	.79	35.17
Groupement V	32.51	10.58[c]	.90	.29	1.17	.38	17.42	5.67[c]	3.07	.00	.00	2.42	.13	16.01	.85	21.83	1.17	18.73
Groupement VI	.90	.29	4.49	1.45	6.55	2.11	.62	.20	3.10	35.53	1.70	4.34	.21	6.86	.33	12.26	.59	20.93
Groupement VII	100.43	14.48[c]	.01	.001	14.54	2.10	14.53	2.10	6.93	.75	.03	2.54	.10	17.22	.65	27.39	1.03	26.57
Groupement VIII	80.72	13.46[c]	1.20	.20	.22	.04	5.96	.99	6.00	10.78	.41	14.28	.54	56.93	2.15	36.37	1.37	26.52
Class Composition	43.10	1.09	22.29	.56	35.20	.89	6.14	.15	39.66	496.75	3.11	2.86	.02	208.62	1.31	17.15	.11	159.57
Class Negation	48.96	4.74[c]	.003	.0003	15.43	1.49	29.55	2.86	10.33	29.62	.59	6.65	.13	.88	.02	49.71	.98	50.56
Combined Classes	.19	.004	22.80	.44	66.22	1.27	14.60	.28	52.08	768.96	3.33	18.23	.08	203.20	.88	38.47	.17	230.78
Relations Composition	247.19	26.73[c]	4.36	.47	3.02	.33	6.36	.69	9.25	26.87	.59	1.20	.03	45.23	.99	44.22	.97	45.74
Relations Reciprocity	64.54	9.14[c]	4.74	.67	4.40	.62	42.33	5.99[c]	7.06	48.96	1.90	.41	.02	21.01	.82	2.64	.10	25.74
Combined Relations	564.37	24.31[c]	18.18	.78	14.27	.61	76.29	3.29[b]	23.21	3.29	.03	3.01	.03	124.11	1.16	67.52	.63	107.14

[a] Degrees of freedom: Sex = 1, Grade = 2, Sex × Grade = 1, Error = 128, Linear = 1, Linear × Sex = 1, Linear × Grade = 2, Linear × Sex × Grade = 2, Quadratic = 1, Quadratic × Sex = 1, Quadratic × Grade = 2, Quadratic × Sex × Grade = 2, Cubic = 1, Cubic × Sex = 1, Cubic × Grade = 2, Cubic × Sex × Grade = 2, Error = 128.

[b] $p < .05$

[c] $p < .01$.

Table A-5-18. Trend Analyses Summary for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span, Auditory Sequential Memory, and Visual Orientation Memory for the Four Assessment Years ($N = 134$)

	Sources of Variance and Trend Components[a]															
	Sex		*Grade*		*Sex × Grade*			*Linear*		*Linear × Sex*		*Linear × Grade*		*Linear × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Class Inclusion	27.65	3.33	175.92	21.18[c]	21.14	2.54	8.31	1347.95	84.19[c]	87.27	5.45[b]	199.75	12.48[c]	27.40	1.71	16.01
Combinatorial Reasoning	9.21	.03	16160.68	54.33[c]	245.68	.83	297.48	83850.02	125.84[c]	4117.93	6.18[b]	11410.58	17.12[c]	624.43	.94	666.33
Auditory Memory Span	149.58	.15	19460.77	19.64[c]	1973.83	1.99	990.90	106404.29	197.56[c]	478.93	.89	226.53	.42	1568.10	2.91	538.59
Auditory Sequential Memory	10.13	.01	30677.19	17.80[c]	1858.70	1.08	1723.12	182190.15	148.12[c]	981.20	.80	10644.09	8.65[c]	978.56	.80	1230.05
Visual Orientation Memory	17.10	.29	4215.79	70.85[c]	181.12	3.04	59.50	31026.27	357.93[c]	20.45	.24	4321.44	49.85[c]	17.97	.21	86.68

	Sources of Variance and Trend Components[a]																
	Quadratic		*Quadratic × Sex*		*Quadratic × Grade*		*Quadratic × Sex × Grade*			*Cubic*		*Cubic × Sex*		*Cubic × Grade*		*Cubic × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Class Inclusion	5.44	1.66	1.11	.34	.66	.20	3.57	1.09	3.28	.19	.01	.06	.01	1.41	.10	31.18	2.31	13.48
Combinatorial Reasoning	636.30	4.93[b]	76.39	.59	479.12	3.71[b]	30.85	.24	11.36	1279.07	1.81	7.92	.01	341.37	.48	171.90	.24	706.86
Auditory Memory Span	3955.10	24.36[c]	81.81	.50	1154.34	7.11[c]	130.72	.81	162.38	1457.94	1.65	1332.97	1.51	3943.38	4.45[b]	1080.86	1.22	885.69
Auditory Sequential Memory	10.22	.03	49.48	.14	634.73	1.85	514.66	1.50	342.63	2072.60	1.78	18.25	.02	11329.97	9.74[c]	160.17	.14	1163.09
Visual Orientation Memory	56.49	3.25	64.53	3.71	55.12	3.17[b]	36.33	2.09	17.38	132.01	1.56	5.47	.06	18.46	.22	214.09	2.52	84.89

[a] Degrees of freedom: Sex = 1, Grade = 2, Sex × Grade = 1, Error = 128, Linear = 1, Linear × Sex = 1, Linear × Grade = 2, Linear × Sex × Grade = 2, Quadratic = 1, Quadratic × Sex = 1, Quadratic × Grade = 2, Quadratic × Sex × Grade = 2, Cubic = 1, Cubic × Sex = 1, Cubic × Grade = 2, Cubic × Sex × Grade = 2, Error = 128.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-19. Trend Analyses Summary for the Number Conservation, Length Conservation, and Weight Conservation Tasks for the Four Assessment Years ($N = 88$)

	Sources of Variance and Trend Components^a															
	Sex		*Grade*		*Sex × Grade*			*Linear*		*Linear × Sex*		*Linear × Grade*		*Linear × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Conservation of Number Identity	2.64	.20	558.28	43.10^c	.70	.05	12.95	1919.56	42.46^c	3.55	.08	1214.18	26.85^c	1.82	.04	45.21
Conservation of Length Equivalence	6.57	.40	296.21	18.21^c	2.64	.16	16.27	1985.50	57.03^c	32.54	.93	286.10	8.22^c	343.44	9.86^c	34.81
Conservation of Length Identity	45.52	1.61	374.59	13.26^c	27.85	.99	28.24	5070.73	85.68^c	17.13	.29	829.27	14.01^c	531.61	8.98^c	59.18
Conservation of Weight Equivalence	44.57	2.15	318.67	15.39^c	19.07	.92	20.71	1290.56	36.80^c	97.62	2.78	193.04	5.51^b	154.20	4.40^b	35.07
Conservation of Weight	26.82	.82	775.82	23.79^c	91.13	2.79	32.61	3627.56	55.12^c	108.84	1.65	622.20	9.45^c	337.74	5.13^b	65.82

Table A-5-19 continued

	Sources of Variance and Trend Components[a]																		
	Quadratic		*Quadratic × Sex*		*Quadratic × Grade*		*Quadratic × Sex × Grade*			*Cubic*		*Cubic × Sex*		*Cubic × Grade*		*Cubic × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Conservation of Number	63.92	17.71[c]	8.11	2.25	48.24	13.37[c]	13.60	3.77	3.61	12.38	.52	4.43	.19	1.75	.07	5.06	.21	23.68
Identity Conservation of Length	14.73	3.47	1.98	.47	.93	.22	2.13	.50	4.24	6.55	.22	3.46	.12	30.45	1.04	48.15	1.64	29.28
Equivalence Conservation of Length	52.55	4.76[b]	.01	.01	24.90	2.26	7.54	.68	11.04	22.00	.51	19.11	.44	27.69	.64	.47	.01	43.46
Identity Conservation of Weight	4.10	.78	2.71	.51	.06	.01	3.36	.64	5.27	54.10	1.66	12.71	.39	1.99	.06	2.29	.07	32.59
Equivalence Conservation of Weight	34.38	3.06	9.07	.81	30.35	2.70	3.41	.30	11.24	13.92	.22	46.87	.74	16.70	.26	154.66	2.44	63.49

[a] Degrees of freedom: Sex = 1, Grade = 1, Sex × Grade = 1, Error = 84, Linear = 1, Linear × Sex = 1, Linear × Grade = 1, Linear × Sex × Grade = 1, Quadratic = 1[illegible]uadratic × Sex = 1, Quadratic × Grade = 1, Quadratic × Sex × Grade = 1, Cubic = 1, Cubic × Sex = 1, Cubic × Grade = 1, Cubic × Sex × Grade [illegible], Error = 84.

[b] $p < .05$.

[c] $p < .01$.

Table A-5-20. Trend Analyses Summary for the Some-all, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks for the Four Assessment Years ($N = 88$)

	Sources of Variance and Trend Components[a]															
	Sex		Grade		Sex × Grade			Linear		Linear × Sex		Linear × Grade		Linear × Sex × Grade		
Variable	MS	F Value	MS	F Value	MS	F Value	Error	MS	F Value	MS	F Value	MS	F Value	MS	F Value	Error
Some-All	3.82	3.73	9.11	8.86[c]	.42	.41	1.02	27.28	7.08[c]	1.67	.43	2.18	.57	.17	.04	3.85
Dichotomies	1.18	.29	164.45	41.12[c]	1.73	.43	4.00	1237.50	153.07[c]	2.07	.26	181.34	22.43[c]	.00	.00	8.08
Cardinality	16.28	1.91	251.03	29.52[c]	13.72	1.61	8.50	1649.56	103.99[c]	16.12	1.02	944.33	59.53[c]	12.49	.79	15.86
Arithmetic Addition	81.66	2.02	3391.74	83.76[c]	18.62	.46	40.50	19710.10	135.47[c]	238.82	1.64	21232.98	145.94[c]	465.71	3.20	145.49

	Sources of Variance and Trend Components[a]																
	Quadratic		Quadratic × Sex		Quadratic × Grade		Quadratic × Sex × Grade			Cubic		Cubic × Sex		Cubic × Grade		Cubic × Sex × Grade		
Variable	MS	F Value	MS	F Value	MS	F Value	MS	F Value	Error	MS	F Value	MS	F Value	MS	F Value	MS	F Value	Error
Some-All	.92	1.43	.12	.18	.01	.01	.01	.01	.64	8.28	2.86	.46	.16	2.62	.90	.47	.16	2.89
Dichotomies	26.18	16.54[c]	.01	.01	4.03	2.55	.81	.51	1.58	10.23	1.63	15.83	2.52	.79	.13	4.58	.73	6.29
Cardinality	34.38	16.39[c]	1.57	.75	9.80	4.67[b]	1.10	.52	2.10	12.38	.77	44.06	2.73	14.84	.92	8.52	.53	16.13
Arithmetic Addition	1127.56	58.60[c]	26.37	1.37	1059.65	55.07[c]	23.20	1.21	19.24	595.92	17.02[c]	18.38	.52	876.73	25.04[c]	28.54	.81	35.02

[a] Degrees of freedom: Sex = 1, Grade = 1, Sex × Grade = 1, Error = 84, Linear = 1, Linear × Sex = 1, Linear × Grade = 1, Linear × Sex × Grade = 1, Quadratic = 1, Quadratic × Sex = 1, Quadratic × Grade = 1, Quadratic × Sex × Grade = 1, Cubic = 1, Cubic × Sex = 1, Cubic × Grade = 1, Cubic × Sex × Grade = 1, Error = 84.

[b] $p < .05$

[c] $p < .01$.

Table A-5-21. Trend Analyses Summary for the Seriation, Length Transitivity, and Weight Transitivity Tasks for the Four Assessment Years ($N = 88$)

	Sources of Variance and Trend Components[a]															
	Sex		*Grade*		*Sex × Grade*			*Linear*		*Linear × Sex*		*Linear × Grade*		*Linear × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Seriation	1.92	.35	306.05	55.93[b]	16.41	3.00	5.47	1728.41	159.61[b]	10.83	1.00	665.68	61.47[b]	11.43	1.06	10.83
Transitivity of Length	.06	.01	157.16	16.43[b]	7.38	.77	9.57	301.92	7.99[b]	36.56	.97	104.59	2.77	2.58	.07	37.78
Transitivity of Weight	10.81	2.63	29.56	7.18[b]	.49	.12	4.11	5.01	.25	32.51	1.65	14.89	.75	4.47	.23	19.76

	Sources of Variance and Trend Components[a]															
	Quadratic		*Quadratic × Sex*		*Quadratic × Grade*		*Quadratic × Sex × Grade*			*Cubic*		*Cubic × Sex*		*Cubic × Grade*		*Cubic × Sex × Grade*		
Variable	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*MS*	*F Value*	*Error*
Seriation	22.00	9.48[b]	.01	.01	1.92	.83	3.03	1.31	2.32	28.41	3.33	64.19	7.52[b]	.71	.08	3.45	.41	8.54
Transitivity of Length	.01	.01	3.28	.44	20.36	2.71	.01	.01	7.52	10.92	.26	19.77	.48	18.97	.46	1.08	.03	41.43
Transitivity of Weight	.10	.03	8.33	2.07	.05	.01	1.83	.45	4.03	25.10	1.14	.72	.03	.07	.01	3.07	.14	21.95

[a] Degrees of freedom: Sex = 1, Grade = 1, Sex × Grade = 1, Error = 84, Linear = 1, Linear × Sex = 1, Linear × Grade = 1, Linear × Sex × Grade = 1, Quadratic = 1, Quadratic × Sex = 1, Quadratic × Grade = 1, Quadratic × Sex × Grade = 1, Cubic = 1, Cubic × Sex = 1, Cubic × Grade = 1, Cubic × Sex × Grade = 1, Error = 84.

[b] $p < .01$.

Table A-6-1. Mean ($\bar{X}$), Standard Deviation, Hoyt Reliability Coefficient, and Standard Error of Measurement of 57 Tests, Kindergarten to Grade 9

Test Name, Abbreviation, and Number of Items	*Kindergarten*					*Grade 1*					*Grade 2*					*Grade 3*				
	%	$\bar{X}$	*SD*	*r*	SE_M	%	$\bar{X}$	*SD*	*r*	SE_M	%	$\bar{X}$	*SD*	*r*	SE_M	%	$\bar{X}$	*SD*	*r*	SE_M
Cutting tool, concrete (CTCO) 8						99	7.89	.31	.00	.31	99	7.95	.23	.00	.22	100	8.00	.00	—	—
Cutting tool, identity (CTID) 8						98	7.84	.37	.00	.37	99	7.89	.31	.00	.31	100	8.00	.00	—	—
Cutting tool, classificatory (CTCL) 8						94	7.54	.84	.47	.57	95	7.57	1.01	.69	.53	97	7.78	.71	.67	.38
Cutting tool, formal (CTFO) 18						78	14.03	2.69	.68	1.49	73	13.11	2.67	.64	1.55	82	14.76	2.25	.56	1.46
Cutting tool, principle (CTPR) 10						54	5.41	1.66	.36	1.26	38	3.78	1.64	.30	1.29	52	5.16	1.80	.35	1.38
Cutting tool, taxonomic relations (CTTAX) 8						77	6.14	1.60	.56	1.00	73	5.84	1.61	.45	1.12	73	5.84	2.01	.70	1.03
Cutting tool, problem solving (CTPS) 5						61	3.05	1.13	.43	.76	50	2.51	1.10	.18	.89	62	3.11	.97	.22	.76
Tree, concrete (TRCO) 8						97	7.78	.42	.00	.43	93	7.46	.69	.05	.63	99	7.95	.33	.57	.20
Tree, identity (TRID) 8						96	7.68	.47	.00	.51	94	7.54	.84	.49	.56	98	7.84	.60	.64	.34
Tree, classificatory (TRCL) 8						80	6.43	2.08	.81	.85	92	7.38	.95	.46	.66	94	7.49	1.56	.91	.43
Tree, formal (TRFO) 35						45	15.78	4.36	.78	2.03	57	19.97	3.61	.58	2.31	52	18.35	5.59	.79	2.53
Tree, principle (TRPR) 20						NA					6	1.16	1.14	.46	.82	27	5.46	3.85	.79	1.73
Tree, taxonomic relations (TRTAX) 8						24	1.89	1.05	.07	.95	34	3.70	1.10	.43	.78	48	3.81	1.91	.56	1.19
Tree, problem solving (TRPS) 10						18	1.78	1.34	.30	1.06	29	2.86	1.92	.63	1.10	46	4.59	1.94	.44	1.37
Equilateral triangle, concrete (ETCO) 8	92	7.35	1.06	.55	.67	97	7.76	.55	.31	.43	99	7.89	.31	.00	.30	100	8.00	.00	—	—
Equilateral triangle, identity (ETID) 8	88	7.00	1.39	.65	.78	97	7.73	.61	.38	.45	100	8.00	.00	—	—	100	8.00	.00	—	—
Equilateral triangle, classificatory (ETCL) 3	65	1.95	.88	.38	.56	77	2.32	1.00	.73	.43	95	2.84	.55	.74	.23	97	2.92	.36	.60	.19

Table A-6-1 continued

Test Name, Abbreviation, and Number of Items	Kindergarten					Grade 1					Grade 2					Grade 3				
	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	r	SD	SE_M
Equilateral triangle, formal (ETFO) 18 (11 for Kg)	22	2.41	1.32	.32	1.06	36	6.43	2.92	.63	1.71	46	8.27	2.68	.51	1.82	59	10.70	2.46	.52	1.65
Equilateral triangle, principle (ETPR) 11	NA					NA					NA					43	4.78	1.99	.44	1.42
Equilateral triangle, taxonomic relations (ETTAX) 8	NA					33	2.62	1.06	.00	1.08	38	3.00	1.18	.09	1.05	49	3.92	1.89	.55	1.18
Equilateral triangle, problem solving (ETPS) 6	NA					NA					NA					51	3.05	1.20	.29	.92
Noun, concrete (NNCO) 8						86	6.89	1.22	.51	.80	98	7.84	.44	.25	.36	98	7.86	.67	.81	.28
Noun, identity (NNID) 8						63	5.00	1.94	.69	1.01	95	7.62	.68	.33	.52	96	7.65	1.03	.78	.45
Noun, classificatory (NNCL) 8						6	.49	1.04	.68	.55	51	4.11	1.91	.68	1.00	74	5.92	2.01	.79	.87
Noun, formal (NNFO) 25						NA					7	1.73	.84	.15	.75	32	8.08	3.09	.58	1.97
Noun, principle (NNPR) 10						NA					NA					32	3.16	1.86	.54	1.20
Noun, taxonomic relations (NNTAX) 8						NA					NA					37	2.95	1.51	.30	1.18
Noun, problem solving (NNPS) 5						NA					NA					25	1.27	1.10	.31	.82
Some-all (SA) 4	95	3.81	1.96	.95	.40	93	3.73	.56	.26	.42	97	3.86	.35	.00	.30	97	3.86	.42	.33	.30
Seriation (Ser) 5	38	1.92	.98	.31	.73	61	3.05	1.05	.53	.64	72	3.62	.95	.41	.66	83	4.14	.82	.33	.60
Transitivity of length (TL) 5	76	3.81	1.96	.95	.40	75	3.73	2.16	.99	.21	80	4.00	1.99	.99	.18	92	4.59	1.17	.90	.33
Transitivity of weight (TW) 5	89	4.46	1.17	.82	.44	94	4.68	1.18	.97	.18	95	4.73	1.15	1.00	.00	94	4.68	1.18	.97	.18
Dichotomous sorting (DS) 3	34	1.02	.85	NC*		61	1.84	.99	NC		79	2.38	.79	NC		90	2.70	.62	NC	
Cardinality (Car) 5	36	1.81	1.24	.60	.70	64	3.22	1.16	.41	.80	77	3.86	.92	.20	.73	83	4.16	.99	.44	.66
Conservation of number (CN) 7	57	3.97	2.35	.84	.88	81	5.68	2.10	.89	.65	97	6.81	1.15	1.00	.00	97	6.76	1.19	.97	.20
Identity conservation of length (ICL) 6	58	3.49	1.80	.80	.73	76	4.57	1.77	.82	.69	85	5.11	1.56	.84	.57	91	5.46	1.22	.81	.48

Equivalence conservation of length (ECL) 6	37	2.22	2.25	.86	.76	69	4.14	2.46	.94	.54	84	5.05	1.81	.91	.51	91	5.46	1.52	.94	.34
Identity conservation of weight (ICW) 6	60	3.59	1.89	.73	.89	74	4.46	1.86	.84	.68	81	4.86	1.65	.82	.64	91	5.46	1.24	.82	.49
Equivalence conservation of weight (ECW) 6	33	2.00	2.29	.89	.70	60	3.62	2.69	.96	.51	73	4.35	2.37	.94	.52	80	4.81	2.11	.94	.48
Class inclusion (CI) 5	54	2.68	1.45	.60	.82	68	3.38	1.19	.44	.79	79	3.97	1.01	.30	.76	87	4.35	1.01	.56	.60
Combinatorial reasoning (CR) 28	28	7.88	7.22	NC		58	16.35	8.52	NC		70	19.57	8.19	NC		80	22.32	7.19	NC	
Addition of primary classes (APC) 4	24	.95	1.13	.57	.64	26	1.05	1.22	.63	.65	33	1.30	1.53	.82	.56	34	1.35	1.42	.74	.62
Subtraction of primary classes (SPC) 4	80	3.19	1.15	.67	.57	94	3.76	.72	.74	.32	100	4.00	.00	—	—	100	4.00	.00	—	—
Addition of secondary classes (ASC) 4	33	1.32	1.27	.59	.71	27	1.08	1.44	.82	.53	20	.81	1.22	.74	.54	44	1.76	1.62	.83	.57
Subtraction of secondary classes (SSC) 4	84	3.35	1.09	.72	.50	96	3.84	.69	.88	.20	98	3.92	.36	.53	.22	99	3.97	.16	.00	.14
Bi-univocal multiplication of classes (BMC) 4	26	1.03	1.14	.56	.65	33	1.30	1.41	.73	.63	35	1.41	1.04	.21	.80	46	1.84	1.44	.69	.69
Bi-univocal division of classes (BDC) 4	51	2.05	1.18	.35	.82	55	2.19	1.37	.67	.68	62	2.49	1.59	.83	.56	68	2.70	1.51	.82	.56
Co-univocal multiplication of classes (CMC) 4	33	1.32	1.23	.54	.72	39	1.54	1.50	.76	.64	54	2.14	1.69	.86	.55	73	2.92	1.57	.90	.43
Co-univocal division of classes (CDC) 4	57	2.27	1.19	.42	.79	66	2.62	1.55	.83	.56	80	3.19	1.20	.72	.55	89	3.57	1.12	.91	.29
Addition of difference relations (ADR) 4	74	2.95	.97	.21	.75	87	3.46	.87	.51	.53	91	3.65	.75	.57	.43	91	3.62	.76	.56	.43
Reciprocity of difference relations (RDR) 4	85	3.38	.92	.53	.55	97	3.86	.42	.33	.30	95	3.78	.53	.35	.37	99	3.95	.33	.67	.16
Addition of equivalence relations (AER) 4	74	2.97	1.07	.53	.63	85	3.41	1.04	.70	.49	92	3.68	.71	.53	.42	99	3.97	.16	.00	.14
Reciprocity of equivalence relations (RER) 4	86	3.43	1.04	.76	.44	91	3.65	1.01	.90	.28	93	3.73	.65	.53	.39	99	3.97	.16	.00	.14

Table A-6-1 continued

Test Name, Abbrevattion, and Number of Items	*Kindergarten*					*Grade 1*					*Grade 2*					*Grade 3*				
	%	$\bar{X}$	*SD*	*r*	SE_M	%	$\bar{X}$	*SD*	*r*	SE_M	%	$\bar{X}$	*SD*	*r*	SE_M	%	$\bar{X}$	*r*	*SD*	SE_M
Bi-univocal multiplication of relations (BMR) 4	63	2.51	1.33	.61	.72	78	3.11	1.13	.60	.62	85	3.41	.98	.65	.51	85	3.41	.98	.66	.49
Bi-univocal reciprocity of relations (BRR) 4	73	2.92	1.28	.67	.63	87	3.46	.96	.64	.50	92	3.68	.71	.56	.41	94	3.76	.76	.81	.29
Co-univocal multiplication of relations (CMR) 4	61	2.43	1.12	.42	.74	72	2.89	1.10	.45	.70	76	3.03	.90	.15	.72	80	3.19	1.02	.56	.59
Co-univocal reciprocity of relations (CRR) 4	68	2.70	1.02	.23	.78	83	3.30	.88	.36	.61	90	3.59	.76	.54	.45	89	3.54	.77	.50	.47

Table A-6-1 continued

Test Name, Abbreviation, and Number of Items	Grade 4					Grade 5					Grade 6				
	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	SD	r	SE_M
Cutting tool, concrete (CTCO) 8	99	7.96	.21	.00	.20	99	7.96	.21	.00	.19	100	8.00	.00	—	—
Cutting tool, identity (CTID) 8	99	7.91	.29	.00	.28	99	7.98	.15	.00	.14	100	8.00	.00	—	—
Cutting tool, classificatory (CTCL) 8	93	7.47	1.22	.76	.55	98	7.82	.39	.00	.39	99	7.98	.15	.00	.14
Cutting tool, formal (CTFO) 18	76	13.76	2.96	.72	1.53	81	14.60	2.20	.58	1.39	89	15.96	3.18	.88	1.06
Cutting tool, principle (CTPR) 10	57	5.69	2.48	.67	1.35	54	5.40	2.46	.67	1.34	68	6.84	2.50	.77	1.14
Cutting tool, taxonomic relations (CTTAX) 8	86	6.89	1.32	.54	.84	81	6.47	1.34	.44	.94	88	7.02	1.32	.61	.77
Cutting tool, problem solving (CTPS) 5	72	3.62	1.28	.51	.81	72	3.60	1.25	.44	.84	72	3.58	1.06	.50	.67
Tree, concrete (TRCO) 8	98	7.82	.44	.14	.38	96	7.71	.55	.10	.49	100	8.00	.00	—	—
Tree, identity (TRID) 8	97	7.76	.43	.00	.45	95	7.60	.69	.28	.55	100	8.00	.00	—	—
Tree, classificatory (TRCL) 8	92	7.33	1.40	.78	.61	93	7.40	1.14	.66	.62	98	7.80	.76	.75	.35
Tree, formal (TRFO) 35	63	21.96	5.08	.79	2.29	72	25.18	5.01	.80	2.18	64	22.40	6.93	.88	2.36
Tree, principle (TRPR) 20	41	8.13	3.51	.69	1.90	47	9.38	3.15	.59	1.98	47	9.44	5.13	.86	1.84
Tree, taxonomic relations (TRTAX) 8	51	4.07	1.21	.16	1.04	60	4.76	1.25	.28	.99	67	5.38	1.76	.56	1.09
Tree, problem solving (TRPS) 10	30	3.02	1.48	.26	1.21	37	3.69	1.66	.28	1.34	53	5.16	2.19	.56	1.37
Equilateral triangle, concrete (ETCO) 8	98	7.84	.52	.50	.34	100	8.00	.00	—	—	100	8.00	.00	—	—
Equilateral triangle, identity (ETID) 8	97	7.73	.50	.01	.46	100	8.00	.00	—	—	100	8.00	.00	—	—
Equilateral triangle, classificatory (ETCL) 3	87	2.60	.91	.87	.27	93	2.78	.60	.62	.30	99	2.98	.15	.00	.12
Equilateral triangle, formal (ETFO) 18	55	9.98	3.21	.67	1.80	66	11.82	3.07	.66	1.74	80	14.33	2.25	.63	1.33
Equilateral triangle, principle (ETPR) 11	40	4.40	2.17	.50	1.46	54	5.98	2.38	.58	1.47	79	8.73	1.48	.18	1.29
Equilateral triangle, taxonomic relations (ETTAX) 8	54	4.31	1.64	.44	1.15	56	4.51	1.62	.35	1.22	75	5.96	1.83	.70	.94
Equilateral triangle, problem solving (ETPS) 6	37	2.24	1.23	.27	.96	39	2.33	1.33	.39	.95	68	4.09	1.47	.62	.83
Noun, concrete (NNCO) 8	99	7.98	.15	.00	.14	99	7.98	.15	.00	.14	100	8.00	.00	—	—
Noun, identity (NNID) 8	99	7.89	.38	.29	.30	99	7.93	.25	.00	.24	100	8.00	.00	—	—
Noun, classificatory (NNCL) 8	57	4.58	1.88	.65	1.04	74	5.93	1.97	.74	.94	77	6.18	1.97	.76	.90
Noun, formal (NNFO) 25	32	8.04	2.90	.47	2.06	46	11.49	3.70	.63	2.21	49	12.24	3.74	.64	2.19
Noun, principle (NNPR) 10	28	2.80	1.71	.37	1.30	36	3.58	1.63	.21	1.37	41	4.07	2.25	.63	1.29
Noun, taxonomic relations (NNTAX) 8	33	2.62	1.32	.10	1.17	50	3.98	.94	.00	1.12	52	4.18	1.50	.40	1.09
Noun, problem solving (NNPS) 5	24	1.20	1.08	.30	.81	31	1.53	1.24	.39	.86	40	1.98	1.25	.33	.92
Some-all (SA) 4	97	3.89	.32	.00	.29	99	3.98	.15	.00	.13	98	3.93	.25	.00	.22
Seriation (Ser) 5	83	4.13	.76	.19	.61	83	4.13	.79	.28	.60	86	4.29	.76	.20	.61
Transitivity of length (TL) 5	98	4.91	.42	.63	.23	96	4.80	.84	.91	.23	95	4.76	1.05	.98	.13
Transitivity of weight (TW) 5	99	4.96	.30	.62	.16	96	4.78	1.04	1.00	.00	97	4.84	.80	.95	.16
Dichotomous sorting (DS) 3	90	2.71	.55			95	2.84	.42			93	2.78	.52		
Cardinality (Car) 5	78	3.89	.96	.22	.76	85	4.24	.83	.06	.68	82	4.11	.93	.30	.70
Conservation of number (CN) 7	97	6.80	.46	.11	.40	100	7.00	.00	—	—	100	7.00	.00	—	—
Identity conservation of length (ICL) 6	87	5.24	1.51	.87	.50	96	5.75	.88	.87	.29	96	5.76	.93	.88	.30
Equivalence conservation of length (ECL) 6	82	4.89	2.20	.97	.35	93	5.58	1.14	.83	.42	95	5.71	1.08	.91	.29
Identity conservation of weight (ICW) 6	90	5.40	1.39	.88	.45	91	5.44	1.34	.87	.45	95	5.71	1.01	.88	.32
Equivalence conservation of weight (ECW) 6	82	4.91	2.19	.97	.33	89	5.31	1.70	.95	.36	95	5.69	1.02	.85	.36

Table A-6-1 continued

Test Name, Abbreviation, and Number of Items	Grade 4					Grade 5					Grade 6				
	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	SD	r	SE_M	%	$\bar{X}$	SD	r	SE_M
Class inclusion (CI) 5	82	4.11	1.11	.55	.67	88	4.38	1.05	.64	.56	89	4.44	.97	.60	.55
Combinatorial reasoning (CR) 28	81	22.69	6.83			86	24.20	5.60			91	25.36	4.45		
Addition of primary classes (APC) 4	42	1.69	1.59	.82	.58	42	1.67	1.71	.88	.51	70	2.78	1.59	.88	.47
Subtraction of primary classes (SPC) 4	98	3.93	.33	.52	.20	99	3.98	.15	.00	.13	97	3.87	.63	.89	.18
Addition of secondary classes (ASC) 4	42	1.67	1.49	.75	.65	45	1.78	1.64	.84	.56	67	2.69	1.73	.93	.39
Subtraction of secondary classes (SSC) 4	98	3.91	.42	.66	.21	99	3.98	.15	.00	.13	100	4.00	.00	—	—
Bi-univocal multiplication of classes (BMC) 4	31	1.24	1.43	.77	.60	32	1.27	1.42	.77	.59	46	1.82	1.32	.61	.72
Bi-univocal division of classes (BDC) 4	72	2.87	1.36	.74	.60	75	3.00	1.22	.67	.61	75	2.98	1.37	.30	.53
Co-univocal multiplication of classes (CMC) 4	60	2.38	1.61	.84	.55	77	3.09	1.29	.76	.55	85	3.38	1.25	.88	.38
Co-univocal division of classes (CDC) 4	90	3.60	.99	.83	.35	93	3.73	.89	.91	.24	97	3.87	.66	.93	.15
Addition of difference relations (ADR) 4	88	3.53	.94	.71	.44	95	3.78	.64	.64	.33	92	3.69	.70	.56	.40
Reciprocity of difference relations (RDR) 4	98	3.91	.36	.42	.24	98	3.93	.33	.52	.20	99	3.96	.30	.67	.15
Addition of equivalance relations (AER) 4	93	3.71	.73	.64	.38	95	3.80	.66	.74	.29	100	4.00	.00	—	—
Reciprocity of equivalence relations (RER) 4	97	3.87	.40	.26	.30	96	3.82	.72	.89	.21	100	4.00	.00	—	—
Bi-univocal multiplication of relations (BMR) 4	87	3.49	.76	.34	.53	95	3.78	.52	.30	.38	92	3.69	.67	.46	.42
Bi-univocal reciprocity of relations (BRR) 4	94	3.76	.53	.25	.40	96	3.82	.68	.84	.24	93	3.73	.58	.38	.40
Co-univocal multiplication of relations (CMR) 4	81	3.22	.88	.26	.65	95	3.78	.52	.34	.36	86	3.44	.87	.51	.53
Co-univocal reciprocity of relations (CRR) 4	86	3.42	.81	.40	.54	96	3.82	.49	.42	.32	95	3.80	.50	.37	.35

Table A-6-1 continued

	Grade 7					Grade 8					Grade 9				
Test Name, Abbreviation, and Number of Items	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*
Cutting tool, concrete (CTCO) 8	NA**					NA					NA				
Cutting tool, identity (CTID) 8	NA					NA					NA				
Cutting tool, classificatory (CTCL) 8	96	7.64	.78	.52	.51	96	7.70	.51	.00	.49	100	8.00	.00	—	—
Cutting tool, formal (CTFO) 18	76	13.68	3.84	.85	1.43	82	14.71	3.15	.79	1.41	90	16.11	3.03	.87	1.05
Cutting tool principle (CTPR) 10	58	5.82	2.76	.78	1.23	65	6.52	2.32	.65	1.29	79	7.86	2.44	.86	.88
Cutting tool, taxonomic relations (CTTAX) 8	78	6.27	1.68	.69	.88	84	6.75	1.37	.59	.82	92	7.32	1.20	.67	.65
Cutting tool, problem solving (CTPS) 5	62	3.11	1.65	.72	.78	73	3.66	1.49	.71	.72	75	3.77	.83	.31	.62
Tree, concrete (TRCO) 8	NA					NA					NA				
Tree, identity (TRID) 8	NA					NA					NA				
Tree, classificatory (TRCL) 8	88	7.05	1.87	.87	.63	99	7.89	.32	.00	.32	100	8.00	.00	—	—
Tree, formal (TRFO) 35	74	26.00	5.82	.88	2.02	76	26.48	5.46	.85	2.06	85	29.82	4.66	.85	1.79
Tree, principle (TRPR) 20	56	11.14	4.10	.79	1.83	59	11.77	3.65	.71	1.92	75	15.00	3.66	.78	1.68
Tree, taxonomic relations (TRTAX) 8	61	4.84	1.55	.49	1.03	71	5.64	1.30	.28	1.03	90	7.20	1.25	.63	.71
Tree, problem solvng (TRPS) 10	39	3.91	1.78	.39	1.31	44	4.39	2.19	.62	1.29	77	7.73	1.76	.55	1.11
Equilateral triangle, concrete (ETCO) 8	NA					NA					NA				
Equilateral triangle, identity (ETID) 8	NA					NA					NA				
Equilateral triangle, classificatory (ETCL) 3	95	2.86	.35	.00	.30	95	2.86	.41	.30	.28	100	3.00	.00	—	—
Equilateral triangle, formal (ETFO) 18	71	12.80	3.55	.80	1.53	78	14.05	2.80	.69	1.50	85	15.27	2.51	.81	1.07
Equilateral triangle, principle (ETPR) 11	65	7.11	2.27	.56	1.45	71	7.82	1.88	.40	1.40	76	8.36	1.77	.48	1.22
Equilateral triangle, taxonomic relations (ETTAX) 8	60	4.77	1.72	.50	1.14	68	5.43	1.59	.50	1.06	86	6.84	1.70	.77	.76
Equilateral triangle, problem solving (ETPS) 6	53	3.18	1.37	.42	.95	52	3.14	1.15	.26	.91	71	4.25	1.56	.64	.86
Noun, concrete (NNCO) 8	99	7.98	.15	.00	.14	NA					NA				
Noun, identity (NNID) 8	99	7.95	.21	.00	.20	NA					NA				
Noun, classificatory (NNCL) 8	78	6.27	1.90	.74	.91	86	6.89	1.42	.63	.81	94	7.52	1.19	.80	.50
Noun, formal (NNFO) 25	50	12.41	4.90	.81	2.07	57	14.34	5.59	.86	2.07	65	16.23	5.41	.86	1.99
Noun, principle (NNPR) 10	49	4.89	1.98	.47	1.36	59	5.89	2.43	.71	1.23	56	5.55	2.54	.71	1.30
Noun, taxonomic relations (NNTAX) 8	50	4.02	1.56	.45	1.08	63	5.00	2.14	.75	1.01	68	5.41	2.05	.70	1.05
Noun, problem solving (NNPS) 5	45	2.27	1.23	.40	.85	48	2.39	1.24	.43	.84	68	3.41	1.15	.40	.79
Some-all (SA) 4	NA					NA					NA				
Seriation (Ser) 5	NA					NA					NA				
Transitivity of length (TL) 5	NA					NA					NA				
Transitivity of weight (TW) 5	NA					NA					NA				
Dichotomous sorting (DS) 3	NA					NA					NA				
Cardinality (Car) 5	NA					NA					NA				
Conservation of number (CN) 7	NA					NA					NA				
Identity conservation of length (ICL) 6	NA					NA					NA				
Equivalence conservation of length (ECL) 6	NA					NA					NA				
Identity conservation of weight (ICW) 6	NA					NA					NA				

Table A-6-1 continued

Test Name, Abbreviation, and Number of Items	Grade 7					Grade 8					Grade 9				
	%	Mean	S.D.	Hoyt	S.E.M.	%	Mean	S.D.	Hoyt	S.E.M.	%	Mean	S.D.	Hoyt	S.E.M.
Equivalence conservation of weight (ECW) 6	NA					NA					NA				
Class inclusion (CI) 5	90	4.50	.95	.65	.51	92	4.61	.75	.47	.49	95	4.75	.58	.35	.41
Combinatorial reasoning (CR) 28	90	25.09	5.30	NC		92	25.68	5.13	NC		99	27.80	1.07	NC	
Addition of primary classes (APC) 4	61	2.45	1.52	.80	.59	68	2.70	1.55	.85	.52	78	3.11	1.33	.83	.47
Subtraction of primary classes (SPC) 4	98	3.91	.36	.42	.24	99	3.95	.30	.67	.15	100	4.00	.00	—	—
Addition of secondary classes (ASC) 4	65	2.59	1.57	.84	.54	68	2.73	1.58	.86	.51	76	3.05	1.51	.91	.39
Subtraction of secondary classes (SSC) 4	100	4.00	.00	—	—	98	3.93	.33	.53	.20	100	4.00	.00	—	—
Bi-univocal multiplication of classes (BMC) 4	49	1.95	1.51	.76	.64	63	2.50	1.64	.86	.53	63	2.50	1.45	.75	.63
Bi-univocal division of classes (BDC) 4	77	3.07	1.35	.83	.49	80	3.20	1.27	.80	.50	85	3.39	1.17	.83	.41
Co-univocal multiplication of classes (CMC) 4	76	3.05	1.46	.88	.44	83	3.30	1.19	.80	.46	90	3.61	.99	.85	.33
Co-univocal division of classes (CDC) 4	95	3.80	.70	.80	.27	96	3.82	.62	.73	.28	98	3.93	.25	.00	.23
Addition of difference relations (ADR) 4	93	3.73	.69	.67	.35	97	3.89	.44	.59	.25	94	3.77	.60	.55	.35
Reciprocity of difference relations (RDR) 4	96	3.84	.48	.44	.31	99	3.95	.21	.00	.18	98	3.93	.33	.53	.20
Addition of equivalence relations (AER) 4	98	3.93	.33	.52	.20	99	3.98	.15	.00	.13	99	3.95	.30	.67	.15
Reciprocity of equivalence relations (RER) 4	99	3.98	.15	.00	.13	97	3.89	.44	.57	.25	99	3.98	.15	.00	.13
Bi-univocal multiplication of relations (BMR) 4	84	3.34	1.16	.78	.47	94	3.75	.65	.59	.36	94	3.75	.69	.67	.34
Bi-univocal reciprocity of relations (BRR) 4	91	3.64	.84	.71	.39	96	3.84	.53	.61	.28	96	3.84	.57	.70	.27
Co-univocal multiplication of relations (CMR) 4	92	3.68	.60	.28	.44	94	3.77	.57	.42	.37	88	3.50	1.00	.77	.41
Co-univocal reciprocity of relations (CRR) 4	92	3.68	.71	.57	.40	95	3.80	.59	.62	.32	95	3.80	.70	.81	.26

NC = Not computable

NA = Not administered

Table A-6-2. Statistically Significant Negative *rs* Between Domains Based on Annual Measurements and Longitudinal Measurements

Grades of Annual Measurement	*CLD and Piagetian*	*CLD and Groupement*	*Piagetian and Groupement*	*Grades of Longitudinal Measurement*	*CLD and Piagetian*	*CLD and Groupement*	*Piagetian and Groupement*
1		TRfo-ASC TRfo-BMR TRps-APC ETfo-CMC ETtax-CRR	Car-APC CN-APC CN-ASC	1-2	ETfo 1-TW 2	CTfo 1-BMC 2 CTfo 2-ASC 1 CTtax 2-APC 1 TRcl 1-APC 2 TRcl 1-ASC 2 TRcl 1-BMC 2 TRps 1-APC 2 NNco 2-APC 1 NNco 2-ASC 1 NNID 2-APC 1	Ser 2-APC 1 Car 2-ASC 1 Car 2-BMR 1 CN 2-APC 1 CN 2-ASC 1 ICL 2-APC 1 ECL 2-APC 1
2		ETcl-APC		2-3	CTfo 3-TL 2 CTpr 2-TW 3 CTpr 2-ICL 3 TRtax 3-ICW 2 TRps 3-TL 2 NNcl 2-Car 3	CTtax 3-BRR 2 TRtax 3-ADR 2 TRtax 3-BRR 2	
3	CTfo-Ser TRtax-Ser TRtax-Car TRtax-ICW TRtax-ECW TRps-CN	TRtax-CRR ETfo-CDC		1-3	TRps 3-ICW 1	CTfo 3-CRR 1 CTpr 3-CRR 1 TRtax 3-APC 1 TRtax 3-ASC 1 TRps 1-RDR 3 ETcl 1-ADR 3 ETfo 1-BDC 3 NNco 3-APC 1 NNid 3-APC 1 NNid 3-ASC 1	CN 1-BMR 3 ECL 3-CMC 1 ICW 3-CMC 1

Table A-6-2 continued

Grades of Annual Measurement	*CLD and Piagetian*	*CLD and Groupement*	*Piagetian and Groupement*	*Grades of Longitudinal Measurement*	*CLD and Piagetian*	*CLD and Groupement*	*Piagetian and Groupement*
4	NNfo-TW NNps-ICW NNps-ECW	TRps-CRR		4–5	ETps 5-ECW 4 NNpr 5-TW 4 NNps 5-TW 4	NNfo 4-RDR 5 NNps 5-RDR 4	
5		TRps-BMR		5–6		ETps 6-APC 5	
6						NNps 5-ADR 6	
7		TRps-ADR		4–6	CTpr 4-DS 6 TRps 4-TW 6 TRps 4-CR 6 NNpr 6-ICW 4 NNpr 6-CR 4	TRps 4-BRR 6 NNfo 4-ADR 6 NNpr 4-BMR 6	

Table A-6-3. Mean, Standard Deviation, Hoyt Reliability Coefficient, and Standard Error of Measurement for Each of 12 Tests, Combined Score, Grades 1-9

Test Name and Number of Items	*Grade 1*					*Grade 2*					*Grade 3*				
	%	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*
Cutting tool (65)	80	51.89	4.68	.68	2.64	75	48.65	5.23	.72	2.74	81	52.65	5.11	.74	2.60
Tree (97	43	41.68	6.21	.77	2.95	51	49.08	6.47	.76	3.12	57	55.49	9.11	.82	3.87
Equilateral triangle (62)	44	27.08	3.95	.63	2.38	49	30.38	3.40	.52	2.33	67	41.38	4.55	.60	2.84
Noun (72)	17	12.51	3.19	.73	1.66	30	21.30	2.92	.71	1.56	51	36.89	8.43	.87	3.06
Conservation (24)	70	16.78	7.26	.95	1.62	81	19.38	5.68	.93	1.51	88	21.19	4.56	.92	1.26
Classification (8)	65	5.22	1.95	NC*		79	6.35	1.40	NC		88	7.05	1.31	NC	
Relations (15)	76	11.46	2.74	.80	1.19	82	12.35	2.74	.83	1.09	89	13.41	2.05	.77	.95
Number (12)	74	8.89	2.94	.84	1.11	89	10.68	1.70	.75	.81	91	10.92	1.61	.72	.81
Combinatorial reasoning (28)	58	16.35	8.52	NC		70	19.57	8.19	NC		80	22.32	7.19	NC	
Composition of classes (16)	31	4.97	4.15	.85	1.55	35	5.65	3.60	.78	1.64	49	7.86	4.07	.83	1.62
Inversion of classes (16)	78	12.41	2.63	.72	1.34	85	13.60	2.37	.75	1.16	89	14.24	2.17	.77	1.00
Composition and inversion of relations (32)	85	27.14	4.18	.80	1.85	89	28.54	3.11	.72	1.63	92	29.41	2.93	.78	1.37

Test Name and Number of Items	*Grade 4*					*Grade 5*					*Grade 6*				
	%	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*
Cutting tool (65)	82	53.29	6.96	.85	2.66	83	53.82	5.91	.81	2.53	88	57.38	6.10	.88	2.13
Tree (97)	62	60.09	8.38	.81	3.67	68	65.71	8.73	.82	3.68	68	66.18	13.76	.93	3.72
Equilateral triangle (62)	63	39.11	5.79	.72	3.06	70	43.42	6.03	.75	2.97	84	52.09	4.94	.75	2.43
Noun (72)	49	35.11	6.36	.75	3.18	59	42.42	6.61	.75	3.30	62	44.64	7.76	.82	3.26
Conservation (24)	85	20.44	5.19	.93	1.36	92	22.11	3.67	.91	1.08	95	22.87	2.83	.90	.87

Table A-6-3 continued

Classification (8)	85	6.82	1.27	NC		90	7.22	1.06	NC		90	7.22	1.17	NC	
Relations (15)	93	14.00	.93	.31	.75	91	13.71	1.56	.68	.85	93	13.89	1.50	.65	.85
Number (12)	89	10.69	1.06	.17	.93	94	11.24	.83	.05	.73	93	11.11	.93	.26	.77
Combinatorial reasoning (28)	81	22.69	6.83	NC		86	24.20	5.60	NC		91	25.36	4.45	NC	
Composition of classes (16)	44	6.98	4.69	.88	1.55	49	7.80	4.95	.91	1.42	67	10.67	4.69	.91	1.36
Inversion of classes (16)	89	14.31	2.13	.75	1.03	92	14.69	1.74	.70	.92	92	14.71	1.74	.71	.91
Composition and inversion of relations (32)	90	28.91	2.78	.68	1.55	95	30.53	2.04	.69	1.12	95	30.31	2.48	.78	1.15

Test Name and Number of Items	*Grade 7*					*Grade 8*					*Grade 9*				
	%	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*	*%*	*Mean*	*S.D.*	*Hoyt*	*S.E.M.*
Cutting tool (65)	81	52.52	8.91	.92	2.56	85	55.34	7.42	.89	2.46	91	59.07	6.43	.92	1.86
Tree (97)	71	68.93	13.03	.93	3.42	74	72.16	11.01	.90	3.43	86	83.75	9.53	.90	2.94
Equilateral triangle (62)	75	46.73	6.84	.83	2.80	80	49.30	5.43	.76	2.66	87	53.73	5.82	.86	2.19
Noun (72)	64	45.80	8.62	.86	3.21	70	50.50	11.00	.92	3.08	75	54.11	10.18	.91	2.99
Combinatorial reasoning (28)	90	25.09	5.30	NC		92	25.68	5.13	NC		99	27.80	1.07	NC	
Composition of classes (16)	63	10.05	4.88	.91	1.43	70	11.23	5.22	.94	1.25	77	12.27	4.38	.92	1.21
Inversion of classes (16)	92	14.77	1.57	.64	.91	93	14.91	1.54	.64	.89	96	15.32	1.20	.64	.70
Composition and inversion of relations (32)	93	29.82	3.02	.81	1.29	96	30.86	2.48	.85	.94	95	30.52	3.32	.91	1.00

NC = Not computable.

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